## SECTION 1.4

**#14**

Solve by the zero-factor property.

$$x^2+2x-8=0$$

$$(x+4)(x-2)=0$$

$$x+4=0 \Rightarrow x=-4$$

or

$$x-2=0 \Rightarrow x=2$$

The solution set is $\{-4,2\}$.

**#26**

Solve by the square root property

$$(4x+1)^2=20$$

$$\sqrt{(4x+1)^2}=\sqrt{20}$$

$$|4x+1|=2\sqrt{5}$$

$$4x+1=\pm 2\sqrt{5}$$
$$4x=-1\pm 2\sqrt{5}$$
$$x=\frac{-1\pm 2\sqrt{5}}{4}$$

The solution set is $\left\{\frac{-1\pm 2\sqrt{5}}{4}\right\}$.

**#32**

Solve by completing the square.

$$3x^2+2x-5=0$$

Isolate the constant: $3x^2+2x=5$

Make the leading coefficient 1: $x^2+\frac{2}{3}x=\frac{5}{3}$

Find the missing term:

$$\left(\frac{1}{2} \textit{coefficient of } x\right)^2=\left(\frac{1}{2}\cdot\frac{2}{3}\right)^2=\frac{1}{9}$$

$$x^2+\frac{2}{3}x+\frac{1}{9}= {}^{3/}\ \frac{5}{3}+\frac{1}{9}$$

$$\left(x+\frac{1}{3}\right)^2=\frac{16}{9}$$

$$\sqrt{\left(x+\frac{1}{3}\right)^2}=\sqrt{\frac{16}{9}}$$

$$\left|x+\frac{1}{3}\right|=\frac{4}{3}$$

$$x+\frac{1}{3}=\pm\frac{4}{3}$$

$$x=-\frac{1}{3}\pm\frac{4}{3}$$

$$x=-\frac{1}{3}+\frac{4}{3}=1$$

or

$$x=-\frac{1}{3}-\frac{4}{3}=-\frac{5}{3}$$

The solution set is $\left\{1,-\frac{5}{3}\right\}$.

**#54**

Solve by the quadratic formula.

$$ {}^{4/}\ \frac{2}{3}x^2+ {}^{3/}\ \frac{1}{4}x= {}^{12/}3$$

Eliminate all fractions: $LCD=12$

$$8x^2+3x=36$$
$$8x^2+3x-36=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_{1,2}=\frac{-3\pm\sqrt{3^2-4(8)(-36)}}{2(8)}$$

$$x_{1,2}=\frac{-3\pm\sqrt{1161}}{16}=\frac{-3\pm 3\sqrt{129}}{16}$$

**#58**

Solve using the quadratic formula.

$$3-\frac{4}{x}-\frac{2}{x^2}=0$$

Condition: $x\neq 0$

Eliminate all fractions: $LCD=x^2$

$$3x^2-4x-2=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_{1,2}=\frac{-(-4)\pm\sqrt{(-4)^2-4(3)(-2)}}{2(3)}$$

$$=\frac{4\pm\sqrt{40}}{6}=\frac{4\pm 2\sqrt{10}}{6}$$

$$=\frac{2\left(2\pm\sqrt{10}\right)}{6}=\frac{2\pm\sqrt{10}}{3}$$

The solution set is $\left\{\frac{2\pm\sqrt{10}}{3}\right\}$.

**#62**

$$x^3+64=0$$

$$x^3+4^3=0$$

Sum of cubes: $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$

$$(x+4)\left(x^2-4x+16\right)=0$$

$x+4=0\Rightarrow x=-4$

or

$x^2-4x+16=0$

$$x=\frac{4\pm\sqrt{16-4(16)}}{2}=\frac{4\pm\sqrt{-3(16)}}{2}$$

$$=\frac{4\pm 4\sqrt{3}i}{2}=2\pm 2\sqrt{3}i$$

The solution set is $\left\{-4, 2\pm 2\sqrt{3}i\right\}$.

**#66**

$s=s_0+gt^2+k$ solve for $t$.

Isolate the variable $t$: $gt^2=s-s_0-k$

$$t^2=\frac{s-s_0-k}{g}$$

$$t=\pm\sqrt{\frac{s-s_0-k}{g}}$$

**#70**

$$3y^2+4xy-9x^2=-1$$

a) Solve for $x$ in terms of $y$.

Therefore $x$ is the unknown. Write the equation

in standard form: $9x^2-4yx-3y^2-1=0$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ with $a=9$, $b=-4y$,

and $c=-3y^2-1$

$$x_{1,2}=\frac{-(-4y)\pm\sqrt{(-4y)^2-4(9)\left(-3y^2-1\right)}}{2(9)}$$

$$=\frac{4y\pm\sqrt{16y^2+4\left(27y^2+9\right)}}{18}$$

$$=\frac{4y\pm\sqrt{4\left(4y^2+27y^2+9\right)}}{18}$$

$$=\frac{4y\pm 2\sqrt{31y^2+9}}{18}=\frac{2\left(2y\pm\sqrt{31y^2+9}\right)}{18}$$

$$x_{1,2}=\frac{2y\pm\sqrt{31y^2+9}}{9}$$

b) Solve for y in terms of x.
Therefore, $y$ is the unknown.

$$3y^2+4xy-9x^2=-1$$
$$3y^2+4xy-9x^2+1=0$$
$$y_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

with $a=3$, $b=4x$, and $c=-9x^2+1$

$$y_{1,2}=\frac{-4x\pm\sqrt{(4x)^2-4(3)(-9x^2+1)}}{2(3)}$$
$$=\frac{-4\pm\sqrt{4\left(4x^2-3(-9x^2+1)\right)}}{6}$$
$$=\frac{-4\pm2\sqrt{4x^2+27x^2-3}}{6}$$
$$=\frac{\not{2}\left(-2\pm\sqrt{4x^2+27x^2-3}\right)}{\not{6}\,3}$$

$$y_{1,2}=\frac{-2\pm\sqrt{31x^2-3}}{3}$$

**#78**

$$3x^2=4x-5$$
$$3x^2-4x+5=0$$
$$\Delta=b^2-4ac$$
$$\Delta=(-4)^2-4(3)(5)=-44$$

$\Delta<0$, therefore there are two distinct complex conjugate solutions

**#86**

$$x_1=i$$
$$x_2=-i$$
$$(x-x_1)(x-x_2)=0$$
$$(x-i)(x+i)=0$$
$$x^2-i^2=0$$

$$x^2+1=0$$

So $a=1, b=0, c=1$.

**SECTION 1.6**

**#4**

$$\frac{2}{x+3}-\frac{5}{x-1}=\frac{-1}{x^2+2x-3}$$
$$\frac{2}{x+3}-\frac{5}{x-1}=\frac{-1}{(x+3)(x-1)}$$

Conditions: $x\neq-3, x\neq1$

Therefore, the values of the variable that cannot be solutions are -3 and 1

**#16**

$$\frac{4x+3}{x+1}+\frac{2}{x}=\frac{1}{x^2+x}$$
$$^{x/}\frac{4x+3}{x+1}+{}^{(x+1)/}\frac{2}{x}=\frac{1}{x(x+1)}$$

Conditions: $x\neq-1, x\neq0$

$$LCD=x(x+1)$$
$$x(4x+3)+2(x+1)=1$$
$$4x^2+3x+2x+2-1=0$$
$$4x^2+5x+1=0$$
$$x=\frac{-5\pm\sqrt{5^2-4(4)}}{2(4)}=\frac{-5\pm\sqrt{9}}{8}=\frac{-5\pm3}{8}$$
$$x_1=\frac{-5+3}{8}=\frac{-1}{4}\quad\text{or}\quad x_2=\frac{-5-3}{8}=\not{-1}$$

The solution set is $\left\{-\frac{1}{4}\right\}$.

**#40**

$$\sqrt{4x+1}=\sqrt{x-1}+2$$

$$\left(\sqrt{4x+1}\right)^2=\left(\sqrt{x-1}+2\right)^2$$

$$4x+1=x-1+4\sqrt{x-1}+4$$

$$4x+1-x-3=4\sqrt{x-1}$$

$$3x-2=4\sqrt{x-1}$$

$$(3x-2)^2=\left(4\sqrt{x-1}\right)^2$$

$$9x^2-12x+4=16(x-1)$$

$$9x^2-12x+4=16x-16$$

$$9x^2-28x+20=0$$

$$x_{1,2}=\frac{28\pm\sqrt{28^2-4(9)(20)}}{2(9)}=\frac{28\pm 8}{18}$$

$$x=\frac{36}{18}=2$$

or

$$x=\frac{20}{18}=\frac{10}{9}$$

Check $x=2$:

$\sqrt{4(2)+1}=\sqrt{2-1}+2$ true

Check $x=\frac{10}{9}$:

$$\sqrt{4\cdot\frac{10}{9}+1}=\sqrt{\frac{10}{9}-1}+2$$

$$\sqrt{\frac{49}{9}}=\sqrt{\frac{1}{9}}+2$$

$\frac{7}{3}=\frac{1}{3}+2$ true

The solution set is $\left\{2,\frac{10}{9}\right\}$.

**#64**

$$3x^4+10x^2-25=0$$

Let $x^2=t$. Then $x^4=t^2$.

$$3t^2+10t-25=0$$

$$t=\frac{-10\pm\sqrt{100+300}}{6}=\frac{-10\pm 20}{6}$$

$t=\frac{10}{6}=\frac{5}{3}$ or $t=\frac{-30}{6}=-5$

If $t=\frac{5}{3}$, $x^2=\frac{5}{3}$ and $x=\pm\sqrt{\frac{5}{3}}$.

If $t=-5$, $x^2=-5$ and $x=\pm i\sqrt{5}$

The solution set is $\left\{\pm\sqrt{\frac{5}{3}},\pm i\sqrt{5}\right\}$.

**#70**

$$(2x-1)^{\frac{2}{3}}+2(2x-1)^{\frac{1}{3}}-3=0$$

Let $(2x-1)^{\frac{1}{3}}=t$. Then $(2x-1)^{\frac{2}{3}}=t^2$.

$$t^2+2t-3=0$$

$$(t+3)(t-1)=0$$

$t=-3$ or $t=1$

If $t=-3$, $(2x-1)^{\frac{1}{3}}=-3$

$$\left((2x-1)^{\frac{1}{3}}\right)^3=(-3)^3$$

$$2x-1=-27$$

$2x=-26$, $x=-13$

If $t=1$, $(2x-1)^{\frac{1}{3}}=1$

$$\left((2x-1)^{\frac{1}{3}}\right)^3=1^3$$

$$2x-1=1$$

$2x=2$, $x=1$

Check $x=-13$ and $x=1$ into the original eq.

The solution set is $\{-13,1\}$.

**#76**

$7x^{-2}-10x^{-1}-8=0$

Condition: $x \neq 0$ ($x^{-1}=\frac{1}{x}$)

Let $x^{-1}=t$. Then $x^{-2}=t^2$.

$7t^2-10t-8=0$

$t=\frac{10\pm\sqrt{100+224}}{14}=\frac{10\pm 18}{14}$

$t=\frac{28}{14}=2$ or $t=\frac{-8}{14}=\frac{-4}{7}$

If $t=2$, $x^{-1}=2$

$\frac{1}{x}=2$

$x=\frac{1}{2}$

If $t=\frac{-4}{7}$, $x^{-1}=\frac{-4}{7}$

$\frac{1}{x}=\frac{-4}{7}$

$x=-\frac{7}{4}$

The solution set is $\left\{-\frac{7}{4},\frac{1}{2}\right\}$.

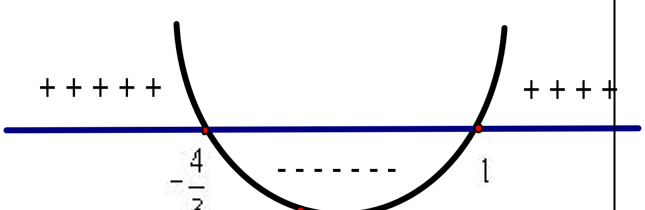
**SECTION 1.7**

**#42**

$3x^2+x-4\le 0$

$(3x+4)(x-1)\le 0$

$y=3x^2+x-4$ is a parabola that opens upward with x-intercepts $x=-\frac{4}{3}$ and $x=1$




Therefore, $3x^2+x-4\le 0$ iff $x\in\left[-\frac{4}{3},1\right]$.

**#62**

$16x-x^3\ge 0$

$x(16-x^2)\ge 0$

$x(4-x)(4+x)\ge 0$

Study the sign of each factor:

| $x$ | $-\infty$ -4 0 4 $\infty$ |
|---|---|
| $x$ | - - - - - - - - - - - - - 0 + + + + + + + |
| $4-x$ | + + + + + + + + + + + + + + 0 - - - - |
| $4+x$ | - - - - - - - 0 + + + + + + + + + + + + |
| $x(4-x)(4+x)$ | + + + + + 0 - - - 0 + + 0 - - - - |

Therefore, $x(4-x)(4+x)\ge 0$ iff $x\in(-\infty,-4]\cup[0,4]$

**#82**

$\frac{-5}{3x+2}\ge\frac{5}{x}$

Bring both terms to the same side:

$\overset{(3x+2)/}{\frac{5}{x}}+\overset{x/}{\frac{5}{3x+2}}\le 0$

Do the addition: $LCD=x(3x+2)$

$\frac{5(3x+2)+5x}{x(3x+2)}\le 0$

$\frac{20x+10}{x(3x+2)}\le 0$

$\frac{10(2x+1)}{x(3x+2)}\le 0$

Study the sign of each factor:

| $x$ | $-\infty$ $-\frac{2}{3}$ $-\frac{1}{2}$ 0 $\infty$ |
|---|---|
| $2x+1$ | - - - - - - - - - - - - 0 + + + + + + + |
| $x$ | - - - - - - - - - - - - - - - - - - - 0 + + + + |
| $3x+2$ | - - - - - - - 0 + + + + + + + + + + + + + |
| $\frac{10(2x+1)}{x(3x+2)}$ | - - - - - - - \| + + + 0 - - - - \| + + + |

Therefore, $\frac{10(2x+1)}{x(3x+2)}\le 0$ iff $x\in(-\infty,-\frac{2}{3})\cup[-\frac{1}{2},0)$.