Sections 4.3 & 4.4 Logarithmic Functions and Properties of Logarithms
Sections 4.4 & 4.5 Exponential and Logarithmic Equations and Their Applications

1) Find the following:

a) $\log_3 27$ b) $\log_4 \frac{1}{16}$ c) $\log_{1/2} 8$ d) $\log_2 \sqrt{2}$

e) $\log_2(\log_4 16)$ f) $\log(\ln e)$ g) $\log(\log_3(\log_5 125))$ h) $\log 70 - \log 7$

i) $2^{\log_2 5} - 3\log_5 \sqrt[3]{5}$ j) $\dfrac{\log_3 81 - \log_p 1}{\log_{2\sqrt{2}} 8 - \log 0.001}$ k) $(\log_2 10)(\log 2)$

l) $5e^{\ln(A^2)}$ m) $\ln(e^{2ab})$

2) Expand as much as possible. Simplify the result if possible. Assume all variables represent positive real numbers

a) $\log_3 \dfrac{4p}{q}$ b) $\log_5 \dfrac{5\sqrt{7}}{3}$ c) $\log_6(7m+3q)$ d) $\log_m \sqrt{\dfrac{5r^3}{z^5}}$

e) $\log_3 \dfrac{\sqrt{x}\cdot\sqrt[3]{y}}{w^2\sqrt{z}}$ f) $\ln \dfrac{5x\sqrt{1+3x}}{(x-4)^3}$

3) Write as a single logarithm with coefficient 1. Assume all variables represent positive real numbers

a) $\log_a x + \log_a y - \log_a m$ b) $2\log_m a - 3\log_m b^2$

c) $\log_b(2y+5) - \frac{1}{2}\log_b(y+3)$

4) Graph the following functions using transformations. Find domain, range, exact intercepts, asymptote, and inverse for each function.

a) $f(x) = -\log_3(x-2)+1$ b) $f(x) = 3^{x-1}-2$ c) $f(x) = e^{x+1}-4$

d) $f(x) = \ln(x+3)-1$

5) Find the domain of each function:

a) $f(x) = \ln(2x+1)$ b) $f(x) = \log_3(x-7)^2$ c) $f(x) = \log(16-x^2)$

6) Solve for *x*.

a) $10^x = 25$ b) $10^{x^2} = 40$ c) $\log_x 1 = 0$ d) $\log_x \sqrt[3]{5} = \frac{1}{3}$

e) $10^{x+3} = 5e^{7-x}$ f) $2e^{3x} = 4e^{5x}$ g) $2x-1 = e^{\ln x^2}$ h) $9^x = 2e^{x^2}$

j) $5^x = 3^{2x-1}$ k) $3^{x^2-4} = 27$ l) $\log_8(x+5) - \log_8 2 = 1$

m) $10^{2x} + 3(10^x) - 10 = 0$ n) $\log_2(\log_3 x) = -1$ o) $e^x - e^{-x} = 1$

p) $\ln(-x) + \ln 3 = \ln(2x-15)$ r) $\ln 5x - \ln(2x-1) = \ln 4$

s) $\log x = \sqrt{\log x}$ t) $5(1.2)^{3x-2} + 1 = 7$ v) $\log_2 x - \dfrac{\log_2 2}{\log_2 x} = 0$

7) Solve for the indicated variable.

a) $r = p - k\ln t$, for t b) $T = T_0 + (T_1 - T_0)10^{-kt}$, for t c) $y = \dfrac{k}{1+ae^{-bx}}$, for b

d) $m = 6 - 2.5\log(M/M_0)$, for M e) $\log A = \log B - C\log x$, for A

f) $P = P_0e^{kt}$, for t g) $ae^{kt} = e^{bt}$, where $k \neq b$; solve for t h) $I = \dfrac{E}{R}\left(1 - e^{-\frac{Rt}{2}}\right)$, for t

8) Convert the functions into the form $P = P_0a^t$. Which represent exponential growth and which represent exponential decay?

a) $P = 2e^{-0.5t}$ b) $P = 15e^{0.25t}$

Note: $P = P_0e^{kt}$ models exponential growth (k>0) or decay (k<0).

k = growth constant

P_0 = initial population

9) Convert the functions into the form $P = P_0e^{kt}$.

a) $P = 15(1.5)^t$ b) $P = 4(0.55)^t$

10) Find the inverse of each function.

a) $f(t) = 50e^{0.1t}$. b) $f(t) = 1 + \ln t$.

11) The air in a factory is being filtered so that the quantity of pollutant, P (measured in mg/liter) is decreasing according to the equation $P = P_0e^{-kt}$, where t represents time in hours. If 10% of the pollution is removed in the first five hours:

a) What percentage of the pollution is left after 10 hours?

b) How long will it take before the pollution is reduced by 50%?

12) If the size of a bacteria colony doubles in 5 hours, how long will it take for the number of bacteria to triple?

13) Suppose a certain radioactive substance has a half-life of 5 years. An object starts with 20 kg of the radioactive material.

a) How much of the radioactive material is left after 10 years?

b) The object can be moved safely when the quantity of the radioactive material is 0.1 kg or less. How much time must pass before the object can be moved?

14) The number of bacteria present in a culture after t hours is given by the formula $N = 1000e^{0.69t}$.

a) How many bacteria will be there after ½ hour?

b) How long will it be before there are 1,000,000 bacteria?

c) What is the doubling time?

15) You place $800 in an account that earns 4% annual interest, compounded annually. How long will it be until you have $2000

16) At the World Championship races held at Rome's Olympic Stadium in 1987, American sprinter Carl Lewis ran the 100-m race in 9.86 sec. His speed in meters per second after t seconds is closely modeled by the function defined by $f(t) = 11.65\left(1 - e^{-\frac{t}{1.27}}\right)$.

a) How fast was he running as he crossed the finish line?

b) After how many seconds was he running at the rate of 10 m per sec?

17) As age increases, so does the likelihood of coronary heart disease (CHD). The fraction of people x years old with some CHD is modeled by $f\left(x\right)=\dfrac{0.9}{1+271e^{-0.122x}}$.

a) Evaluate $f\left(25\right)$, $f\left(65\right)$. Interpret the results.
b) At what age does this likelihood equal 50%?

18) Find the doubling time of an investment earning 2.5% interest if interest is compounded continuously.

19) In 2000 India's population reached 1 billion, and in 2025 it is projected to be 1.4 billion.

a) Find values for P_0 and a so that $f\left(x\right)=P_0a^{x-2000}$ models the population of India in year x.
b) Estimate India's population in 2010.
c) When will India's population might reach 1.5 billion?

20) Assume the cost of a loaf of bread is $4. With continuous compounding, find the time it would take for the cost to triple at an annual inflation rate of 6%.

Answers:

1) a) 3; b) -2; c) -3; d) ½; e) 1; f) 0; g) 0; h) 1; i) 4; j) 4/5; k) 1; l) $5A^2$; m) 2ab

2) c)cannot be simplified; d) $\frac{1}{2}\left(\log_m 5+\log_m r-5\log_m z\right)$; e) $\frac{1}{2}\log_3 x+\frac{1}{3}\log_3 y-2\log_3 w-\frac{1}{2}\log_3 z$; f) $\ln 5+\ln x+\frac{1}{2}\ln\left(1+3x\right)-3\ln\left(x-4\right)$

3) a) $\log_a\frac{xy}{m}$; b) $\log_m\frac{a^2}{b^6}$; c) $\log_b\frac{2y+5}{\sqrt{y+3}}$

4) a) $f^{-1}\left(x\right)=3^{1-x}+2$; b) $f^{-1}\left(x\right)=1+\log_3\left(x+2\right)$; c) $f^{-1}\left(x\right)=\ln\left(x+4\right)-1$; d) $f^{-1}\left(x\right)=e^{x+1}-3$

5) a) $x>-1/2$; b) $x\neq 7$; c) $\left(-4,4\right)$

6) a) $\log 25$; b) $\pm\sqrt{\log 40}$; c) $x>0, x\neq 1$; d) 5; e) 0.515; f) -0.347; g)1; h) 1.81, 0.38; j) $\frac{\log_5 3}{2\log_5 3-1}$; k) $\pm\sqrt{7}$; l)11; m) $\log 2$; n) $\sqrt{3}$; o) $\ln\frac{1+\sqrt{5}}{2}$; p) $\varnothing$; r) 4/3; s) 1, 10; t) 1; u) $\sqrt{3}$; v) 2, 1/2

7) a) $e^{(p-r)/k}$; b) $-\frac{1}{k}\log\left(\frac{T-T_0}{T_1-T_0}\right)$; c) $\frac{\ln\left(\left(k-y\right)/\left(ay\right)\right)}{-x}$; d) $M_0\cdot 10^{(6-m)/2.5}$; e) $\frac{B}{x^C}$; f) $\ln\left(P/P_0\right)/k$; g) $\frac{\ln a}{b-k}$; h) $-\frac{2}{R}\ln\left(1-\frac{RI}{E}\right)$

8) a) $P=2\left(0.61\right)^t$; decay; b) $P=15\left(1.284\right)^t$ growth; **9)** a) $P=15e^{0.41t}$; b) $P=4e^{-0.6t}$

10) a) $f^{-1}\left(t\right)=10\ln\left(\frac{t}{50}\right)$; b) $f^{-1}\left(t\right)=e^{t-1}$; **11)** a) 81%; b) 33 hours; **12)** 7.925 hours;

13) a) 5 kg; b) 38.2 years; **14)** a) 1412 bacteria; b) 10 hours; c) 1 hour; **15)** 23.4 years

16) a) 11.6451 m per sec; b) 2.4823 sec **17)** a) 0.065; 0.82; b) about 48 years. **18)** about 27.73 years

19) a) 1 and a=1.01355; b) about 1.14 billion ; c) 2030; **20)** 18.3 yr.
