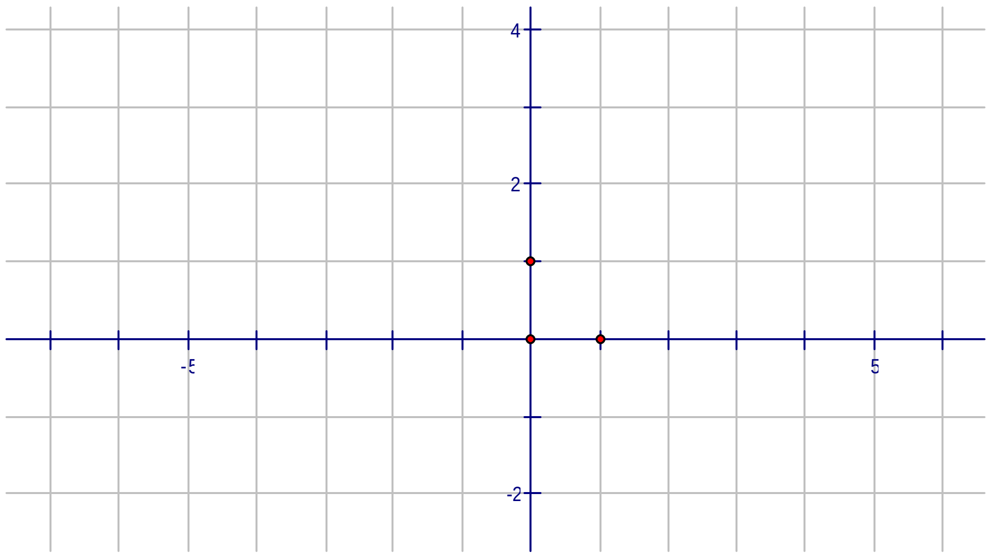
Section 3.1
Quadratic Functions and Models

Quadratic Function: $f\left(x\right)=ax^2+bx+c \quad (a\neq 0)$
The graph of a quadratic function is called a **parabola.**

Graphing Parabolas: Special Cases
The "basic" parabola is the graph of the simplest quadratic function $y=x^2$.

x	y
-3	
-2	
-1	
0	
1	
2	
3	



All parabolas share certain features.

Vertex – the lowest point (if the parabola opens up) or the highest point (if the parabola opens down).

The vertex of the basic parabola is ___________________.

Axis of symmetry – the parabola is symmetric about the vertical line that runs through the vertex.

The axis of symmetry of the basic parabola is ___________________.

***y*-intercept** – the point where the parabola intersects the *y*-axis.

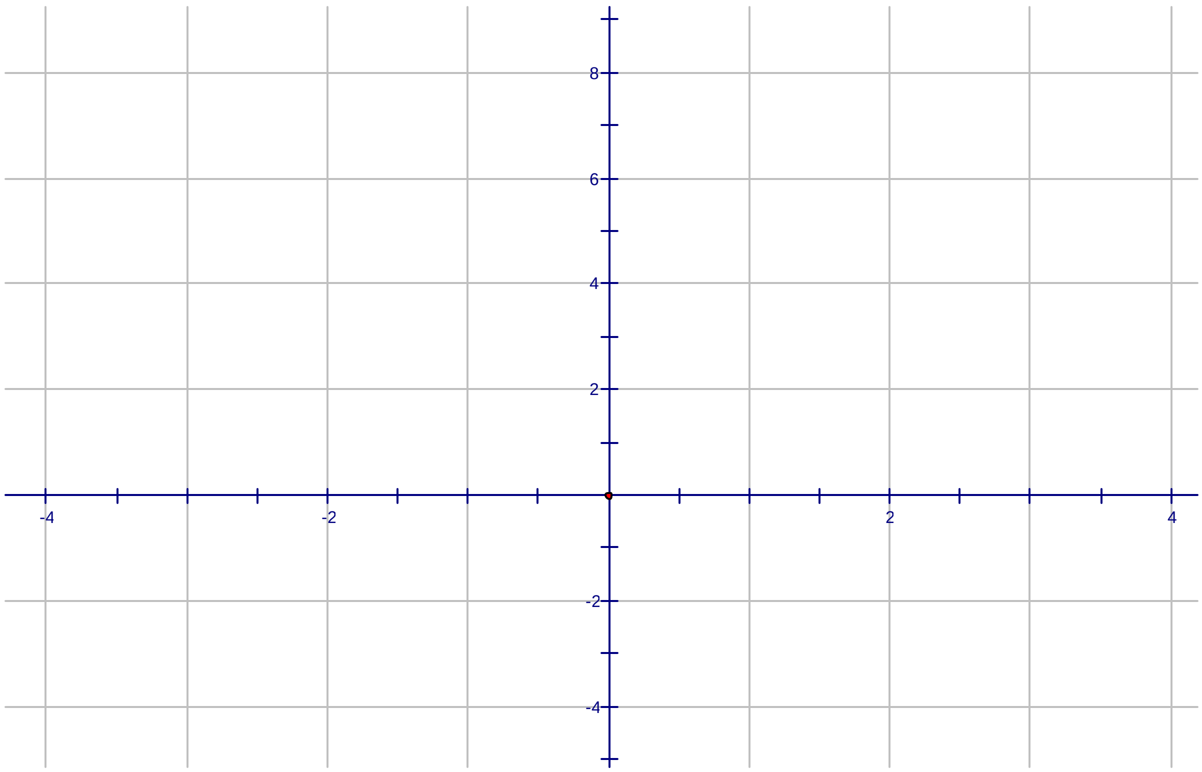
***x*-intercept(s)** – the point(s) where the parabola intersects the *x*-axis.

The x- and y-intercept of the basic parabola is ___________________.

Example #1 Graph the following parabolas on the same coordinate system:

1) $y = x^2$ 2) $y = 2x^2$ 3) $y = \frac{1}{2}x^2$ 4) $y = -x^2$ 5) $y = -2x^2$

Investigate the effect of the coefficient of x^2 on the graph.



What are the effects of the coefficient *a* of x^2 on the graph?

If $a > 0$, the parabola opens ___________________________.

If $a < 0$, the parabola opens ___________________________.

How to Graph a Parabola

Standard form: $y = ax^2 + bx + c \quad (a \neq 0)$

Note that if $a > 0$, the parabola opens upward, and if $a < 0$, the parabola opens downward.

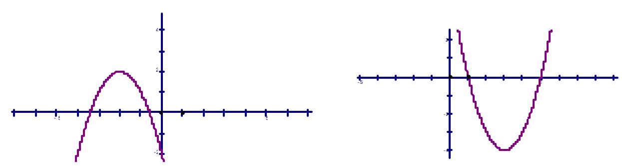
Vertex $V(x_v, y_v)$ $\boxed{x_v = \dfrac{-b}{2a}}$ To find y_v substitute the value of x_v in the equation and solve for y.

y-intercept To find the y-intercept make x=0 and solve for y.

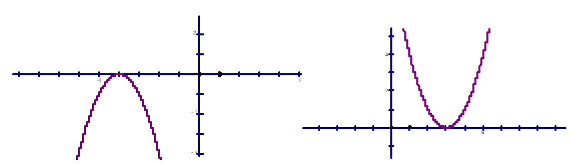
x-intercept(s) (if any) To find the x-intercept(s) make y=0 and solve for x.

How many x-intercepts?

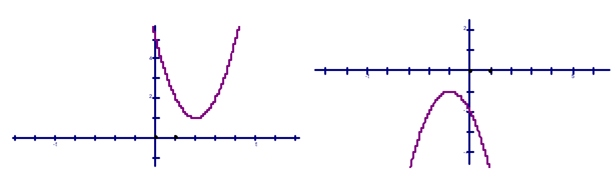
Two $x-\cap$ (when $\Delta > 0$)



One $x-\cap$ (when $\Delta = 0$)



No $x-\cap$ (when $\Delta < 0$)



Note: The parabola is symmetric about the vertical axis that passes through the vertex. If no x-intercept, use the symmetric of the y-intercept about the axis of symmetry to graph the parabola

The Vertex Form of a Parabola: $\boxed{y = a(x - x_v)^2 + y_v}$, where $V(x_v, y_v)$ is the vertex and a is the coefficient of x^2.

Exercise #1:

(a) Graph the following parabola: $y = x^2 + 3x + 2$. Give the domain and range. Solve the following inequalities using the graph: $x^2 + 3x + 2 > 0$ and $x^2 + 3x + 2 \le 0$

(b) Graph the following parabola: $y = -2x^2 + 4x + 1$. Give the domain and range. Solve the following inequalities using the graph: $-2x^2 + 4x + 1 \ge 0$ and $-2x^2 + 4x + 1 < 0$

Exercise #2: Find the vertex of each parabola. Decide whether the vertex is a maximum or a minimum point. Give the domain and the range.

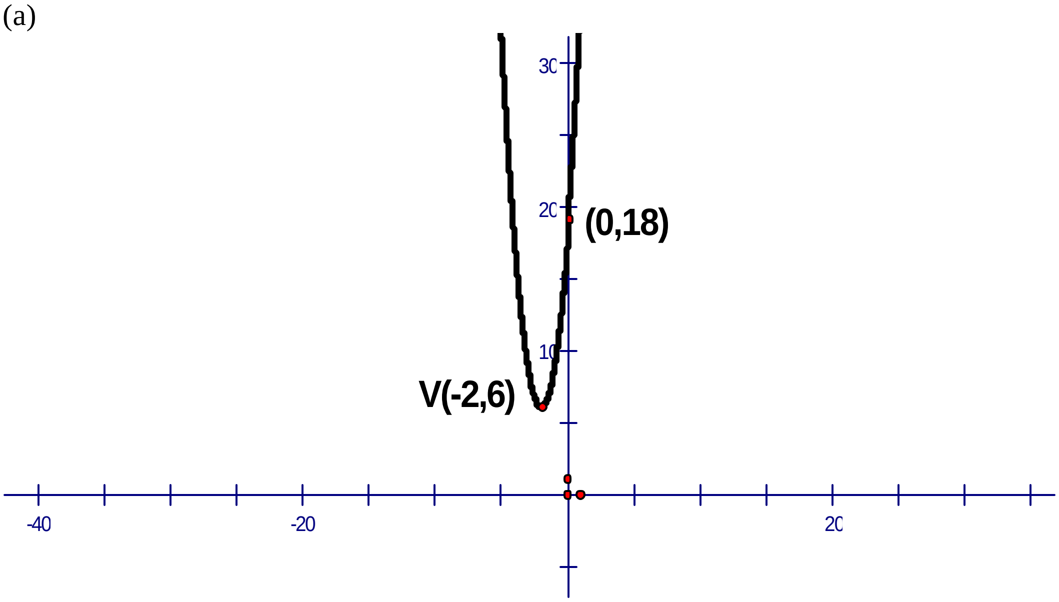
(a) $y = 2(x-3)^2 + 4$. Graph the function explaining how its graph is obtained from the graph of the basic parabola.

(b) $y = -3(x+3)^2 - 5$. Graph the function explaining how its graph is obtained from the graph of the basic parabola.

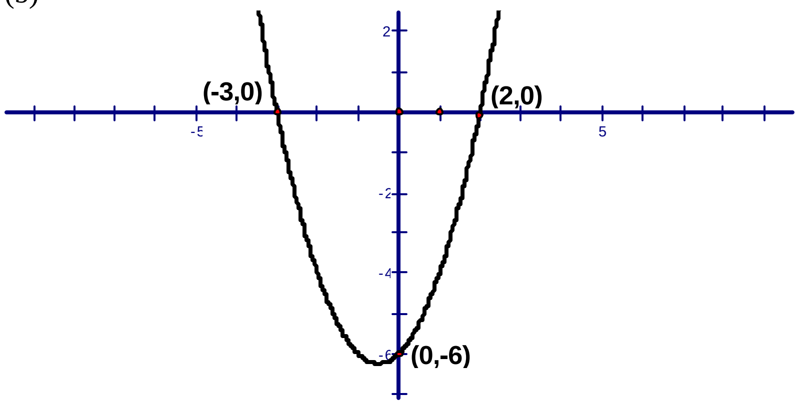
(c) $y = 3x^2 + 4x + 2$. Graph this parabola by writing its equation in vertex form first (by completing the square on x).

Exercise #3: Write an equation for each graph. Give the domain and range. Solve $f(x) > 0$, $f(x) < 0$, $f(x) = 0$

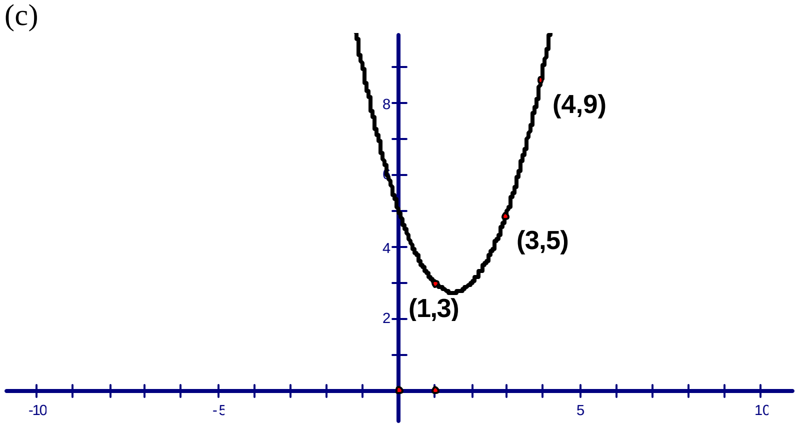
(a)



(b)



(c)



Exercise #4 If air resistance is neglected, the height *s* (in feet) of an object propelled directly upward from a an initial height s_0 feet with initial velocity v_0 feet per second is

$$s(t) = -16t^2 + v_0 t + s_0 ,$$

where t is the number of seconds after the object is propelled.

A toy rocket is launched straight up from the top of a building 50 ft tall at an initial velocity of 200 ft per sec.

a) Give the function that describes the height of the rocket in terms of t.

b) Determine the time at which the rocket reaches its maximum height, and the maximum height in feet.

c) For what interval will the rocket be more than 300 feet above the ground level?

d) After how many seconds will it hit the ground?

Exercise #5 Suppose that x represents one of two positive numbers whose sum is 30.

a) Represent the other of the two numbers in terms of x.

b) What are the restrictions on x?

c) Determine a function f that represents the product of these two numbers.

d) What are the two such numbers that yield the maximum product? What is their product?

e) For what two such numbers is the product equal to 140?

Exercise #6 One campus has plans to construct a rectangular parking lot on land bordered on one side by a highway. There are 640 ft of fencing available to fence the other three sides. Let x represent the length of each of the two parallel sides of fencing.

a) Represent the length of the remaining side to be fenced in terms of x.

b) What are the restrictions on x?

c) Determine a function A that represents the area of the parking lot in terms of x.

d) Determine the values of x that will give an area between 30,000 and 40,000 sq.ft.

e) What dimensions will give a maximum area, and what will this area be?

Exercise #7 A frog leaps from a stump 3 ft high and lands 4 ft from the base of the stump. We can consider the initial position of the frog to be (0,3) and its landing position to be at (4,0). It is determined that the height of the frog as a function of its horizontal distance x from the base of the stump is given by $h(x) = -0.5x^2 + 1.25x + 3$, where x and $h(x)$ are both in feet.

a) How high was the frog when its horizontal distance from the base of the stump was 2 ft?

b) At what horizontal distances from the base of the stump was the frog 3.25 ft above the ground?

c) At what horizontal distance from the base of the stump did the frog reach its highest point?

d) What was the maximum height reached by the frog?

Exercise #8 Find a value of c so that $y = x^2 - 10x + c$ has exactly one x-intercept.