

Sections 4.5 & 4.6

Exponential and Logarithmic Equations and Their Applications

Solve for x in Problems 1 – 9.

- 1) $10^{x+3} = 5e^{7-x}$ 2) $2e^{3x} = 4e^{5x}$ 3) $2x-1 = e^{\ln x^2}$ 4) $9^x = 2e^{x^2}$
- 5) $5^x = 3^{2x-1}$ 6) $3^{x^2-4} = 27$ 7) $\log_8(x+5) - \log_8 2 = 1$
- 8) $10^{2x} + 3(10^x) - 10 = 0$ 9) $\log_2(\log_3 x) = -1$ 10) $e^x - e^{-x} = 1$
- 11) $\ln(-x) + \ln 3 = \ln(2x-15)$ 12) $\ln 5x - \ln(2x-1) = \ln 4$
- 13) $\log x = \sqrt{\log x}$

For problems 11 – 12, solve for t . Assume a and b are positive constants and k is nonzero.

- 14) $P = P_0 e^{kt}$ 15) $ae^{kt} = e^{bt}$, where $k \neq b$. 16) $r = p - k \ln t$

$$17) I = \frac{E}{R} \left(1 - e^{-\frac{Rt}{2}} \right)$$

Simplify the expressions in 13 – 16 completely.

- 18) $5e^{\ln(A^2)}$ 19) $\ln(e^{2ab})$ 20) $\ln\left(\frac{1}{e}\right) + \ln AB$ 21) $2\ln(e^A) + 3\ln B^e$

Convert the functions in 17 – 18 into the form $P = P_0 a^t$. Which represent exponential growth and which represent exponential decay?

- 22) $P = 2e^{-0.5t}$ 23) $P = 15e^{0.25t}$

Convert the functions in 19 – 20 into the form $P = P_0 e^{kt}$.

- 24) $P = 15(1.5)^t$ 25) $P = 4(0.55)^t$

26) Find the inverse of $f(t) = 50e^{0.1t}$.

27) Find the inverse of $f(t) = 1 + \ln t$.

Note: $P = P_0 e^{kt}$ models exponential growth ($k > 0$) or decay ($k < 0$).

k = growth constant

P_0 = initial population

28) The air in a factory is being filtered so that the quantity of pollutant, P (measured in mg/liter) is decreasing according to the equation $P = P_0 e^{-kt}$, where t represents time in hours. If 10% of the pollution is removed in the first five hours:

- a) What percentage of the pollution is left after 10 hours?
 b) How long will it take before the pollution is reduced by 50%?

29) If the size of a bacteria colony doubles in 5 hours, how long will it take for the number of bacteria to triple?

30) Suppose a certain radioactive substance has a half-life of 5 years. An object starts with 20 kg of the radioactive material.

- a) How much of the radioactive material is left after 10 years?
 b) The object can be moved safely when the quantity of the radioactive material is 0.1 kg or less. How much time must pass before the object can be moved?

32) You place \$800 in an account that earns 4% annual interest, compounded annually. How long will it be until you have \$2000

31) The number of bacteria present in a culture after t hours is given by the formula $N = 1000e^{0.69t}$.

- a) How many bacteria will be there after $\frac{1}{2}$ hour?
 b) How long will it be before there are 1,000,000 bacteria?
 c) What is the doubling time?

33) (Textbook 4.5 # 66) At the World Championship races held at Rome's Olympic Stadium in 1987, American sprinter Carl Lewis ran the 100-m race in 9.86 sec. His speed in meters per second after t seconds is closely

modeled by the function defined by $f(t) = 11.65 \left(1 - e^{-\frac{t}{1.27}} \right)$.

- a) How fast was he running as he crossed the finish line?
 b) After how many seconds was he running at the rate of 10 m per sec?

34) (Textbook 4.6 # 45) As age increases, so does the likelihood of coronary heart disease (CHD). The fraction

of people x years old with some CHD is modeled by $f(x) = \frac{0.9}{1 + 271e^{-0.122x}}$.

- a) Evaluate $f(25)$, $f(65)$. Interpret the results.
 b) At what age does this likelihood equal 50%?

35) (Textbook 4.6 #21) Find the doubling time of an investment earning 2.5% interest if interest is compounded continuously.

36) (Textbook 4.6 # 27) In 2000 India's population reached 1 billion, and in 2025 it is projected to be 1.4 billion.

- a) Find values for P_0 and a so that $f(x) = P_0 a^{x-2000}$ models the population of India in year x .
 b) Estimate India's population in 2010.
 c) When will India's population might reach 1.5 billion?

37) (Textbook 4.6 # 42) Assume the cost of a loaf of bread is #4. With continuous compounding, find the time it would take for the cost to triple at an annual inflation rate of 6%.

Answers:

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| | | 20) $-1 + \ln A + \ln B$ |
| 1) 0.515 | 10) $\ln \frac{1 + \sqrt{5}}{2}$ | 21) $2A + 3e \ln B$ |
| 2) -0.347 | 11) \emptyset | 22) $P = 2(0.61)^t$; decay; |
| 3) $x=1$ | 12) $4/3$ | 23) $15(1.284)^t$ |
| 4) 1.81, 0.38 | 13) 1,10 | 24) $15e^{0.41t}$ |
| 5) $\frac{\log_5 3}{2 \log_5 3 - 1}$ | 14) $\ln(P/P_0)/k$ | 25) $P = 4e^{-0.6t}$ |
| 6) $\pm\sqrt{7}$ | 16) $e^{\frac{p-r}{k}}$ | 26) $f^{-1}(t) = 10 \ln \left(\frac{t}{50} \right)$ |
| 7) 11 | 17) $-\frac{2}{R} \ln \left(1 - \frac{RI}{E} \right)$ | 27) $y = e^{t-1}$ |
| 8) $\log 2$ | 18) $5A^2$ | 28) a) 81%; b) 33 hours |
| 9) $\sqrt{3}$ | 19) $2ab$ | 29) 7.925 hours; |
| 30) a) 5 kg; b) 38.2 years; | 31) a) 1412 bacteria; b) 10 hours; c) 1 hour; | 32) 23.4 years |
| 33) a) 11.6451 m per sec; b) 2.4823 sec | 34) a) 0.065; 0.82; b) about 48 years. | 35) about 27.73 years |
| 36) a) 1 and $a=1.01355$; b) about 1.14 billion ; c) 2030. | | 37) 18.3 yr. |