HOMEWORK SOLUTIONS

HANDOUT CHAPTER 1 - REVIEW

(3d) Solve by completing the square:

$x^2 + \sqrt{3}x - \frac{1}{4} = 0$

$x^2 + \sqrt{3}x = \frac{1}{4} \quad / + \frac{3}{4}$

$\left(\frac{1}{2}\text{coef}(x)\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

$x^2 + \sqrt{3}x + \frac{3}{4} = \frac{1}{4} + \frac{3}{4}$

$\left(x + \frac{\sqrt{3}}{2}\right)^2 = 1 \quad / \sqrt{\ }$

$\sqrt{\left(x + \frac{\sqrt{3}}{2}\right)^2} = \sqrt{1}$

$x + \frac{\sqrt{3}}{2} = \pm 1$

$\boxed{x = \frac{-\sqrt{3}}{2} \pm 1}$

(5b) Solve in $\mathbb{C}$:

$2x^7 - 128x = 0$

$2x(x^6 - 64) = 0$

$2x((x^3)^2 - 8^2) = 0$

$2x(x^3 - 8)(x^3 + 8) = 0$

$2x(x-2)(x^2+2x+4)(x+2)(x^2-2x+4) = 0$

I $x = 0$

or

II $x - 2 = 0 \Rightarrow x = 2$

or

III $x^2 + 2x + 4 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm \sqrt{-12}}{2}$

$x = \frac{-2 \pm 2i\sqrt{3}}{2}$

$x = -1 \pm i\sqrt{3}$

or

IV $x + 2 = 0 \Rightarrow x = -2$

or

V $x^2 - 2x + 4 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{2 \pm \sqrt{4-16}}{2} = \frac{2 \pm \sqrt{-12}}{2}$

$x = \frac{2 \pm 2i\sqrt{3}}{2}$

$x = 1 \pm i\sqrt{3}$

$\boxed{x \in \{0, \pm 2, -1 \pm i\sqrt{3}, 1 \pm i\sqrt{3}\}}$

(8b) $A = 2w^2 + 4/w$

Solve for w

This is a quadratic equation in w

$2w^2 + 4/w - A = 0$

Apply the Quadratic Formula

where $\begin{cases} a = 2 \\ b = 4/ \\ c = -A \end{cases}$

$w = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$w = \frac{-4l \pm \sqrt{(4l)^2 - 4(2)(-A)}}{2(2)}$$

$$w = \frac{-4l \pm \sqrt{16l^2 + 8A}}{4}$$

$$w = \frac{-4l \pm \sqrt{4(4l^2 + 2A)}}{4}$$

$$w = \frac{-4l \pm 2\sqrt{4l^2 + 2A}}{4}$$

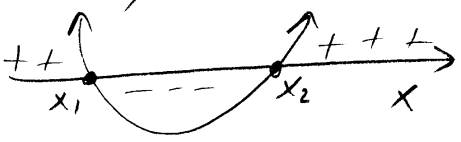
$$\boxed{w = \frac{-2l \pm \sqrt{4l^2 + 2A}}{2}}$$

(12d) $x^2 + 4x > -1$

$x^2 + 4x + 1 > 0$

let $y = x^2 + 4x + 1$

The graph of this equation is a parabola opening up



with x_1 and x_2 x-intercepts

x-int: $x^2 + 4x + 1 = 0$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 4}}{2} = \frac{-4 \pm 2\sqrt{3}}{2}$$

$x_1 = -2 - \sqrt{3}$, $x_2 = -2 + \sqrt{3}$

Therefore, $x^2 + 4x + 1 > 0$ iff

$$\boxed{x \in (-\infty, -2-\sqrt{3}) \cup (-2+\sqrt{3}, \infty)}$$

(12e) $x^3 + 4x^2 - 9x - 36 \geq 0$

$x^2(x+4) - 9(x+4) \geq 0$

$(x+4)(x^2 - 9) \geq 0$

$(x+4)(x+3)(x-3) \geq 0$

Study the sign of each factor:

x	$-\infty$		-4		-3		3		∞
$x+4$	−	−	0	+	+	+	+	+	+
$x+3$	−	−	−	−	0	+	+	+	+
$x-3$	−	−	−	−	−	−	0	+	+
$(x+4)(x+3)(x-3)$	−	−	0	+	0	−	0	+	+

$(x+4)(x+3)(x-3) \geq 0$ iff

$$\boxed{x \in [-4, -3] \cup [3, \infty)}$$

(12f) $(x-5)^2(x+1) < 0$

Note that $(x-5)^2 > 0$ for any $x \neq 5$

So, we need $x + 1 < 0$

$x < -1$

So, $\boxed{x \in (-\infty, -1)}$

(13b) $\frac{x+3}{x-5} \leq 1$

$\frac{x+3}{x-5} - 1 \leq 0$

$\frac{x+3-x+5}{x-5} \leq 0$

$\frac{8}{x-5} \leq 0$ iff $x - 5 < 0$

$x < 5$

So, $\boxed{x \in (-\infty, 5)}$