

REVIEW

Solving Equations

Definition The standard form of a quadratic or second degree equation in one variable is $ax^2 + bx + c = 0$ where $a, b, c \in \mathbb{R}; a \neq 0$.

Solving quadratic equations

(1) THE FACTORING METHOD – used to solve equations of the form

$ax^2 + bx + c = 0$ that are factorable (see factoring methods on page 2)

Zero-Factor Property: The product of two factors equals zero if and only if one of the factors (or both) is zero.

$$AB = 0 \Leftrightarrow A = 0 \text{ or } B = 0$$

(2) EXTRACTION OF ROOTS – used to solve equations of the form

$$\begin{array}{ll} x^2 = k & \text{or} & (x-p)^2 = k \\ \sqrt{x^2} = \sqrt{k} & & \sqrt{(x-p)^2} = \sqrt{k} \\ x = \pm\sqrt{k} & & x-p = \pm\sqrt{k} \\ & & x = p \pm \sqrt{k} \end{array}$$

(3) QUADRATIC FORMULA If $ax^2 + bx + c = 0$, then the solutions are given by:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition The discriminant of a quadratic equation is $\Delta = b^2 - 4ac$

Properties If $a, b, c \in \mathbb{R}$, then:

- If $\Delta > 0$, the equation has two distinct real solutions.
- If $\Delta = 0$, the equation has one real (rational) solution.
- If $\Delta < 0$, the equation has two complex (nonreal) solutions.

Factoring a polynomial(for more about factoring, see Factoring Polynomials Handout on www.timetodare.com)

1. GCF Factor out the greatest common factor (if any).

2. Special products

Two terms

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Three terms

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

3. Factoring techniques to factor out a trinomial $ax^2 + bx + c$ $a = 1$

$$x^2 + bx + c = (x + \square)(x + \square)$$

product = c
sum = b $a \neq 1$ split the middle term bx

$$ax^2 + bx + c = ax^2 + \square x + \square x + c$$

product = ac
sum = b

then factor by grouping

4. If more than four term, factor by grouping.

Exercise #1 Solve (in R) by factoring (the zero-factor property):

a) $x^2 + 2x - 8 = 0$

e) $x^2 + 2x = 0$

i) $p = 2p^3$

b) $5x^2 - 3x = 2$

f) $3t^2 = 6t$

j) $x^3 + 4x^2 - 9x - 36 = 0$

c) $-6x^2 + 7x = -10$

g) $5z^2 = 5z$

k) $9y^3 = 49y$

d) $a^4 - 16 = 0$

h) $m^3 - 8 = 0$

l) $3x^2 \left(x + \frac{1}{2} \right) \left(2x - \frac{1}{3} \right) \left(5x - \frac{1}{2} \right) = 0$

Exercise #2 Solve (in C) by extracting roots (the square root property). Give exact answers.

a) $\frac{2x^2}{3} = 4$

c) $1 - 3(x - 1)^2 = -26$

b) $\left(t - \frac{1}{2} \right)^2 = \frac{3}{4}$

d) $(-2x + 5)^2 = 8$

Exercise #3 Solve (in C) by the quadratic formula. Give exact answers.

a) $x^2 - \frac{x}{2} + 1 = 0$

b) $\frac{1}{2}a^2 - 3 = -\frac{1}{4}a$

c) $3 - \frac{4}{x} - \frac{2}{x^2} = 0$

Exercise #4 Solve the following equations in \mathbb{C} .

a) $x^3 + 27 = 0$

f) $\frac{1}{2y} = 7$

k) $2l^2 + 2l - 1 = 0$

b) $2x^7 - 128x = 0$

g) $2z^2 - z - 1 = 0$

l) $(x-5)^2(x+1) = 0$

c) $2b^2 - 3b + 1 = 0$

h) $2d^2 + d - 1 = 0$

m) $\frac{x+2}{5} = \frac{2x-1}{3}$

d) $4x - \sqrt{3} = 2x$

i) $\frac{1}{x+1} = \frac{x-1}{2}$

n) $x^2 + 4x = -1$

e) $\frac{2c}{3} = 5$

j) $18p^2 + 7p^2 - 9 = 0$

o) $x^2 = 9$

Exercise #5 Solve each equation for the indicated variable:

a) If $x^2 + y^2 = 1$ and $x = \frac{2}{3}$, find y .

d) $2x = \frac{1}{3y}$, for x , then for y .

b) $A = 2w^2 + 4lw$, for w ;

e) $\frac{x}{a} = \frac{y}{\frac{b}{2}}$, for x

c) $a^2 + b^2 = c^2$, for b

f) $x^2 + y^2 = r^2$, for x , then for y

Exercise #6 Solve the following equations:

a) $2x^4 - 7x^2 + 5 = 0$

b) $x^4 - 10x^2 + 9 = 0$

c) $2x - \sqrt{x} - 10 = 0$

Answers: #1 a) -4, 2; b) -2/5, 1; c) -5/6, 2; d) $\pm 2, \pm 2i$; e) 0, -2; f) 0, 2; g) 0, 1; h) $2, -1 \pm \sqrt{3}i$; i) $0, \pm \frac{\sqrt{2}}{2}$;

j) -4, 3, -3; k) $0, -\frac{7}{3}, \frac{7}{3}$; l) $0, -\frac{1}{2}, \frac{1}{6}, \frac{1}{10}$. #2 a) $\pm\sqrt{6}$; b) $\frac{1}{2} \pm \frac{\sqrt{3}}{2}$; c) 4, -2; d) $\frac{5}{2} \pm \sqrt{2}$. #3 a) $\frac{1 \pm \sqrt{15}i}{2}$; b)

$\frac{-1 \pm \sqrt{97}}{4}$; c) $\frac{2 \pm \sqrt{10}}{3}$. #4 a) -3, $\frac{3 \pm 3\sqrt{3}i}{2}$; b) $0, \pm 2, 1 \pm \sqrt{3}i, -1 \pm \sqrt{3}i$; c) 1, 1/2; d) $\sqrt{3}/2$; e) 15/2; f)

1/14; g) 1, -1/2; h) 1/2, -1; i) $\pm\sqrt{3}$; j) $\pm 3/5$; k) $\frac{-1 \pm \sqrt{3}}{2}$; l) 5, -1; n) $-2 \pm \sqrt{3}$; o) ± 3 . #5 a) $\pm \frac{\sqrt{5}}{3}$;

b) $\frac{-2l \pm \sqrt{4l^2 + 2A}}{2}$; c) $\pm\sqrt{c^2 - a^2}$.

#6 a) $\pm 1, \pm \frac{\sqrt{10}}{2}$; b) $\pm 3, \pm 1$; c) 25/4