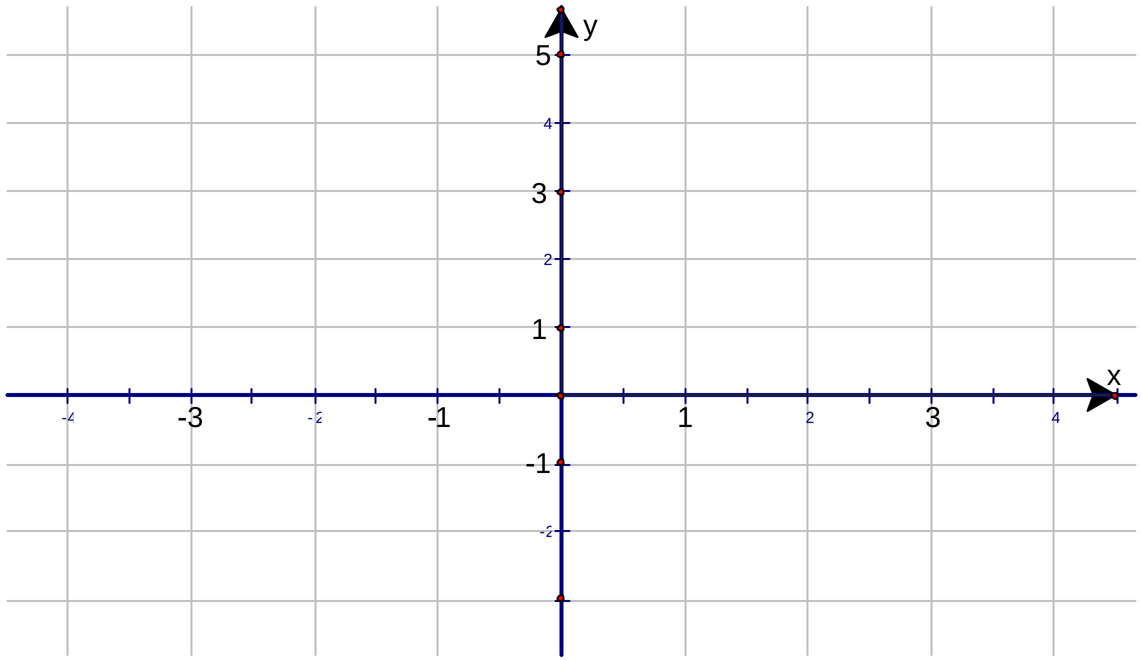
VERTICAL SHIFTING (TRANSLATION)

Example #1

Use the graph of
$f(x) = x^2$
to obtain the graphs of
$g(x) = x^2 + 1$
and
$h(x) = x^2 - 2$.



x	$f(x) = x^2$	$g(x) = x^2 + 1$	$h(x) = x^2 - 2$
-2			
-1			
0			
1			
2			

VERTICAL SHIFTING : A vertical shifting does not change the shape of the graph but simply translates it to another position in the plane.

Equation	**How to obtain the graph**	**Example**
$y = f(x) + k$ $k > 0$	Shift graph of $y = f(x)$ upward k units.	$g(x) = x^2 + 1$
$y = f(x) - k$ $k > 0$	Shift graph of $y = f(x)$ downward k units.	$h(x) = x^2 - 2$

HORIZONTAL SHIFTING (TRANSLATION)

<u>Example #2</u>

Use the graph of

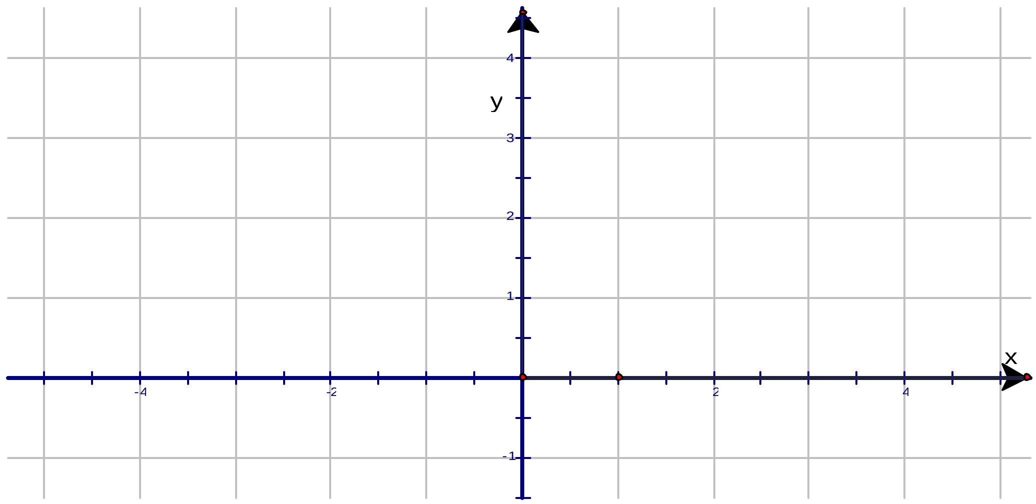
$f(x) = x^2$

to obtain the graphs of

$g(x) = (x-1)^2$

and

$h(x) = (x+1)^2$.



x	$f(x) = x^2$	$g(x) = (x-1)^2$	$h(x) = (x+1)^2$
-2			
-1			
0			
1			
2			

HORIZONTAL SHIFTING : A horizontal shifting doesn't change the shape of the graph but simply translates it to another position in the plane.

Equation	**How to obtain the graph**	**Example**
$y = f(x-h)$ $h > 0$	Shift graph of $y = f(x)$ to the right h units.	$g(x) = (x-1)^2$
$y = f(x+h)$ $h > 0$	Shift graph of $y = f(x)$ to the left h units.	$h(x) = (x+1)^2$

VERTICAL STRETCHING AND SHRINKING

Example #3 .

Use the graph of

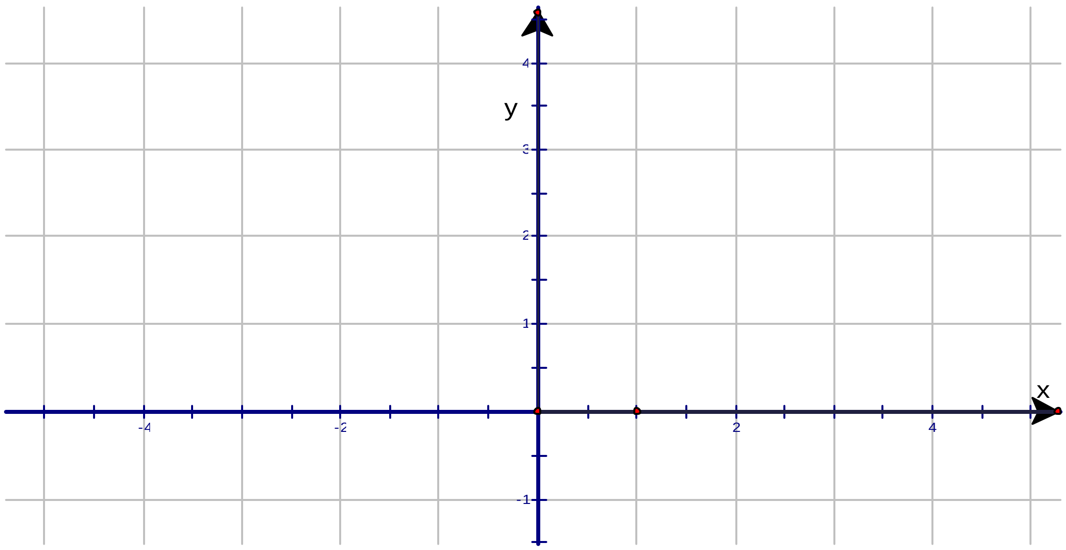
$f(x) = |x|$

to obtain the graphs of

$g(x) = 2|x|$

and

$h(x) = \frac{1}{2}|x|$



x	$f(x) = \|x\|$	$g(x) = 2\|x\|$	$h(x) = \frac{1}{2}\|x\|$
-2			
-1			
0			
1			
2			

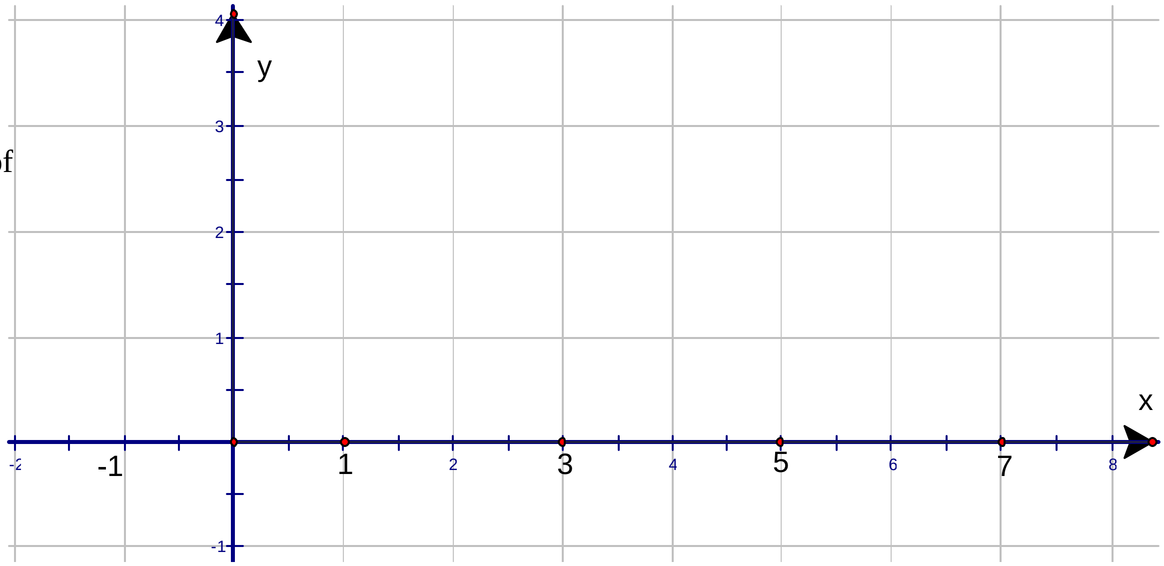
VERTICAL STRETCHING AND SHRINKING

Equation	**How to obtain the graph**	**Example**
$y = af(x)$ $a > 1$	Stretch the graph of $y = f(x)$ vertically by a factor of a.	$g(x) = 2\|x\|$
$y = af(x)$ $0 < a < 1$	Compress the graph of $y = f(x)$ vertically by a factor of $\frac{1}{a}$.	$h(x) = \frac{1}{2}\|x\|$

HORIZONTAL STRETCHING AND SHRINKING

Example #4 .

Use the graph of
$f(x)=\sqrt{x}$
to obtain the graphs of
$g(x)=\sqrt{2x}$
and
$h(x)=\sqrt{\frac{1}{2}x}$.



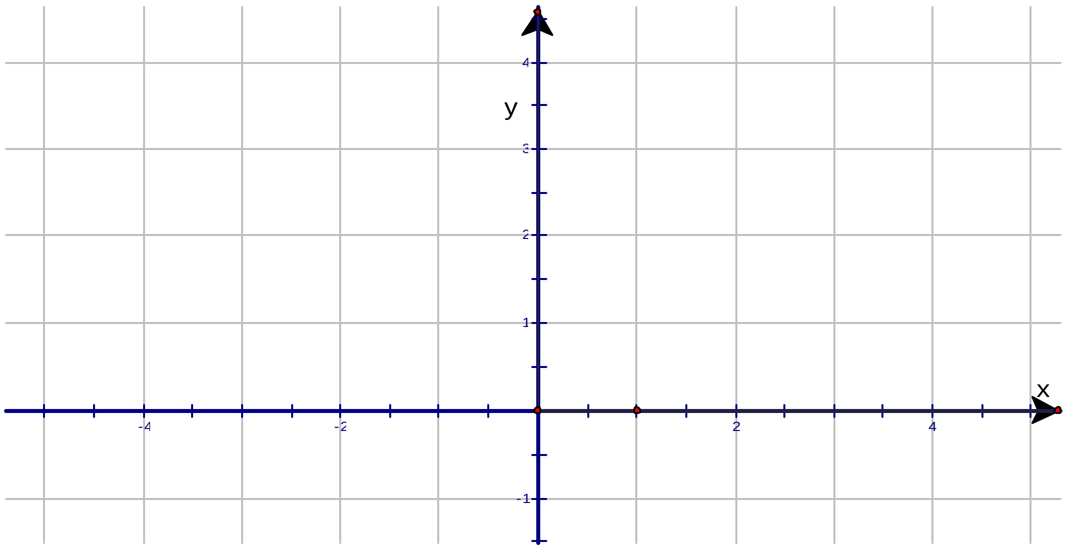
x	$f(x)=\sqrt{x}$	$g(x)=\sqrt{2x}$	$h(x)=\sqrt{\frac{1}{2}x}$
0			
1			
4			
9			

Equation	**How to obtain the graph**	**Example**
$y=f(ax)$ $a>1$	Compress the graph of $y=f(x)$ horizontally by a factor of a.	$g(x)=\sqrt{2x}$
$y=f(ax)$ $0<a<1$	Stretch the graph of $y=f(x)$ horizontally by a factor of $\frac{1}{a}$.	$h(x)=\sqrt{\frac{1}{2}x}$

REFLECTION ABOUT THE AXES

Example #5

Use the graph of
$f(x)=\sqrt{x}$
to obtain the graphs of
$g(x)=-\sqrt{x}$
and
$h(x)=\sqrt{-x}$



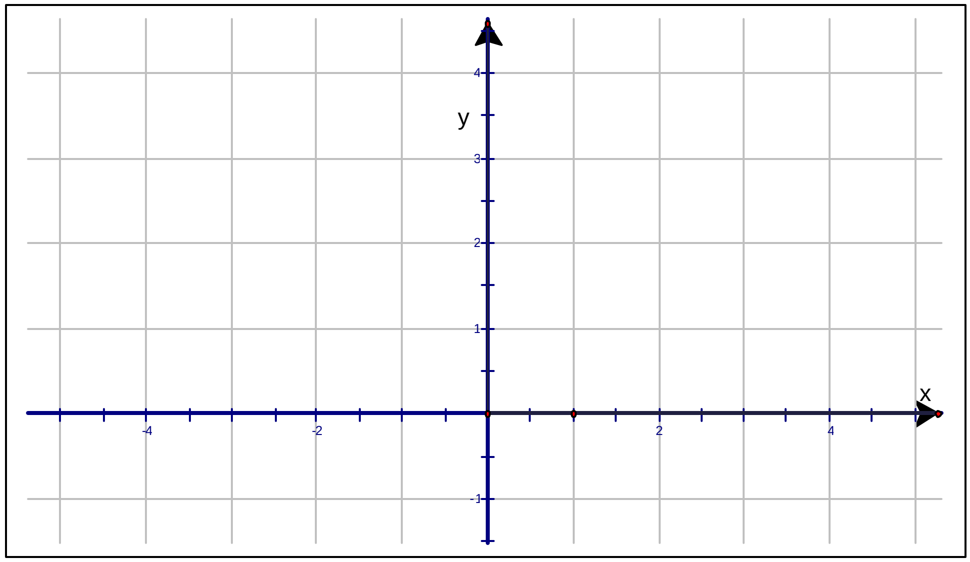
x	$f(x)=\sqrt{x}$	$g(x)=-\sqrt{x}$	$h(x)=\sqrt{-x}$
-4			
-1			
0			
1			
4			

REFLECTION ABOUT THE AXES

Equation	**How to obtain the graph**	**Example**
$y=-f(x)$	Reflect the graph of $y=f(x)$ about the x-axis.	$g(x)=-\sqrt{x}$
$y=f(-x)$	Reflect the graph of $y=f(x)$ about the y-axis.	$h(x)=\sqrt{-x}$

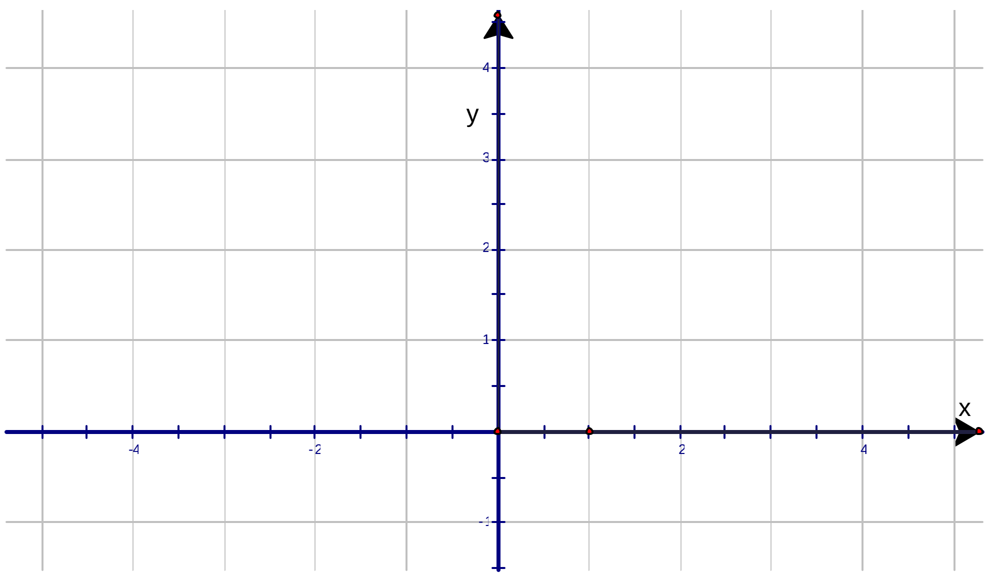
Exercise #1 .

Graph $f(x) = \sin x + 2$ over one period.



Exercise #2 .

Graph $g(x) = \cos\left(x - \frac{p}{2}\right)$ over one period.



Exercise #3 Find the function that is finally graphed after the following transformations are applied to the graph of a) $f(x) = \sqrt{x}$; b) $g(x) = x^3$.

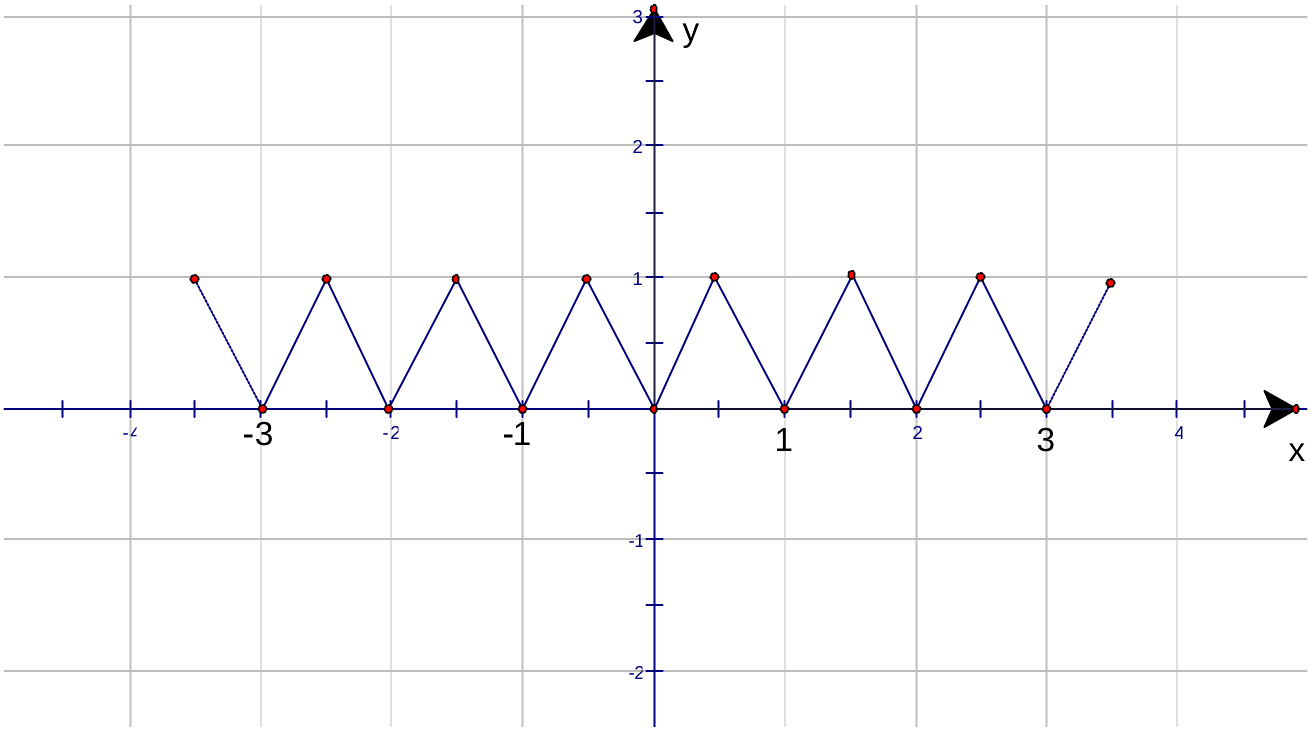
1) Shift left 3 units

2) Shift up 1 unit.

Exercise #4 The graph of $y = f(x)$ is shown. Sketch the graph of each function:

a) $y = f(x+1)$ b) $y = f(x) - 2$. c) $y = -f(x)$

d) $y = f(-x)$



Exercise #5 Suppose the point (8,12) is on the graph of $y = f(x)$. Find a point on the graph of each function.

a) $y = f(x+4)$

c) $y = \frac{1}{4} f(x)$

b) $y = f(x) + 4$

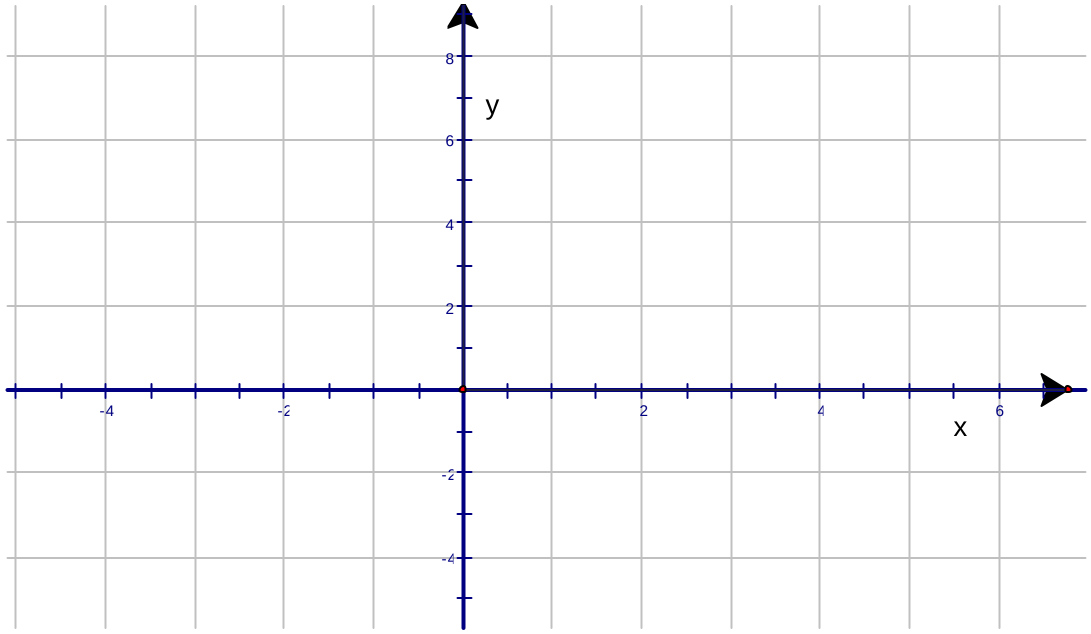
d) $y = 4f(x)$

Exercise #6 Graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages.

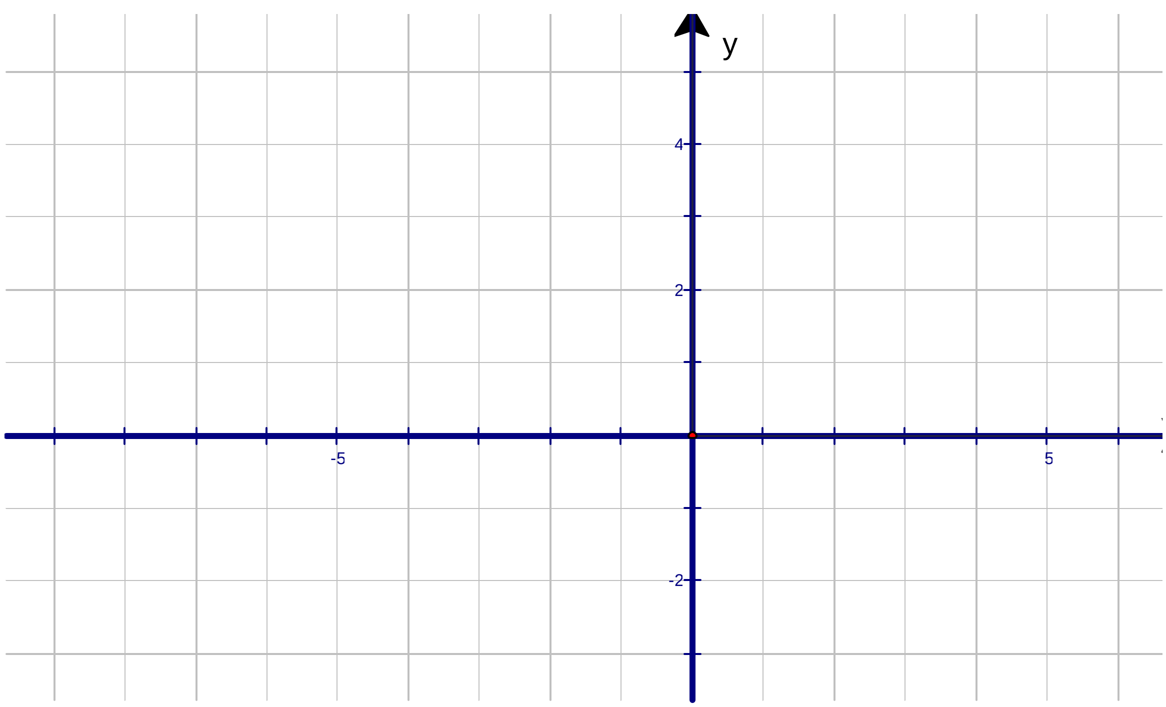
a) $f(x) = 2(x-2)^2 - 4$

Find the exact intercepts.

Find domain and range.



b) $g(x) = -|x+3| + 2$



c) $h(x)=\sqrt{-x-3}+2$

Find the exact intercepts.

Find domain and range.