Section 2.8 – The Algebra and Composition of Functions

Two functions *f* and *g* can be combined to form new functions $f+g, f-g, fg, \frac{f}{g}$ in a manner similar to the way we add, subtract, multiply and divide real numbers.

<u>Definition</u> Let *f* and *g* be two functions. Let D_f be the domain of f and D_g the domain of *g*. Then:

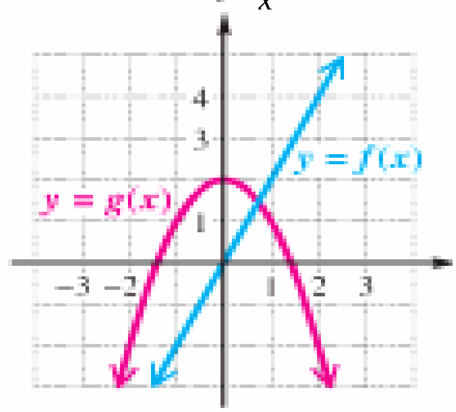
- $(f+g)(x)=f(x)+g(x)$ and the domain of $f+g$ is $D_f \cap D_g$ (all real numbers that are common to the domain of *f* and the domain if *g*.)
- $(f-g)(x)=f(x)-g(x)$ and the domain of $f-g$ is $D_f \cap D_g$
- $(fg)(x)=f(x)\cdot g(x)$ and the domain of fg is $D_f \cap D_g$
- $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$ and the domain of $\frac{f}{g}$ is the set of all real numbers that are common to the domain of f and the domain of g such that $g(x)\neq 0$

Exercise 1 Find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$. Find the domain of each.

a) $f(x)=2x^2-3x, g(x)=x^2-x+3$

b) $f(x)=\sqrt{4x-1}, g(x)=\frac{1}{x}$

c) $f(x)=\frac{4}{x-3}, \ g(x)=\frac{1}{x+5}$



Exercise 2 Use the graph to evaluate each expression.

a) $(f+g)(0)$ b) $(f-g)(-1)$

c) $(fg)(1)$ d) $\left(\frac{f}{g}\right)(2)$

Exercise 3 Suppose the total cost, in dollars, of manufacturing a certain computer component can be modeled by the function $C(n)=0.1n^2$, where n is the number of components made. If each component is sold at a price of \$11.45, the revenue is modeled by $R(n)=11.45n$. Find the following:

a) Find the function that represent the total profit made from sales of the components

b) How much profit is earned if 12 components are made and sold?

Composition of Functions

Definition: If *f* and *g* are function, then the **composite function**, or **composition**, of *f* and *g* is defined as

$$(f \circ g)(x) = f(g(x))$$

where the domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of *f*.

Exercise 4 For each pair of functions below, find $(f \circ g)(x)$, $(g \circ f)(x)$, and their domain.

a) $f(x) = \dfrac{2}{x^3}, g(x) = 1 - x$

b) $f(x) = \sqrt{x+3}, g(x) = 2x - 5$

c) $f(x) = x + 3, g(x) = \sqrt{9 - x^2}$

d) $f(x) = \dfrac{3}{x}, g(x) = \dfrac{1}{x-2}$

Exercise 5 Let $f(x) = x^2$ and $g(x) = 3x + 1$. Show two ways in which you can compute $(f \circ g)(-2)$.

Exercise 6 Suppose that in a certain biology lab experiment, the number of bacteria is related to the temperature *T* of the environment by the function $N(T) = -2T^2 + 240T - 5400$, where $40 \le T \le 90$. Here, $N(T)$ represents the number of bacteria present when the temperature is *T* degrees Fahrenheit. Also, suppose that *t* hours after the experiment begins, the temperature is given by $T(t) = 10t + 40$, where $0 \le t \le 5$

a) Compute $N(T(t))$.

b) How many bacteria are present when $t = 0$ hr? When $t = 2$ hr? When $t = 5$ hr?

Exercise 7 Given $f(x) = 2x + 3, g(x) = \dfrac{x-3}{2}$, and $h(x) = 5 - x$ find :

a) $(f \circ f)(x)$

b) $(f \circ f)(-1)$

c) $(g \circ g)(x)$

d) $f(g(h(x)))$

e) $h^2(x)$

f) $(h \circ h)(x)$

Exercise 8 Due to a lighting strike, a forest fire begins to burn and is spreading outward in shape that is roughly circular. The radius of the circle is modeled by the function $r(t) = 2t$, where *t* is the time in minutes and *r* is measured in meters.

a) Write a function for the area burned by the fire directly as a function of time *t*.

b) Find the area of the circular burn after 60 minutes.

Exercise 9 Decomposition of functions

Let $s(x) = \sqrt{1 + x^4}$. Express the function *s* as a composition of two simpler functions *f* and *g*.

Exercise 10 Let $g(x) = 4x - 1$. Find $f(x)$, given that the equation $(g \circ f)(x) = x + 5$ is true for all values of x.