HANDOUT MORE PRACTICE 2.1-2.4

(1) $9x^2+9y^2+12x-18y-23=0$

(a) We'll write the equation in standard form
$$(x-h)^2+(y-k)^2=r^2$$
by completing the square on x and on y.

1st • make the leading coefficients equal to 1
$$x^2+\frac{12}{9}x+y^2-2y-\frac{23}{9}=0$$

2nd • isolate the constant
$$x^2+\frac{4}{3}x+y^2-2y=\frac{23}{9}$$

3rd • find the missing terms
$$\left(\tfrac{1}{2}\,\text{coef. } x\right)^2=\left(\tfrac{1}{2}\cdot\tfrac{4}{3}\right)^2=\left(\tfrac{2}{3}\right)^2=\frac{4}{9}$$
$$\left(\tfrac{1}{2}\,\text{coef. } y\right)^2=\left(\tfrac{1}{2}(-2)\right)^2=(-1)^2=1$$

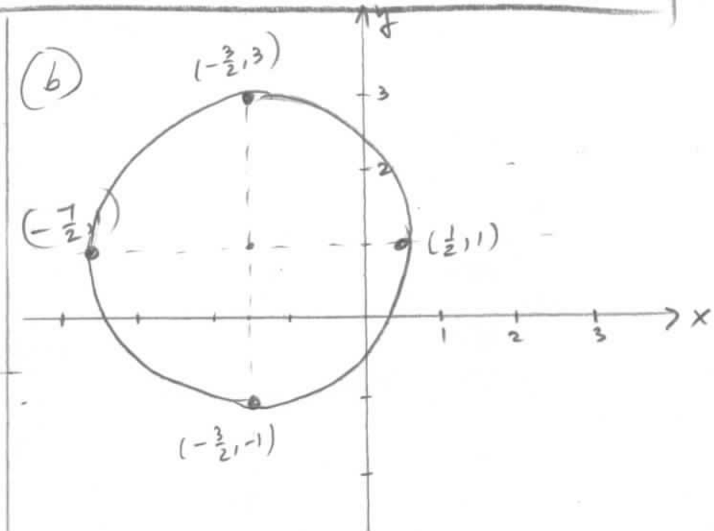
• add $\frac{4}{9}$ and 1 to both sides of the equation
$$\underbrace{x^2+\frac{4}{3}x+\frac{4}{9}}+\underbrace{y^2-2y+1}=\frac{23}{9}+\frac{4}{9}+1$$

$$\boxed{\left(x+\frac{2}{3}\right)^2+(y-1)^2=4}$$

Therefore, the circle has

center $\left(-\frac{2}{3},1\right)$
radius $r=2$

(b)



(c) x-n: let $y=0$ in equation (1)
$$9x^2+9y^2+12x-18y-23=0$$
OR in equation (2):
$$\left(x+\frac{2}{3}\right)^2+(y-1)^2=4$$

say $y=0$ in eq. (2):
$$\left(x+\frac{2}{3}\right)^2+(0-1)^2=4$$
$$\left(x+\frac{2}{3}\right)^2+1=4$$
$$\left(x+\frac{2}{3}\right)^2=3 \quad \Big/ \sqrt{\ }$$
$$\sqrt{\left(x+\frac{2}{3}\right)^2}=\sqrt{3}$$
$$x+\frac{2}{3}=\pm\sqrt{3}$$
$$x=-\frac{2}{3}\pm\sqrt{3}$$

$$\boxed{x\text{-n: } \left(-\frac{2}{3}\pm\sqrt{3},0\right)}$$

y-n: let $x=0$ in eq.(2):

$$\left(0+\frac{2}{3}\right)^2+(y-1)^2=4$$

$$\frac{4}{9}+(y-1)^2=4$$

$$(y-1)^2=\frac{4}{1}-\frac{4}{9}$$

$$(y-1)^2=\frac{32}{9}$$

$$\sqrt{(y-1)^2}=\sqrt{\frac{32}{9}}$$

$$y-1=\pm\sqrt{\frac{32}{9}}$$

$$y=1\pm\frac{\sqrt{32}}{3}=1\pm\frac{4\sqrt{2}}{3}$$

$$\boxed{\text{y-n: } \left(0,\ 1\pm\frac{4\sqrt{2}}{3}\right)}$$

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(2) $\left(-\frac{3}{4}, \frac{-1}{3}\right)$ and $\left(\frac{3}{8}, \frac{5}{6}\right)$

(a) let d = distance

$$d^2=(\Delta x)^2+(\Delta y)^2$$

$$d^2=\left[\frac{3}{8}-\left(-\frac{3}{4}\right)\right]^2+\left[\frac{5}{6}-\left(\frac{-1}{3}\right)\right]^2$$

$$d^2=\left(\frac{9}{8}\right)^2+\left(\frac{7}{6}\right)^2$$

$$d^2=\frac{81}{64}+\frac{49}{36}$$

$$d^2=\frac{1513}{576}$$

$d=\pm\sqrt{\frac{1513}{576}}$, but $d \geq 0$

so $d=\frac{\sqrt{1513}}{\sqrt{576}}=\frac{\sqrt{1513}}{24}$

$$\boxed{d=\frac{\sqrt{1513}}{24}}$$

(b) let $M(x_M, y_M)$ = midpoint

$$x_M=\frac{x_1+x_2}{2}=\frac{-\frac{3}{4}+\frac{3}{8}}{2}=\frac{-\frac{3}{8}}{2}$$

$$x_M=\frac{-3}{16}$$

$$y_M=\frac{y_1+y_2}{2}=\frac{-\frac{1}{3}+\frac{5}{6}}{2}=\frac{\frac{3}{6}}{2}=\frac{\frac{1}{2}}{2}$$

$$y_M=\frac{1}{4}$$

so $\boxed{M\left(\frac{-3}{16}, \frac{1}{4}\right)}$

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(3) $(2,6), (-4,r)$

(a) the line is parallel to $2x-3y=6$, so they both have the same slope

m_1 = slope of line through $(2,6)$ and $(-4,r)$

$$m_1=\frac{\Delta y}{\Delta x}=\frac{6-r}{2+4}=\frac{6-r}{6}$$

m_2 = slope of line $2x-3y=6$

$$2x-3y=6$$

$$2x-6=3y \quad /\div 3$$

$$y=\frac{2}{3}x-2$$

$$m_2=\frac{2}{3}$$

Then,

$m_1=m_2 \Rightarrow \frac{6-r}{6}=\frac{2}{3}$, $\cdot 18$

$$3(6-r)=6\cdot 2$$

$$6-r=4$$

$$\boxed{r=2}$$

(b) The line is perpendicular to line $x+2y=1$

let m_3 = slope of $x+2y=1$

then, $m_1 \cdot m_3 = -1$

$$x+2y=1$$
$$2y=-x+1 \quad / \div 2$$
$$y=\frac{-1}{2}x+\frac{1}{2}$$

so $m_3 = \frac{-1}{2}$

$m_1 = \frac{6-r}{6}$ $\Big\}$ $\Rightarrow$

$$\frac{6-r}{6}\cdot\frac{-1}{2}=-1$$
$$\frac{r-6}{12}=-1$$
$$r-6=-12$$
$$\boxed{r=-6}$$

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(4) $f(x)=\frac{3}{x-5}$

(a) Domain

condition: $x-5\neq 0$

$x\neq 5$

$$\boxed{x\in \mathbb{R}\setminus\{5\}}$$

(b) x-n: let $y=0$

but $\frac{3}{x-5}\neq 0$

so no x-n.

y-n: let $x=0$, then $y=\frac{-3}{5}$

$$\boxed{y\text{-}n: (0, -\tfrac{3}{5})}$$

$g(a)=\sqrt{3a+5}$

(a) Domain

condition: $3a+5\geq 0$

so $a\geq \frac{-5}{3}$

$$\boxed{a\in[-\tfrac{5}{3},\infty)}$$

(b) a-n: let $y=0$

$$\sqrt{3a+5}=0$$
$$3a+5=0$$
$$a=\frac{-5}{3}$$

$$\boxed{a\text{-}n: (-\tfrac{5}{3},0)}$$

y-n: let $a=0$, then $y=\sqrt{5}$

$$\boxed{y\text{-}n: (0,\sqrt{5})}$$

$v(x)=\frac{x+2}{x^2-25}=\frac{x+2}{(x+5)(x-5)}$

(a) Domain:

condition: $x^2-25\neq 0$

$x\neq \pm 5$

$$\boxed{x\in\mathbb{R}\setminus\{5,-5\}}$$

(b) x-n: $y=0$ iff $x+2=0$

$x=-2$

$$\boxed{x\text{-}n: (-2,0)}$$

y-n: $x=0$, then $y=\frac{-2}{25}$

$$\boxed{y\text{-}n: (0,\tfrac{-2}{25})}$$