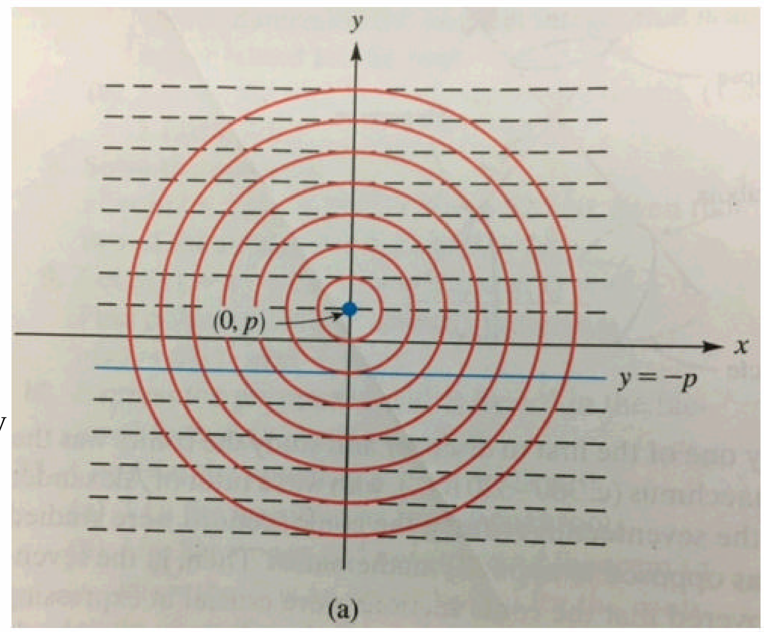
Section 10.2
The Parabola

We have seen that the graph of a quadratic function $y = ax^2 + bx + c$ is a symmetric U-shaped curve called parabola. In this section, we give a more general definition of the parabola, a definition that emphasizes the geometric properties of the curve.

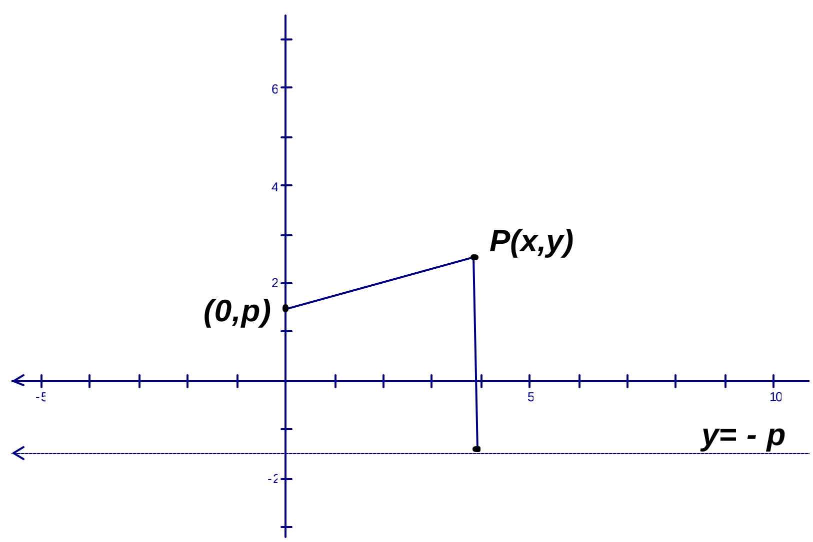
Definition A **parabola** is the set of all points in the plane equally distant from a fixed line and a fixed point not on the line. The fixed line is called the **directrix**; the fixed point is called the **focus**.

Suppose the focus of the parabola is the point $(0, p)$ and the directrix is the line $y = -p$. We will assume that p is positive. To understand the geometric context of the definition of the parabola, the special graph paper displayed below is useful. The common center of the concentric circles is the focus $(0, p)$. Thus, all the points on a given circle are at a fixed distance from the focus. The radii of the circles increase in increments of p units. Similarly, the broken horizontal lines in the figure are drawn at intervals that are multiples of p units from the directrix $y = -p$. By considering the points where the circles intersect the horizontal lines, we can find a number of points equally distant from the focus $(0, p)$ and the directrix $y = -p$. The graphical method just described lets us locate many Points equally distant from the focus $(0, p)$ and the directrix $y = -p$.



To describe the required set of points comopletely, and to show that our new definition is consistent with the old one, we need to bring algebraic methods to bear on the problem.

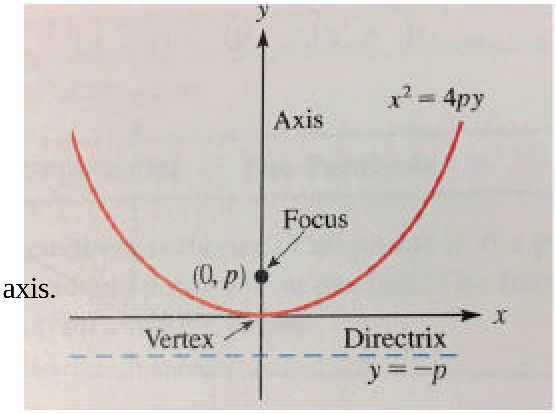
$$\sqrt{(x-0)^2 + (y-p)^2} = y - (-p)$$



This equation is the parabola with focus $(0, p)$ and directrix $y = -p$.

Let's summarize the properties of the parabola $x^2 = 4py$. The terminology introduced applies equally well to an arbitrary parabola for which the axis of symmetry is not necessarily parallel to one of the coordinate axes and the vertex is not necessarily the origin.

<u>Property Summary The Parabola</u>



1.The parabola is the set of all points in the plane equally distant from a fixed line called the directrix and a fixed point, not on the line, called the focus.

2. The axis of a parabola is the line drawn through the focus and perpendicular to the directrix.

3. The vertex of a parabola is the point where the parabola intersects its axis. The vertex is located halfway between the focus and the directrix.

Definitions A **chord** of a parabola is a straight line segment joining any two points on the curve. If the curve passes through the focus, it is called a **focal chord**.

For purposes of graphing, it is useful to know the length of the focal chord perpendicular to the axis of the parabola. We will call its length the **focal width.**

Find the focal width of the parabola $x^2 = 4py$

Example1 Analyzing a parabola

a) Determine the focus and the directrix of the parabola $x^2 = 16y$.

b) Graph the parabola.

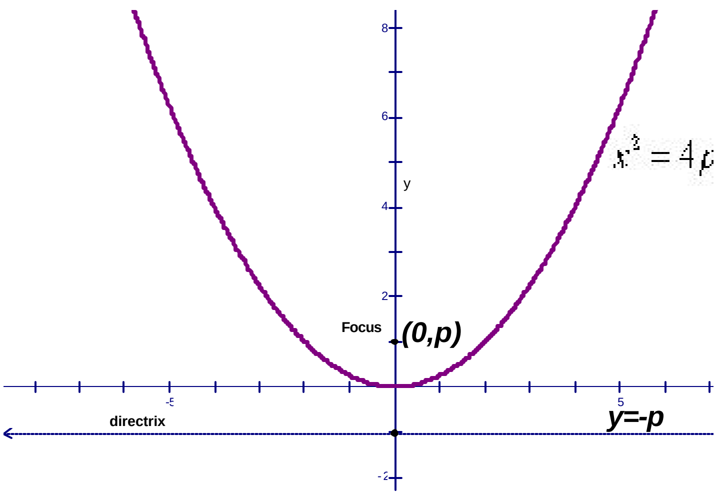
Example 2 Finding the equation of a parabola

Determine the equation of a parabola with vertex at $(0,0)$ passing through $(3,5)$. Specify the focus and directrix.

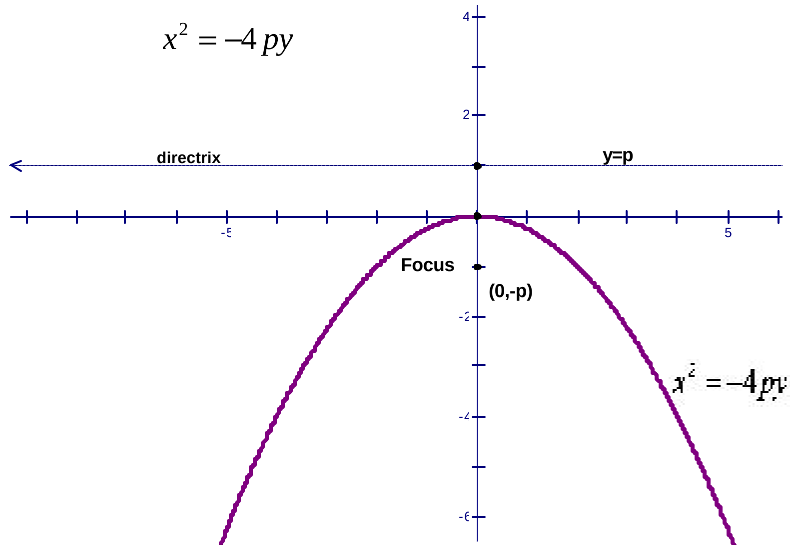
We have seen that the equation of a parabola with focus $(0, p)$ and directrix $y = -p$ is $x^2 = 4py$. By following the same method, we can obtain general equations for parabolas with other orientations. The basic results are summarized below.

<u>Property Summary Basic Equation for the Parabola</u>

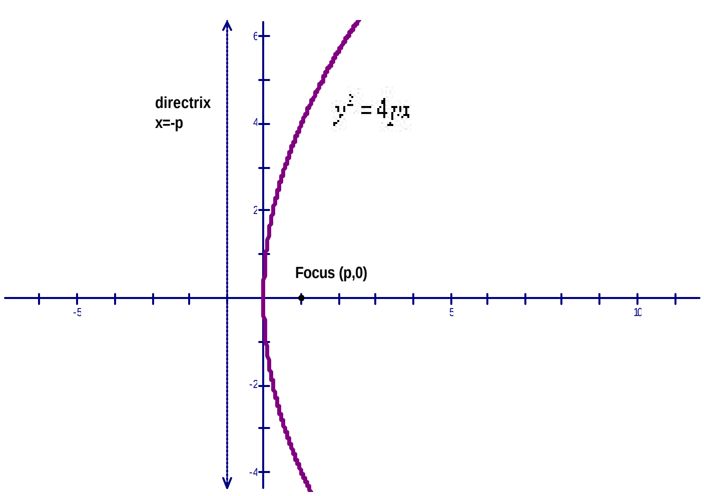
$x^2 = 4py$



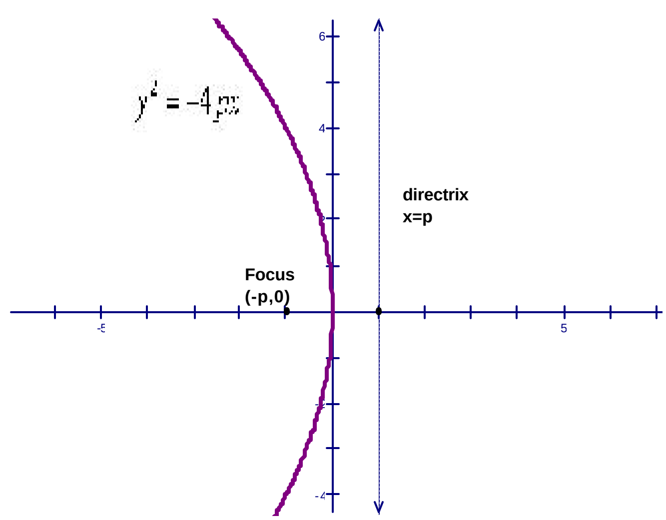
$x^2 = -4py$



$y^2 = 4px$



$y^2 = -4px$



Example3 Determining the focus and directrix of a parabola.

Find the focus and the directrix of the parabola $y^2 = -4x$, and sketch the graph.

Example 4 A parabola with a vertex other than the origin

Graph the parabola $(y+1)^2 = -4(x-2)$. Specify the vertex, the focus, the directrix, the axis of symmetry.

Solution

The given equation is obtained from ___________________ by replacing x and y with _________

and __________, respectively.

Therefore, the required graph is obtained by translating the parabola in Example 3 to

_______________________and ______________________.

The vertex moves from ____________ to ______________

The focus moves from ____________ to ______________

The directrix moves from ____________ to ______________

The axis of symmetry moves from ____________ to ______________

There are numerous applications of the parabola in sciences. Many of these involve parabolic reflectors. Light rays coming in parallel to the axis of the parabola are reflected through the focus. The word “focus” comes from a Latin word meaning “fireplace”. In addition to telescopes and radio telescopes, parabolic reflectors are used in communication systems such as satellite dishes for television, in surveillance systems, and in automobile headlights.

Geometrically, parabolic reflectors are described and designed as follows. We begin with a portion of a parabola and its axis of symmetry. By rotating the parabola about its axis, we obtain the bowl-shaped surface known as a **paraboloid of revolution**. This is the surface used in a parabolic reflector.