Review
II Functions

The word *function*, used casually, expresses the notion of dependence. For example, a person might say that election results are a function of the economy, meaning that the winner of an election is determined by how the economy is doing. Another person may claim that car sales are a function of the weather, meaning that the number of cars sold on a given day is affected by the weather.

In mathematics, the meaning of the word *function* is more precise, but the basic idea is the same.

<u>Definition 1</u> **A function** is a relationship between two quantities. If the value of one quantity uniquely determines the value of the second quantity, we say the second quantity is a function of the first.

Example 1 In the early 1980s, the recording industry introduced the compact disc (CD) as an alternative to vinyl long playing records (LPs). Table 1 gives the number of units (in millions) of CDs sold for the years 1982 through 1987. The year uniquely determines the number of CDs sold. Thus, we say that the number of CDs sold is a function of the year. We may also say that the number of CDs sold depends on the year.

TABLE 1
Millions of CDs sold, by year

Year	**Sales (millions)**
1982	0
1983	0.8
1984	5.8
1985	23
1986	53
1987	102

Note: The quantities described by a function are called *variables.*

<u>Definition 2</u> **A function** is a relationship between two variables: **independent variable (input)** and **dependent variable (output)** that assigns to each independent variable a unique value of the dependent variable.

Function Notation

There is a convenient notation we use when discussing functions. First, we choose a name for the particular function, let's say f. That is, f is the name of the relationship between the two variables. Let's say t (the year) and S (the CD's sold) from Example 1. Then we can write

$$S = f(t)$$

which means " S is a function of t, and f is the name of the function."

Domain and Range

If $y = f(x)$, then

- the **domain** of f is the set of values for the independent variable, x

$$D_f = \{x \mid f(x) \in \mathbb{R}\}$$

- the **range** of f is the set of values for the dependent variable, y

$$R_f = \{y \mid y = f(x),\ x \in D_f\}$$

Functions Defined by Tables

TABLE 2
Millions of CDs sold, by year

Cost of merchandise (M)	Shipping Charges (C)
$0.01 – 10.00	$2.50
10.01 – 20.00	3.75
20.01 – 30.00	4.85
30.01 – 50.00	5.95
50.01 – 75.00	7.95
Over 75.00	8.95

Exercise 1 a) Is C a function of M? Explain.

b) Is M a function of C? Explain.

c) If $C = f(M)$, find $f(3)$. What does it mean?

d) Solve $f(M) = 6.95$. What does it mean?

Functions Defined By Graphs

How to Tell if a Graph Represents a Function: the Vertical Line Test

In general, for y to be a function of x, each value of x must be associated with exactly one value of y. Let us think of what this requirement means graphically. In order for a graph to represent a function, each x-value must correspond to exactly one y-value. This means that the graph must not intersect any vertical line at more than one point. Otherwise, the curve would contain two points with different y-values but the same x-value.

The vertical line test:

If any vertical line intersects a graph in more than one point, then the graph does not represent a function.

Exercise 2

Use the vertical line test to identify the graphs in which y is a function of x. For those graph, identify the domain and range.

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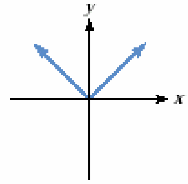

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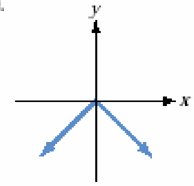

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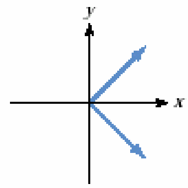

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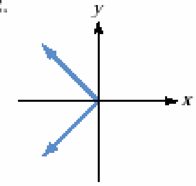

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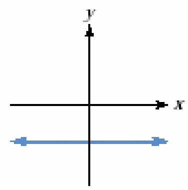

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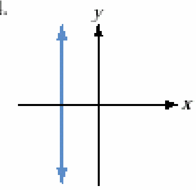

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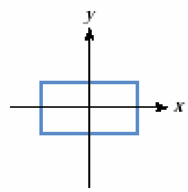

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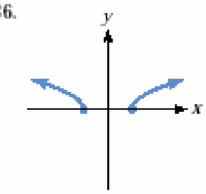
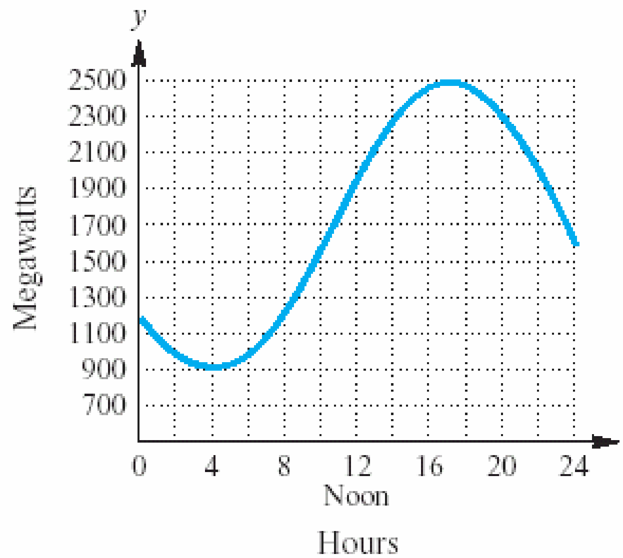


Exercise 3

The graph shows the daily megawatts of electricity used on a record-breaking summer day in Sacramento.

Electricity Use



Source: Sacramento Municipal Utility District.

a) Is this the graph of a function?

b) What is the domain? What is the range?

c) Estimate the number of megawatts used at 8am.

d) At what time was the most electricity used? the least?

d) Call this function f. What is $f(12)$? What does it mean?

f) During what time intervals is electricity usage increasing? Decreasing?

Increasing, Decreasing, and Constant Functions

Suppose that a function f is defined over an interval I. If x_1 and x_2 are in I,

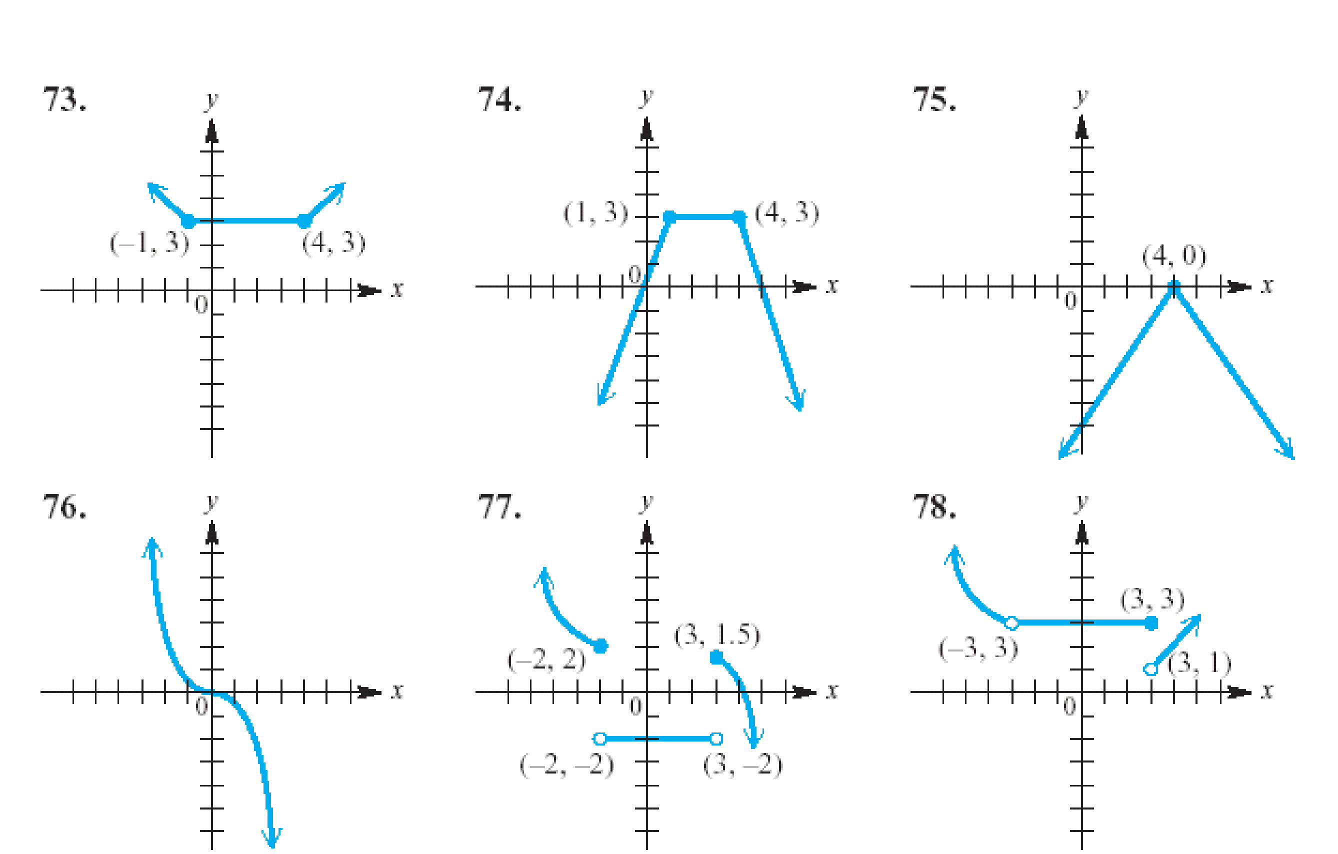
a) f **increases** on I if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$;

b) f **decreases** on I if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$;

c) f **is constant** on I if, for every x_1 and x_2, $f(x_1) = f(x_2)$.

Exercise 4

a) Find the domain and range of each function.
b) Determine the intervals of the domain for which each function is (i) increasing, (ii) decreasing, and (iii) constant.



Functions Defined by Equations

Exercise 5 Recall from geometry that if we know the radius of a circle, we can find its area. If we let $A = q(r)$ represent the area of a circle as a function of its radius, then a formula for $q(r)$ is

$$A = q(r) = p r^2 .$$

Use the above formula, where r is in cm, to evaluate $q(10)$ and $q(20)$. Explain what your results tell you about circles.

Exercise 6 Let $f(x) = 2x + 3$. Answer the following:

a) Is y a function of x? Why?

b) Find the domain and the range.

c) Find $f(0)$, $f(2)$, $f(-x)$, $f(x+1)$, $f(x-2)$

Exercise 7

$y=\sqrt{4x+2}$ $\quad$ $y=-\sqrt{x}$ $\quad$ $y=-6x+8$ $\quad$ $y=x^2-2x+5$

Answer the following:

a) Decide whether each relation defines y as a function of x.

b) Give the domain and the range.

c) Rewrite the equations (when possible) using function notation.

d) Find $f(-x)$ and $f(x+1)$ for each function.

The Inverse of a Function

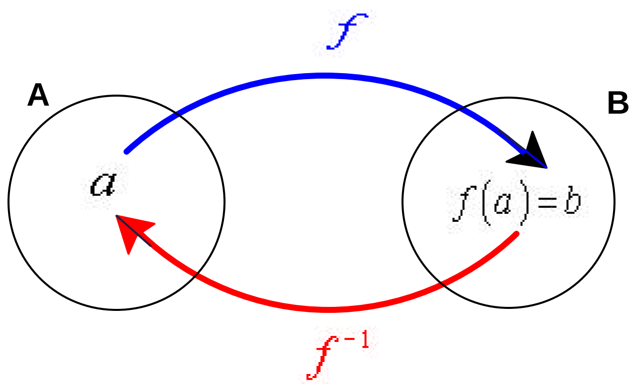
Let $y = f(x)$. Recall that y is a function of x if and only if for every input x, there is only one output y.

<u>Definition 3</u>

i) A function f with domain D is **one-to-one** if no two elements of the domain have the same image.

ii) A function is one-to-one if and only if its graph passes the **horizontal line test** – that is, no horizontal line intersects the graph more than once.

<u>Definition 4</u>



Let f be a one-to-one function. Then,

$f^{-1}(b) = a$ if and only if $f(a) = b$

Note that $f : A \to B$

$f^{-1} : B \to A$

Property

Let f be a one-to-one function $f : A \to B$.

The function f^{-1} is the inverse of f if and only if it satisfies the following properties:

i) $f\left(f^{-1}(b)\right) = b$ for any $b \in B$ and

ii) $f^{-1}\left(f(a)\right) = a$ for any $a \in A$.

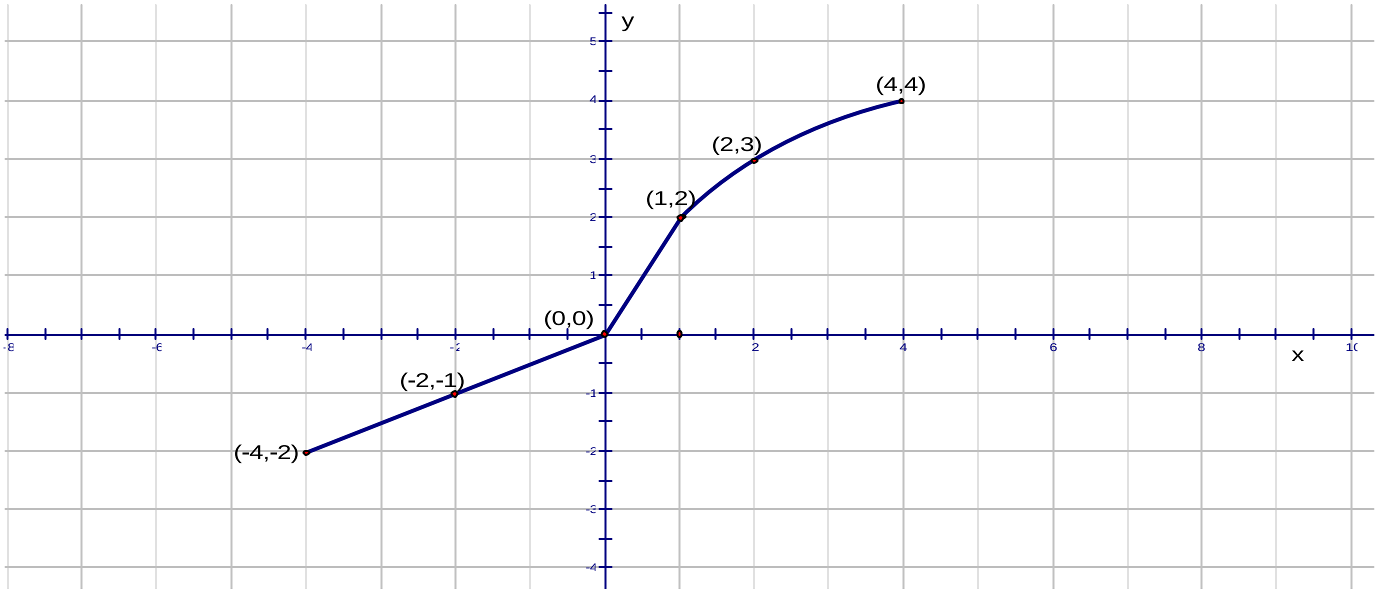
The Graphs of a Function and its Inverse

Property

i) $(x, y) \in Graph_f \leftrightarrow (y, x) \in Graph_{f^{-1}}$

ii) The graph of f^{-1} is a reflection of the graph of f about the bisector line $y = x$

Exercise 8



Using the graph $y = f(x)$ shown, answer the following.
You may use the above grid to graph. Write all the answers and show ALL your work on separate paper.

a) Is *y* a function of *x*? Explain.

b) Find the domain and range of *f*.

c) Find $f(-2)$.

d) For what values of *x* does $f(x) = 2$?

e) Solve $f(x) < 0$.

f) Does *f* have an inverse? Explain.

g) Find the domain and range of f^{-1}.

h) Graph $y = f^{-1}(x)$ showing the symmetry through the line $y = x$.

i) Find $f^{-1}(-2)$ $f^{-1}(3)$.