Section 2.3
Linear Functions. Equations of Lines. Curve Fitting

In class work: Complete all statements. Solve all exercises.

Linear Equation in Two Variables

Standard form: $ax + by = c$

Slope –Intercept form: $y = mx + b$, where m is the slope of the line, b is the y-intercept

Slope –Point form: $y - y_1 = m\left(x - x_1\right)$, where m is the slope of the line and $\left(x_1, y_1\right)$ a point on the line.

Vertical Line: $x = k$, where k is a constant

Horizontal Line: $y = k$, where k is a constant.

Slope of a Line

$$m = \frac{change\ in\ y}{change in\ x}$$ as we move from one point to another on the line.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$$

The slope m is the rate of change of y with respect to x.

Properties of Lines

Two distinct lines are parallel if and only if they have the same slope.

$$l_1 \,||\, l_2 \Leftrightarrow m_1 = m_2$$

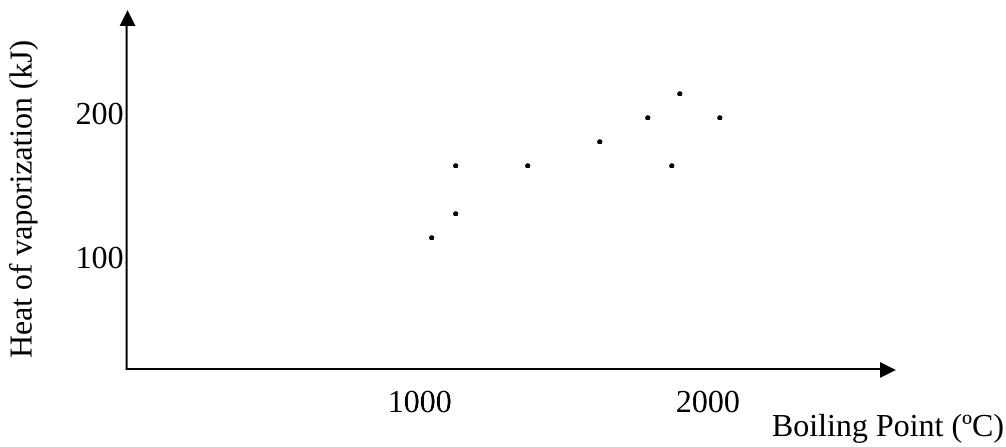
Two lines are perpendicular if and only if the product of their slopes is -1.

$$l_1 \perp l_2 \Leftrightarrow m_1 \cdot m_2 = -1$$

Lines of best fit. Linear regression
Linear Interpolation and Extrapolation

An equation that relates two variables can be used to find values of one variable from the value of the other. We will consider methods for fitting a linear equation to a collection of data points.

For example, the figure below is called a **scatterplot.** Each point on a scatterplot exhibits a pair of measurements about a single event. The points on a scatterplot may or may not show some sort of a pattern. In our example, although the points do not lie on a straight line, they seem to be clustered around some imaginary line.



Linear regression:

If the data in a scatterplot are roughly linear, we can estimate the location of an imaginary "line of best fit" that passes as close as possible to the data points. We can use this line to make predictions about the data (when drawing the line that "fits" the data points as best as we can, we try to end up with roughly equal numbers of data points above and below our line). The process of predicting a value of y based on a straight line that fits the date is called **linear regression**, and the line itself is called **the regression line.** The equation of the regression line is usually used (instead of the graph) to predict values.

Linear Interpolation:

The process of estimating between known data points is called interpolation.

Linear Extrapolation:

The process of making predictions beyond the range of known data is called extrapolation.

1. (#1) Sketch the line through A and B and find its slope: $A(-3,2), B(5,-4)$.

2. (#8) Show that the following points are vertices of a trapezoid.

$$A(2,3), B(5,-1), C(0,-6), D(-6,2)$$

3. (#10) Show that the following points form a right triangle.

$$A(1,4), B(6,-4), C(-15,-6).$$

4. (#15)Sketch the graph of the line through $P(3,1)$ for each value of m:

$$\text{a) } m=\frac{1}{2}\text{; b) } m=-1\text{; c) } m=-\frac{1}{5}.$$

5. Write an equation for the line described (# 23, 28, 29,32):

a) Through $(5,-3)$ and slope $m=-4$.

b) Through $(-1,6)$ and x –intercept 5.

c) Through $(2,-4)$, parallel to $5x-2y=4$.

d) Through $(4,5)$, perpendicular to $3x+2y=7$.

6. (#37) Write a general form of an equation for the perpendicular bisector of the segment AB, where $A(3,-1), B(-2,6)$.

7. (#50) Find an equation of the line that is tangent to the circle $x^2+y^2=25$ at the point $P(3,4)$.

8. (#54) Newborn blue whales are approximately 24 feet long and weigh 3 tons. Young whales are nursed for 7 months, and by the time of weaning they often are 53 feet long and weigh 23 tons. Let *L* and *W* denote the length (in feet) and the weight (in tons), respectively, of a whale that is *t* months of age.

a) If *L* and *t* are linearly related, express *L* in terms of *t*.
b) What is the daily increase in the length of a young whale? (use 1 month = 30 days.)
c) If *W* and *t* are linearly related, express *W* in terms of t.
d) What is the daily increase in the weight of a young whale?

9. (#56) A cheese manufacturer produces 18,000 pounds of cheese from January 1 through March 24. Suppose that this rate of production continues for the remainder of the year.

a) Express the number *y* of pounds of cheese produced in terms of the number *x* of the day in a 365-day year.
b) Predict, to the nearest pound, the number of pounds produced for the year.

10. (#58) A college student receives an interest free loan of $8250 from a relative. The student will repay $125 per month until the loan is paid off.

a) Express the amount P in dollars remaining to be paid in terms of time t (in months).
b) After how many months will the student owe $5000?

c) Sketch, on a rectangular coordinate system, a graph that shows the relationship between P and t for the duration of the loan.

11. (#77) The following table gives the cost (in thousands of dollars) for a 30-second television advertisement during Super Bowl for various years.

Year	**Cost**
1986	550
1996	1085
2001	2100
2005	2400

a) Plot the data.
b) Determine a line that models the data.
Graph this line together with the data on the same coordinate system.
c) Use this line to predict the cost of a 30-second commercial in 2002 and 2003. Compare your answers to the actual values of $2,200,0000 and $2,150,000, respectively.
d) Interpret the slope of this line.