3.5 Graphs of Rational Functions

Example 1 Graph the reciprocal function $f(x) = \frac{1}{x}$ Answer the following questions:

a) What is the domain of the function? ______________________________

b) What is the range of the function? ______________________________

c) What are the x-and y-intercepts? ______________________________

d) What is the end- behavior of the function, that is, what happens with the values of y as x goes to ∞ and $-\infty$?

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e) What is the behavior of the function when x approaches 0 ?

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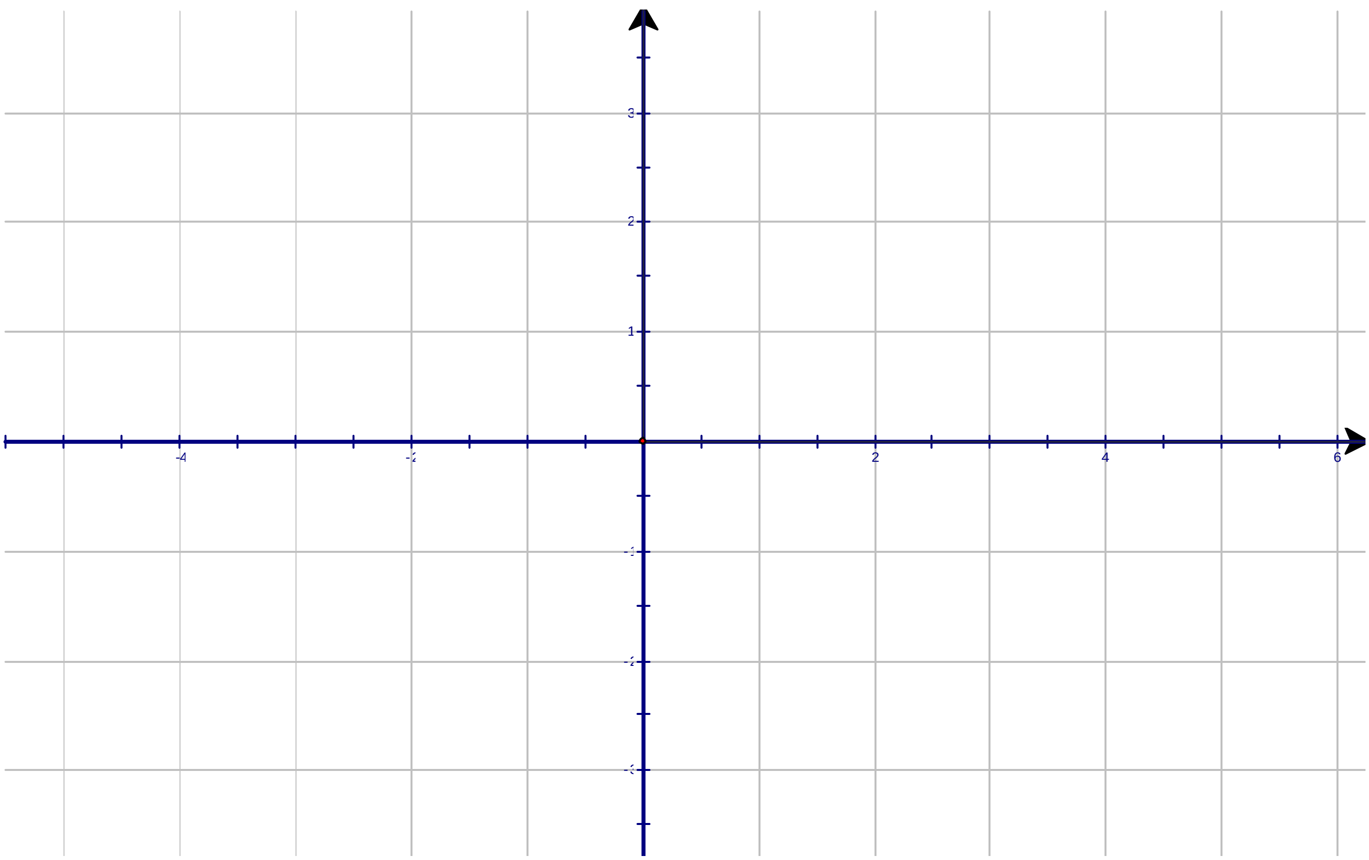
x	



Definition A **rational function** is a function f of the form $f(x)=\dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials, with $q(x)\neq 0$.

Notations:

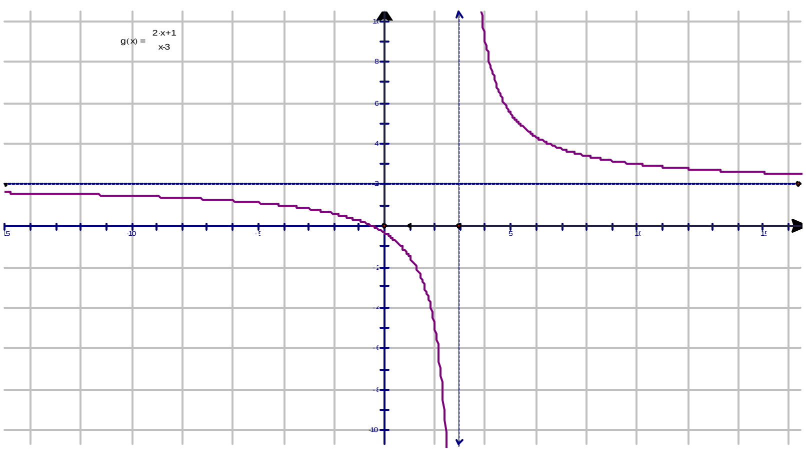
$x\to\infty$	x approaches infinity (x increases without bound)
$x\to-\infty$	x approaches negative infinity (x decreases without bound)
$x\to a^{+}$	x approaches a from the right
$x\to a^{-}$	x approaches a from the left

Definition The line $\boxed{x=a}$ is a **vertical asymptote** for the graph of $f(x)$ if, when $x\to a$, $y\to\pm\infty$.

The line $\boxed{y=b}$ is a **horizontal asymptote** for the graph of $f(x)$ if, when $x\to\pm\infty$, $y\to b$.

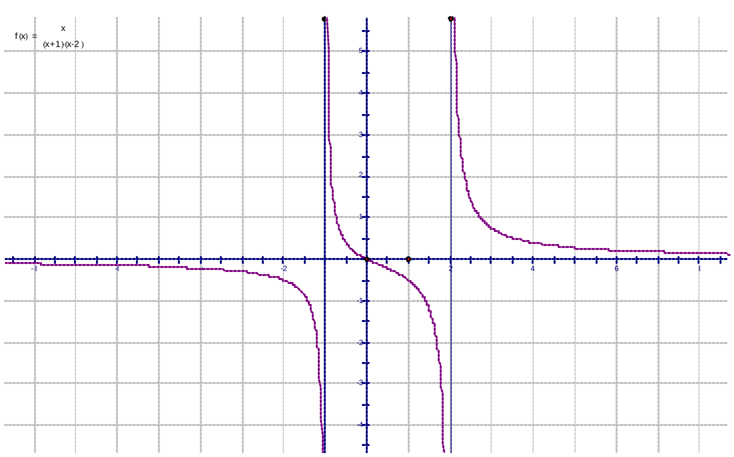
Exercise #1 Identify all the vertical and horizontal asymptotes of the following graphs.
How can the vertical asymptotes be found? What about the horizontal asymptotes?

a)



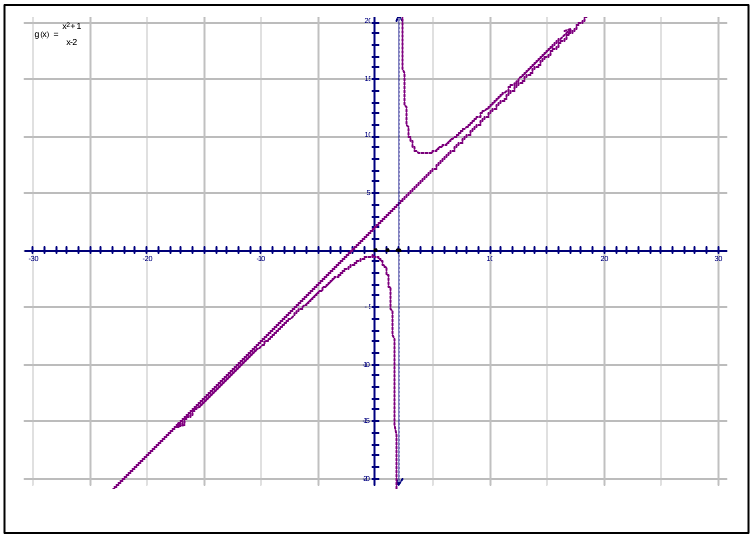
$$f(x)=\frac{2x+1}{x-3}$$

b)



$$f(x)=\frac{x}{(x+1)(x-2)}$$

c)



$$f(x) = \frac{x^2+1}{x-2}$$

Asymptotes for a rational function $f(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + \ldots + a_0}{b_m x^m + \ldots b_0}$

1. The vertical asymptotes are the lines $x = c$, where c is a zero of the denominator.

2. If $n < m$, then $y = 0$ (the x-axis) is the **horizontal asymptote.**

If $n = m$, then $y = \frac{a_n}{b_n}$ is the **horizontal asymptote.**

If $n > m$, there are **no horizontal asymptotes.**

If, however, $n = m+1$, then there is an oblique asymptote . Divide the numerator by the denominator and disregard the remainder.

y = quotient is the oblique asymptote

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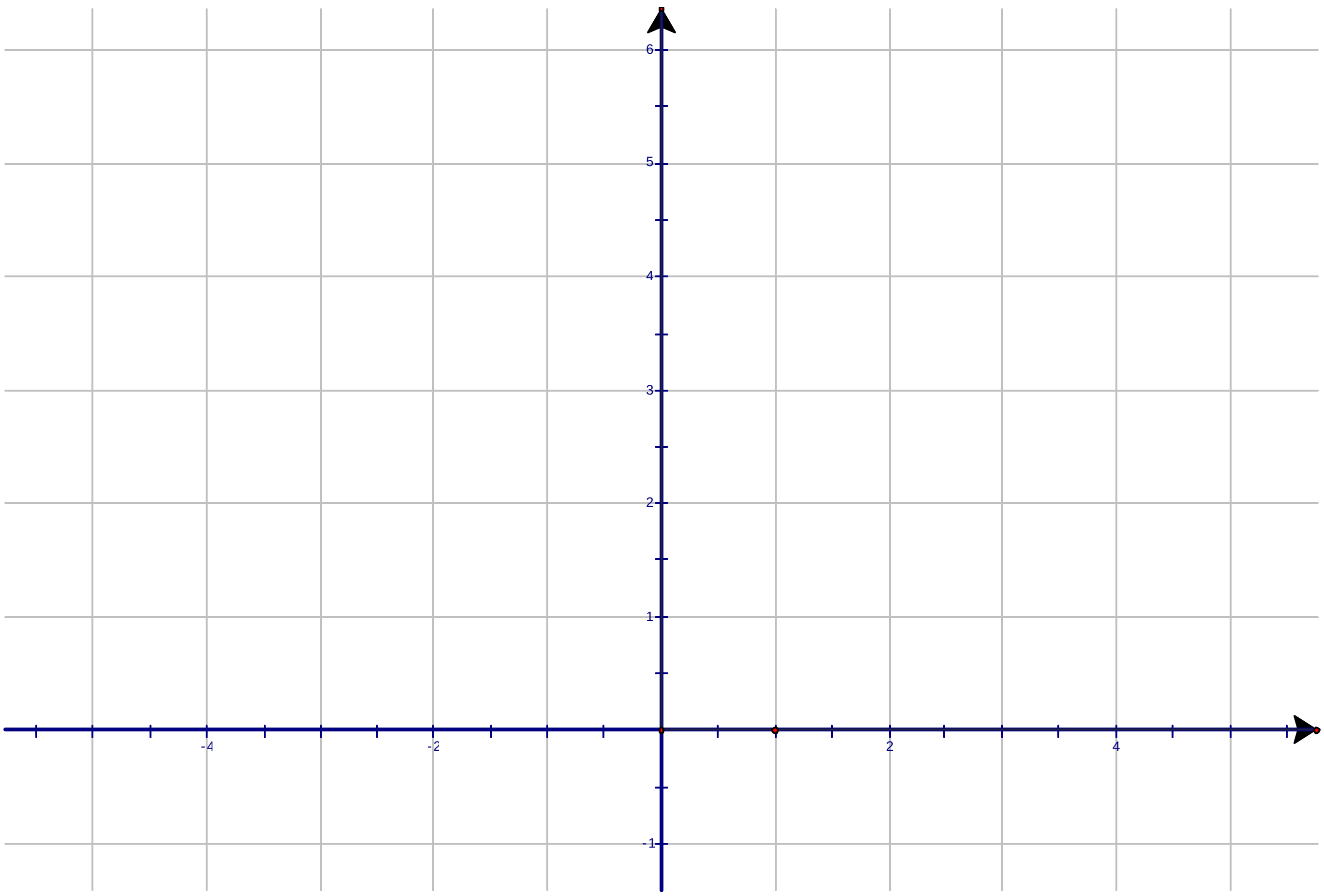
<u>Exercise #2</u> Identify all the asymptotes for the following functions:

$f(x) = \frac{2x+7}{x-5}$ | $g(x) = \frac{4x^2+x-5}{2x^2-3x-5}$ | $h(x) = \frac{x^2+6}{x-3}$ | $l(x) = \frac{1}{2x^2-2}$

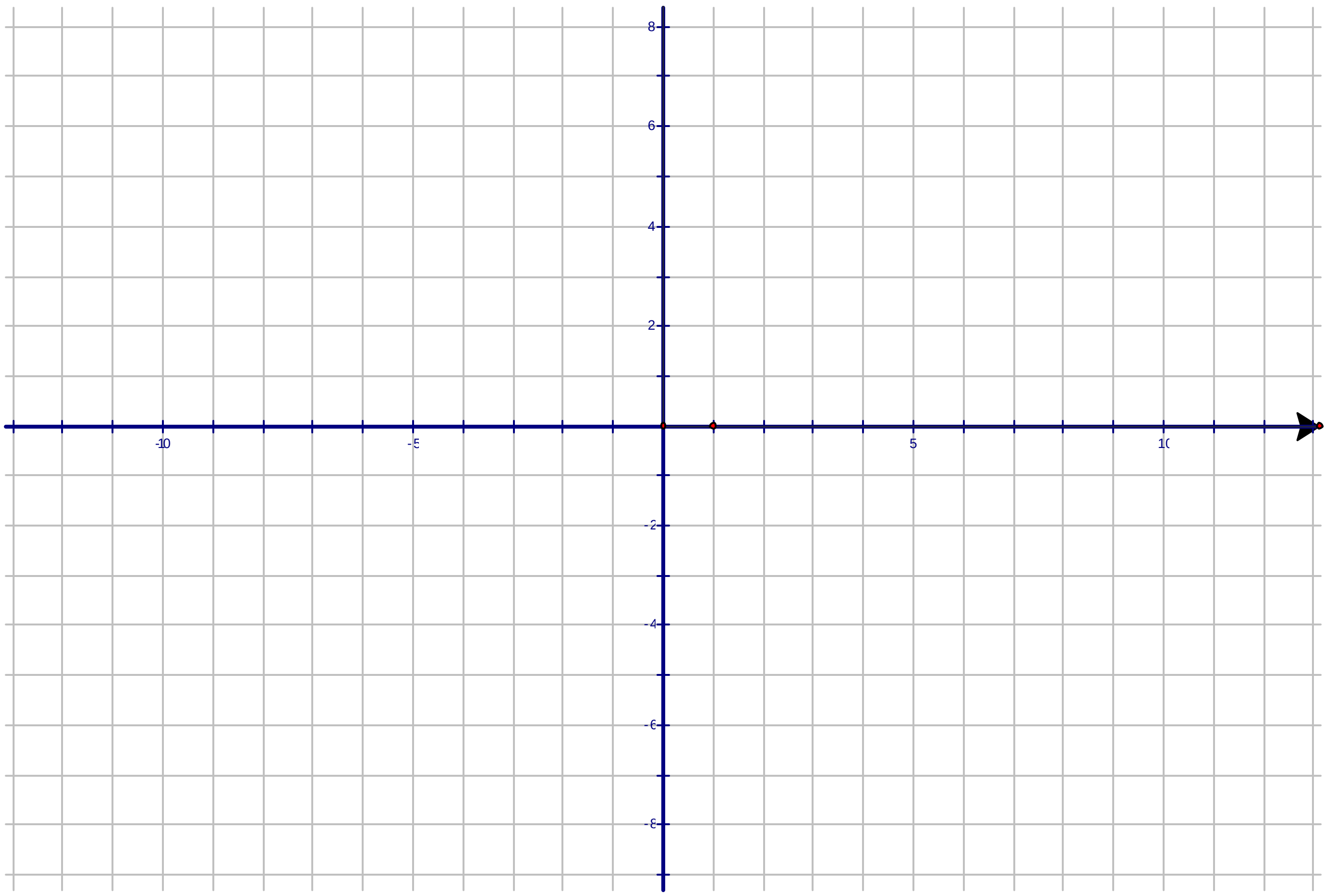
Exercise #3 Graph the function $f(x) = \frac{1}{x^2}$. Find the domain, the asymptotes, and the *x*- and *y*-intercepts.



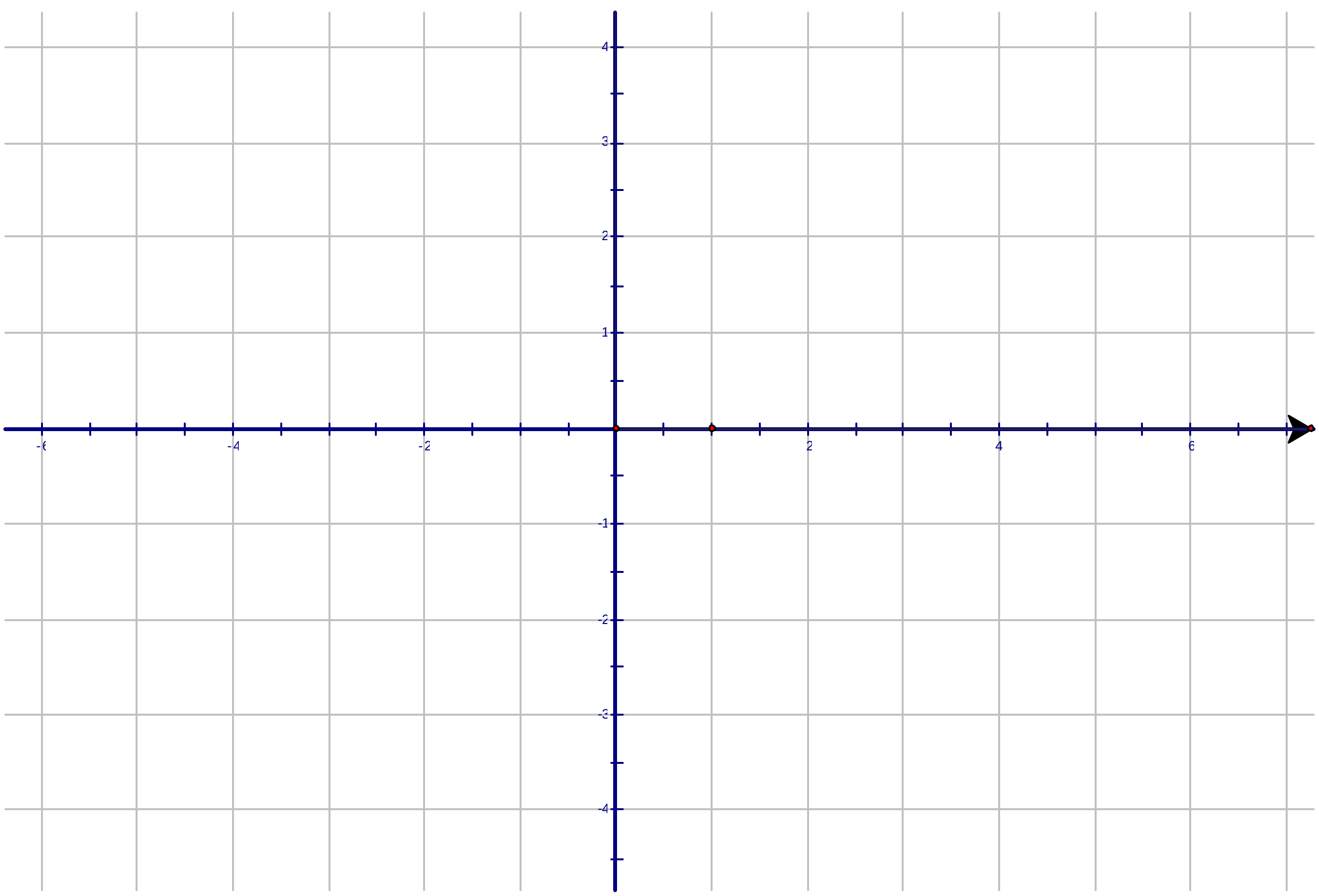
Exercise #4 Show how to obtain the graph of $g(x) = \frac{1}{(x+1)^2} + 1$ from the graph of $f(x) = \frac{1}{x^2}$.

What are the asymptotes of $g(x)$?

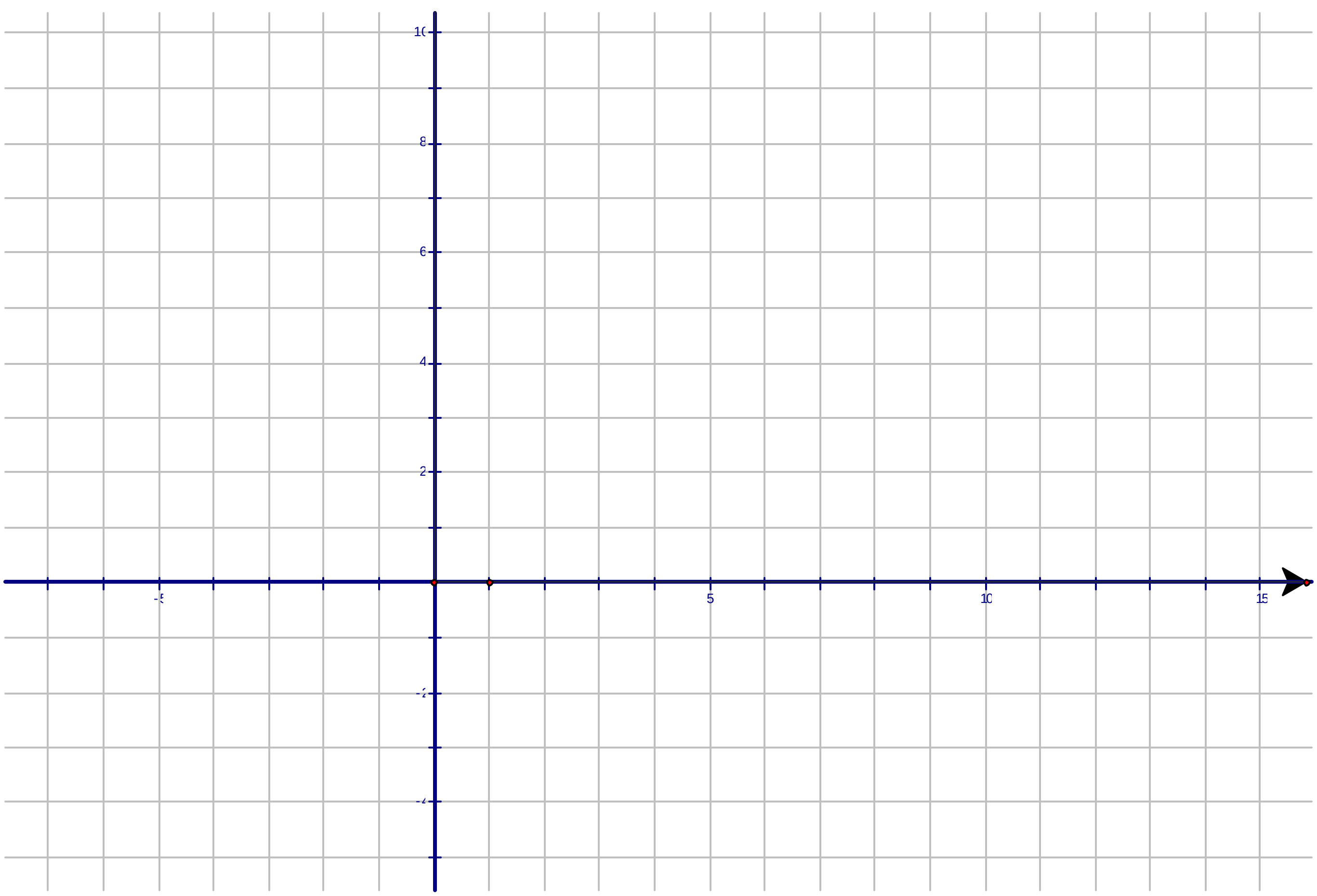
Exercise #5 Sketch the graph of $f\left(x\right)=\dfrac{x+1}{x-4}$. Find the domain, all the asymptotes, the *x*- and *y*-intercepts Determine if the graph intersects its nonvertical asymptote. Plot additional test points, as needed.



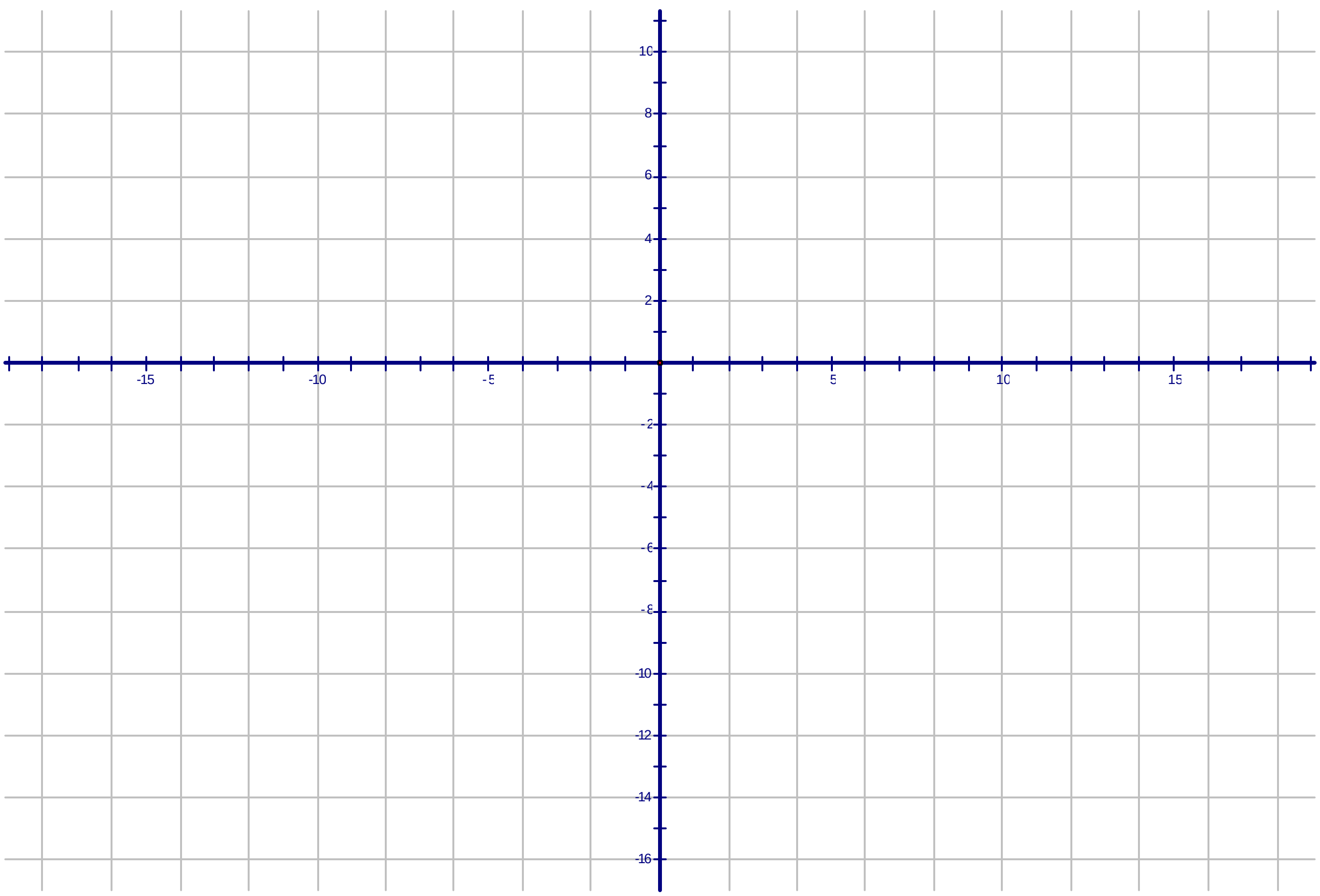
<u>Exercise #6</u> Sketch the graph of $f(x) = \dfrac{x-2}{x^2-1}$. Find the domain, all the asymptotes, the *x*- and *y*-intercepts Determine if the graph intersects its nonvertical asymptote. Plot additional test points, as needed.



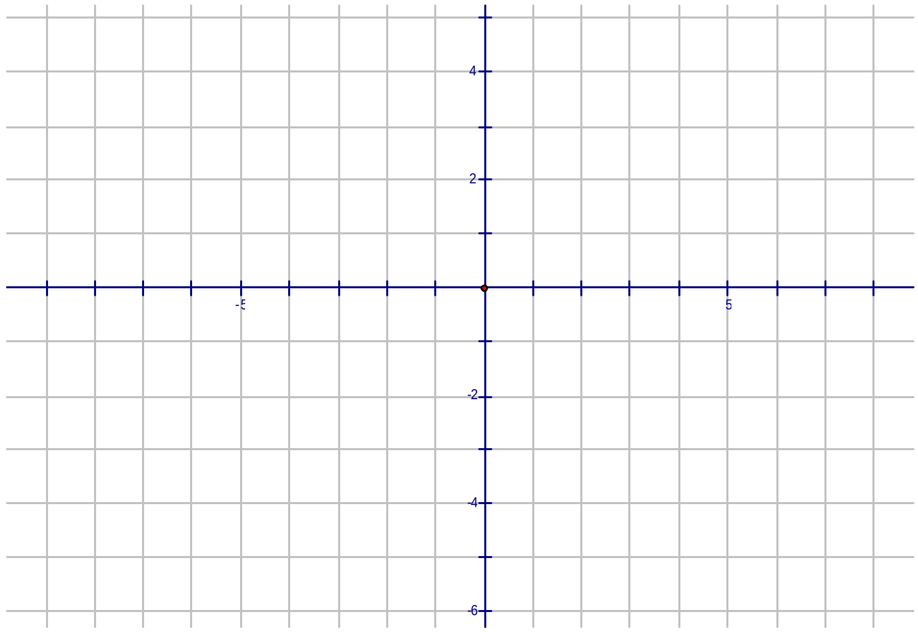
Exercise #7 Sketch the graph of $f\left(x\right)=\dfrac{x^2-2x-8}{x^2-4x+3}$. Find the domain, all the asymptotes, the *x*- and *y*-intercepts Determine if the graph intersects its nonvertical asymptote. Plot additional test points, as needed.

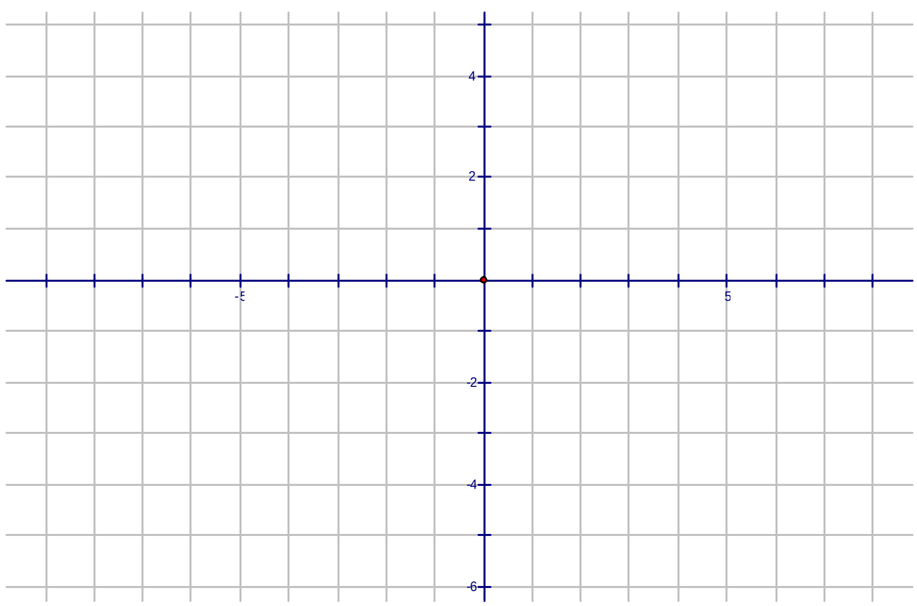


Exercise #8 Sketch the graph of $f\left(x\right)=\dfrac{x^2+1}{x+3}$. Find the domain, all the asymptotes, the *x*- and *y*-intercepts Determine if the graph intersects its nonvertical asymptote. Plot additional test points, as needed.

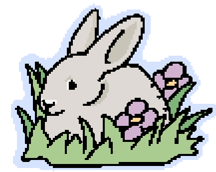


Exercise #9 Graph the following functions: $f(x) = \frac{x+2}{x+2}$ and $g(x) = \frac{x^2-9}{x+3}$.





3.5 Graphs of Rational Functions - Applications



1. The rabbit population on Mr. Jenkins's farm follows the formula

$$p(t) = \frac{3000t}{t+1}$$

where $t \geq 0$ is the time (in months) since the beginning of the year.

a) Sketch a graph of the rabbit population.

b) What eventually happens to the rabbit population?

2. Using rational functions to model bacterial growth

A group of agricultural scientists has been studying how the growth of a particular type of bacteria is affected by the acidity level of the soil. One colony of the bacteria is placed in a soil that is slightly acidic. A second colony of the same size is placed in a neutral soil. Suppose that after analyzing the data, the scientists determine that the size of each population over time can be modeled by the following functions.

Colony of neutral soil: $y = \frac{2t+1}{t+1}, t \geq 0$

Colony of acidic soil: $y = \frac{4t+3}{t^2+3}, t \geq 0$

In both cases, y represents the population, in thousands, after t hours.

a) What is the initial population for each colony?

b) Determine the long-term behavior of each colony.

3. Electrical Resistance

When two resistors with resistances R_1 and R_2 are connected in parallel, their combined resistance R is given by the formula

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Suppose that a fixed 8-ohm resistor is connected in parallel with a variable resistor. If the resistance of the variable resistor is denoted by x, then the combined resistance R is a function of x. Graph R and give a physical interpretation of the graph.