REVIEW
Solving Equations

Definition The standard form of a quadratic or second degree equation in one variable is $ax^2+bx+c=0$ where $a,b,c\in\mathbb{R}; a\neq 0$.

Solving quadratic equations

(1) THE FACTORING METHOD – used to solve equations of the form $ax^2+bx+c=0$ that are factorable (see factoring methods on page 2)

Zero-Factor Property: The product of two factors equals zero if and only if one of the factors (or both) is zero.

$$AB=0 \Leftrightarrow A=0 \ \ or \ \ B=0$$

(2) EXTRACTION OF ROOTS – used to solve equations of the form

$x^2=k$ or $(x-p)^2=k$.

$\sqrt{x^2}=\sqrt{k}$ $\sqrt{(x-p)^2}=\sqrt{k}$

$x=\pm\sqrt{k}$ $x-p=\pm\sqrt{k}$

$x=p\pm\sqrt{k}$

(3) QUADRATIC FORMULA If $ax^2+bx+c=0$, then the solutions are given by:

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Definition The discriminant of a quadratic equation is $\Delta=b^2-4ac$

Properties If $a,b,c\in\mathbb{R}$, then:

If $\Delta>0$, the equation has two distinct real solutions.

If $\Delta=0$, the equation has one real (rational) solution.

If $\Delta<0$, the equation has two complex (nonreal) solutions.

Factoring a polynomial

(for more about factoring, see Factoring Polynomials Handout on www.timetodare.com)

1. GCF Factor out the greatest common factor (if any).

2. Special products

Two terms

$$a^2-b^2=(a-b)(a+b)$$
$$a^3-b^3=(a-b)(a^2+ab+b^2)$$
$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

Three terms

$$a^2+2ab+b^2=(a+b)^2$$
$$a^2-2ab+b^2=(a-b)^2$$

3. Factoring techniques to factor out a trinomial ax^2+bx+c

$a=1$

$x^2+bx+c=(x+\square)(x+\square)$

product = c
sum = b

$a\neq 1$

split the middle term bx

$ax^2+bx+c=ax^2+\square x+\square x+c$

product = ac
sum = b
then factor by grouping

4. If more than four term, factor by grouping.

Exercise #1 Solve (in R) by factoring (the zero-factor property):

a) $x^2+2x-8=0$

b) $5x^2-3x=2$

c) $-6x^2+7x=-10$

d) $a^4-16=0$

e) $x^2+2x=0$

f) $3t^2=6t$

g) $5z^2=5z$

h) $m^3-8=0$

i) $p=2p^3$

j) $x^3+4x^2-9x-36=0$

k) $9y^3=49y$

l) $3x^2\left(x+\frac{1}{2}\right)\left(2x-\frac{1}{3}\right)\left(5x-\frac{1}{2}\right)=0$

Exercise #2 Solve $(\text{in}\,\mathbb{C})$ by extracting roots (the square root property). Give exact answers.

a) $\frac{2x^2}{3}=4$

b) $\left(t-\frac{1}{2}\right)^2=\frac{3}{4}$

c) $1-3(x-1)^2=-26$

d) $(-2x+5)^2=8$

Exercise #3 Solve $(\text{in}\,\mathbb{C})$ by the quadratic formula. Give exact answers.

a) $x^2-\frac{x}{2}+1=0$

b) $\frac{1}{2}a^2-3=-\frac{1}{4}a$

c) $3-\frac{4}{x}-\frac{2}{x^2}=0$

Exercise #4 Solve the following equations in $\mathbb{C}$.

a) $x^3+27=0$

b) $2x^7-128x=0$

c) $2b^2-3b+1=0$

d) $4x-\sqrt{3}=2x$

e) $\frac{2c}{3}=5$

f) $\frac{1}{2y}=7$

g) $2z^2-z-1=0$

h) $2d^2+d-1=0$

i) $\frac{1}{x+1}=\frac{x-1}{2}$

j) $18p^2+7p^2-9=0$

k) $2l^2+2l-1=0$

l) $(x-5)^2(x+1)=0$

m) $\frac{x+2}{5}=\frac{2x-1}{3}$

n) $x^2+4x=-1$

o) $x^2=9$

Exercise #5 Solve each equation for the indicated variable:

a) If $x^2+y^2=1$ and $x=\frac{2}{3}$, find y.

b) $A=2w^2+4lw$, for w;

c) $a^2+b^2=c^2$, for b

d) $2x=\frac{1}{3y}$, for x, then for y.

e) $\frac{x}{a}=\frac{y}{\frac{b}{2}}$, for x

f) $x^2+y^2=r^2$, for x, then for y

Exercise #6 Solve the following equations:

a) $2x^4-7x^2+5=0$

b) $x^4-10x^2+9=0$

c) $2x-\sqrt{x}-10=0$

Answers: **#1** a) -4,2; b) -2/5, 1; c)-5/6, 2;d) $\pm2,\pm2i$; e) 0, -2; f) 0,2; g) 0,1; h) $2,-1\pm\sqrt{3}i$; i) $0,\pm\frac{\sqrt{2}}{2}$; j) -4, 3, -3; k) $0,-\frac{7}{3},\frac{7}{3}$; l) $0,-\frac{1}{2},\frac{1}{6},\frac{1}{10}$. **#2** a) $\pm\sqrt{6}$; b) $\frac{1}{2}\pm\frac{\sqrt{3}}{2}$; c) 4, -2; d) $\frac{5}{2}\pm\sqrt{2}$. #3 a) $\frac{1\pm\sqrt{15}i}{2}$; b) $\frac{-1\pm\sqrt{97}}{4}$; c) $\frac{2\pm\sqrt{10}}{3}$. **#4** a) -3, $\frac{3\pm3\sqrt{3}i}{2}$; b) 0, $\pm2,1\pm\sqrt{3}i,-1\pm\sqrt{3}i$; c) 1, ½; d) $\sqrt{3}/2$; e) 15/2; f) 1/14; g) 1, -1/2; h) ½, -1; i) $\pm\sqrt{3}$; j) $\pm3/5$; k) $\frac{-1\pm\sqrt{3}}{2}$; l) 5, -1; n) $-2\pm\sqrt{3}$; o) ±3. **#5** a)$\pm\frac{\sqrt{5}}{3}$; b)$\frac{-2l\pm\sqrt{4l^2+2A}}{2}$; c) $\pm\sqrt{c^2-a^2}$.

#6 a) $\pm1,\pm\frac{\sqrt{10}}{2}$; b) $\pm3,\pm1$; c) 25/4