VERTICAL SHIFTING (TRANSLATION)

<u>Example #1</u>

Use the graph of

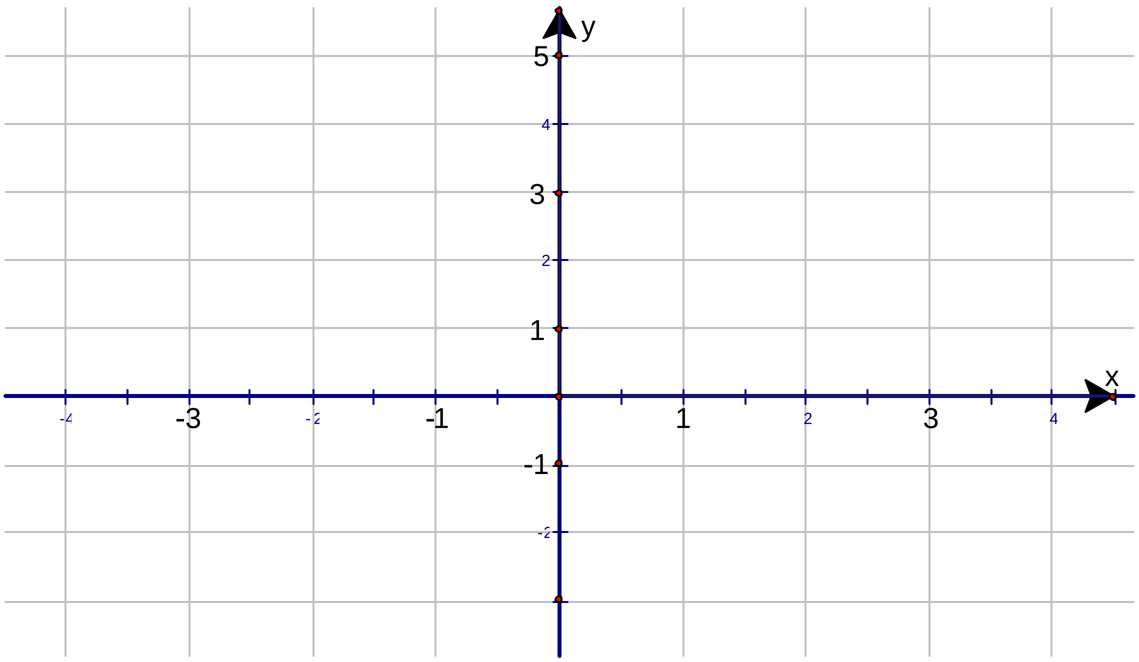
$f(x) = x^2$

to obtain the graphs of

$g(x) = x^2 + 1$

and

$h(x) = x^2 - 2$.



x	$f(x) = x^2$	$g(x) = x^2 + 1$	$h(x) = x^2 - 2$
-2			
-1			
0			
1			
2			

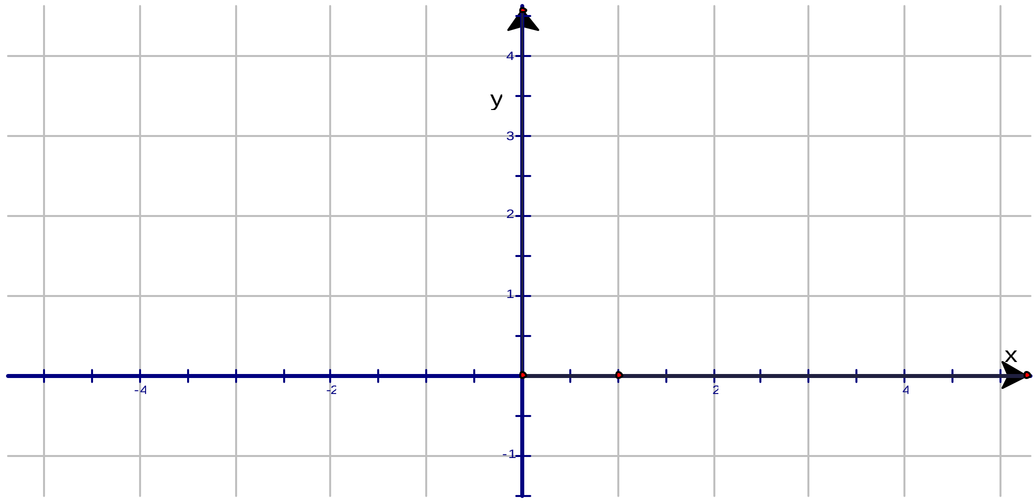
VERTICAL SHIFTING : A vertical shifting does not change the shape of the graph but simply translates it to another position in the plane.

Equation	How to obtain the graph	Example
$y = f(x) + k$ $k > 0$	Shift graph of $y = f(x)$ upward k units.	$g(x) = x^2 + 1$
$y = f(x) - k$ $k > 0$	Shift graph of $y = f(x)$ downward k units.	$h(x) = x^2 - 2$

HORIZONTAL SHIFTING (TRANSLATION)

Example #2

Use the graph of
$f(x) = x^2$
to obtain the graphs of
$g(x) = (x-1)^2$
and
$h(x) = (x+1)^2$.



x	$f(x) = x^2$	$g(x) = (x-1)^2$	$h(x) = (x+1)^2$
-2			
-1			
0			
1			
2			

HORIZONTAL SHIFTING : A horizontal shifting doesn't change the shape of the graph but simply translates it to another position in the plane.

Equation	**How to obtain the graph**	**Example**
$y = f(x-h)$ $h > 0$	Shift graph of $y = f(x)$ to the right h units.	$g(x) = (x-1)^2$
$y = f(x+h)$ $h > 0$	Shift graph of $y = f(x)$ to the left h units.	$h(x) = (x+1)^2$

VERTICAL STRETCH AND COMPRESSION

Example #3 .

Use the graph of

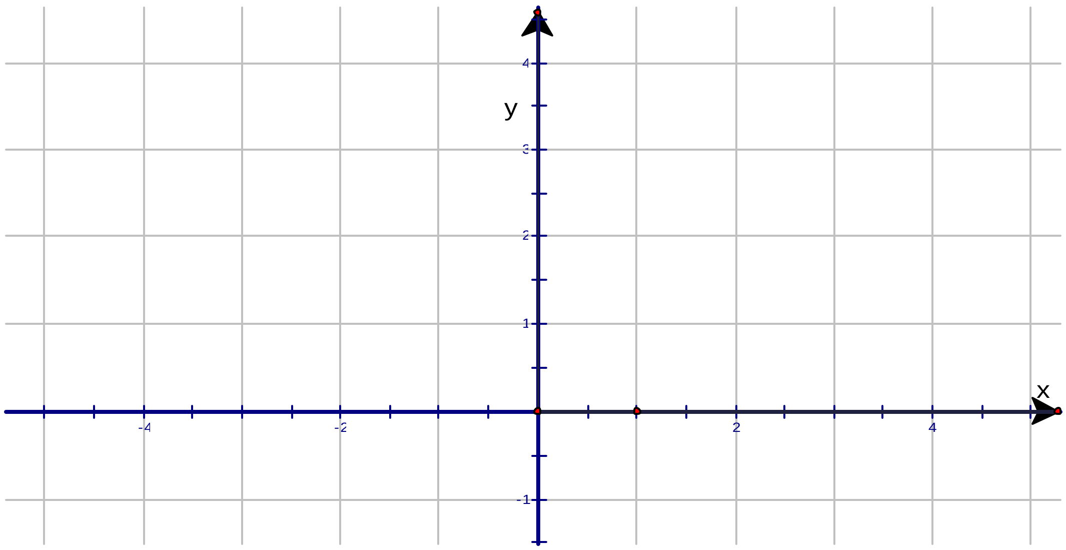
$f(x) = |x|$

to obtain the graphs of

$g(x) = 2|x|$

and

$h(x) = \frac{1}{2}|x|$



x	$f(x) = \lvert x \rvert$	$g(x) = 2\lvert x \rvert$	$h(x) = \frac{1}{2}\lvert x \rvert$
-2			
-1			
0			
1			
2			

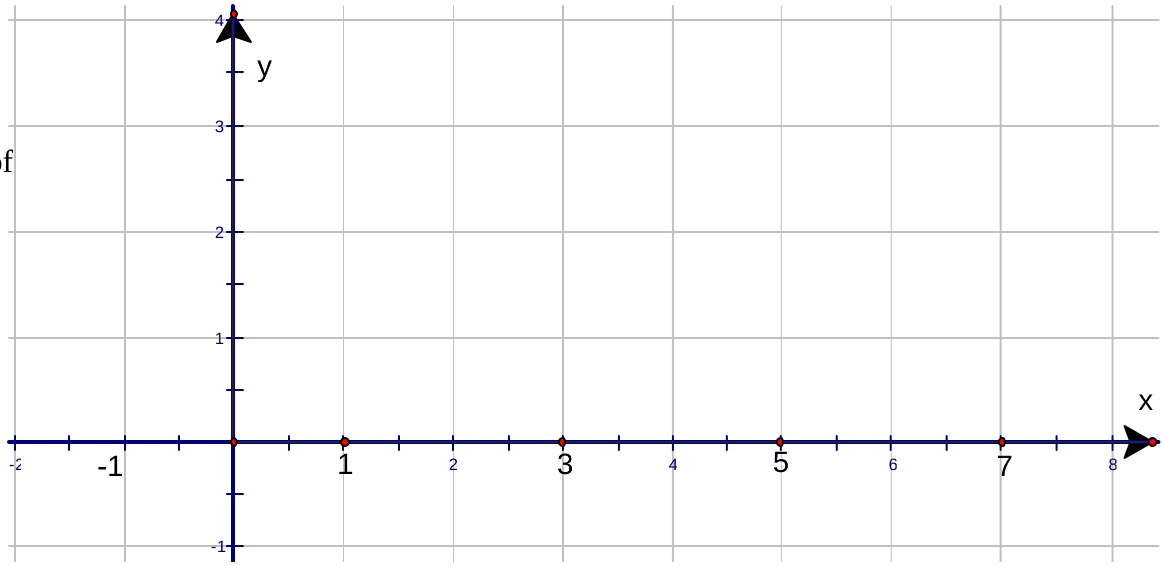
VERTICAL STRETCH AND COMPRESSION

Equation	**How to obtain the graph**	**Example**
$y = af(x)$ $a > 1$	Stretch the graph of $y = f(x)$ vertically by a factor of a.	$g(x) = 2\lvert x \rvert$
$y = af(x)$ $0 < a < 1$	Compress the graph of $y = f(x)$ vertically by a factor of $\frac{1}{a}$.	$h(x) = \frac{1}{2}\lvert x \rvert$

HORIZONTAL COMPRESSION AND STRETCH

<u>Example #4</u> .

Use the graph of
$f(x) = \sqrt{x}$
to obtain the graphs of
$g(x) = \sqrt{2x}$
and
$h(x) = \sqrt{\frac{1}{2}x}$.



x	$f(x) = \sqrt{x}$	$g(x) = \sqrt{2x}$	$h(x) = \sqrt{\frac{1}{2}x}$
0			
1			
4			
9			

Equation	**How to obtain the graph**	**Example**
$y = f(ax)$ $a > 1$	Compress the graph of $y = f(x)$ horizontally by a factor of a .	$g(x) = \sqrt{2x}$
$y = f(ax)$ $0 < a < 1$	Stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{a}$.	$h(x) = \sqrt{\frac{1}{2}x}$

REFLECTION ABOUT THE AXES

Example #5

Use the graph of

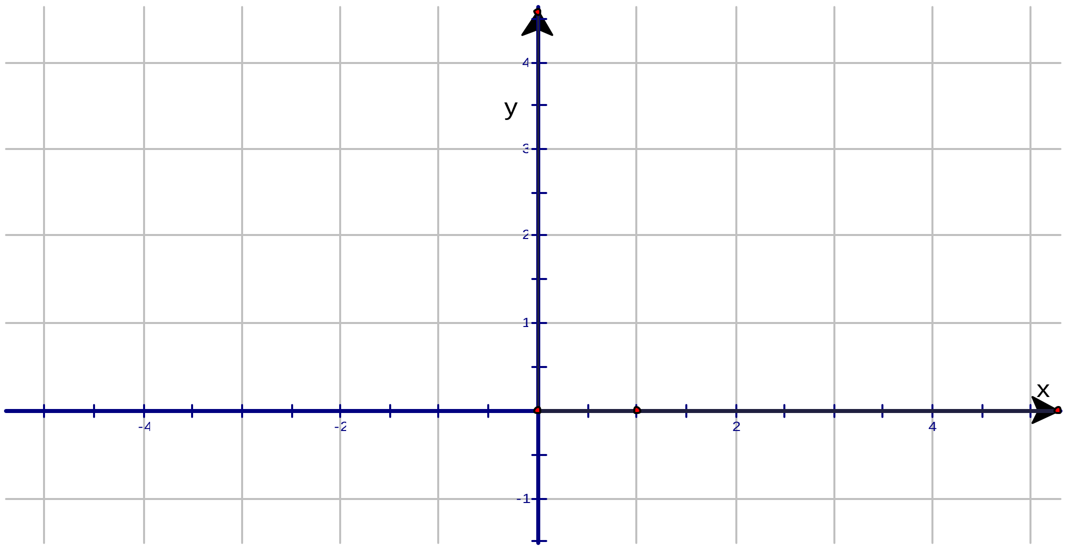
$f(x) = \sqrt{x}$

to obtain the graphs of

$g(x) = -\sqrt{x}$

and

$h(x) = \sqrt{-x}$



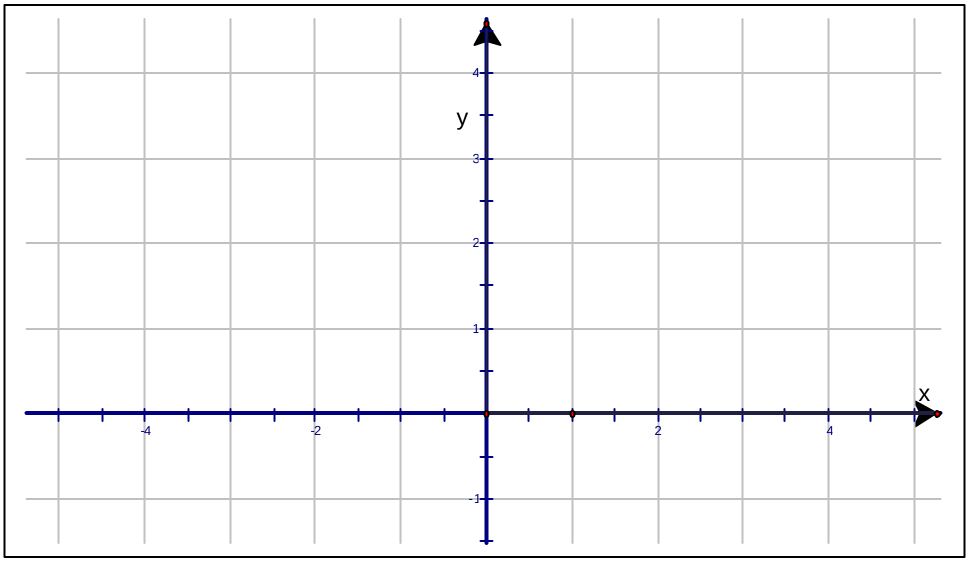
x	$f(x) = \sqrt{x}$	$g(x) = -\sqrt{x}$	$h(x) = \sqrt{-x}$
-4			
-1			
0			
1			
4			

REFLECTION ABOUT THE AXES

Equation	**How to obtain the graph**	**Example**
$y = -f(x)$	Reflect the graph of $y = f(x)$ about the x-axis.	$g(x) = -\sqrt{x}$
$y = f(-x)$	Reflect the graph of $y = f(x)$ about the y-axis.	$h(x) = \sqrt{-x}$

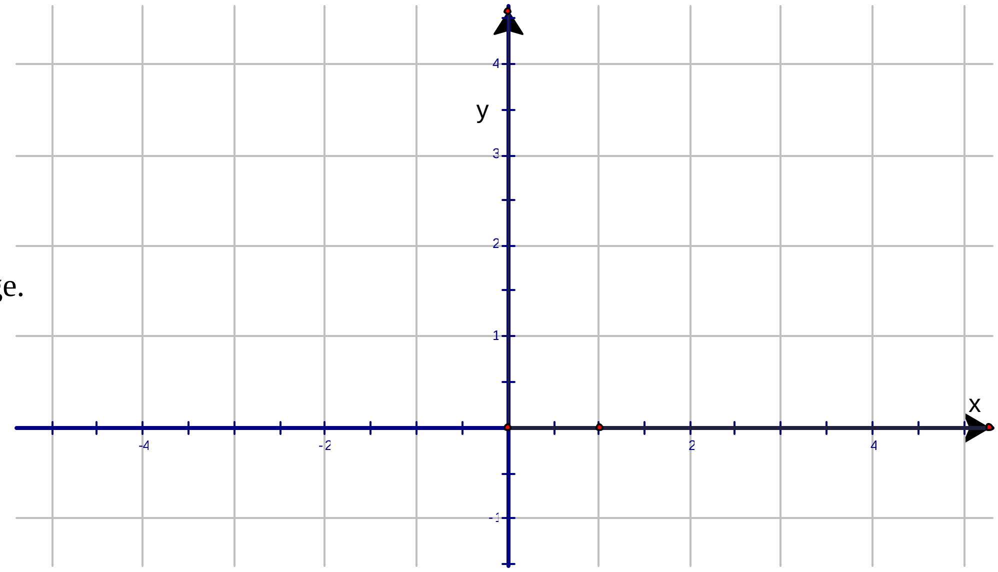
Exercise #1 .

a) Use the graph of
$f(x) = |x|$
to obtain the graph of
$g(x) = |x| + 2$.
b) Find the domain and range.



Exercise #2 .

a) Use the graph of
$f(x) = \sqrt{x}$
to obtain the graph of
$g(x) = \sqrt{x-3}$.
b) Find the domain and range.



Exercise #3 Find the function that is finally graphed after the following transformations are applied to the graph of a) $f(x) = \sqrt{x}$; b) $g(x) = x^3$.

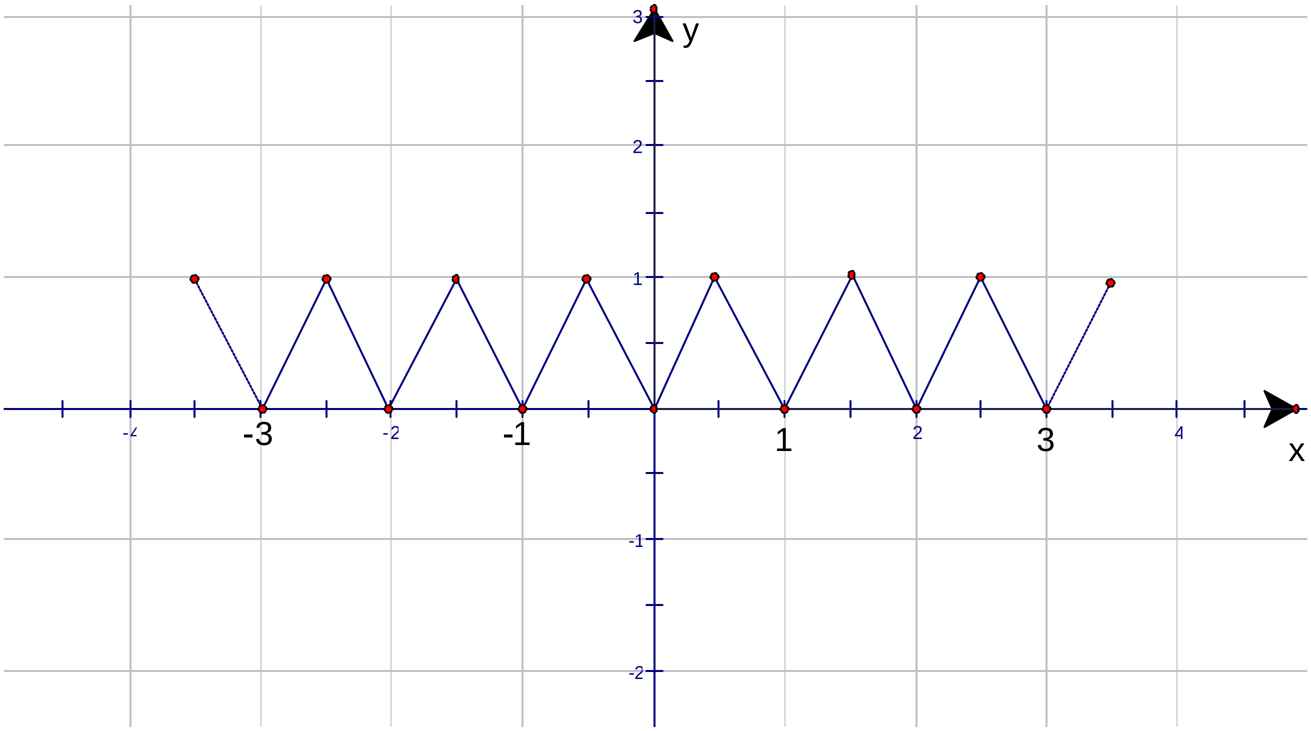
1) Shift left 3 units

2) Shift up 1 unit.

Exercise #4 The graph of $y = f(x)$ is shown. Sketch the graph of each function:

a) $y = f(x+1)$ b) $y = f(x)-2$. c) $y = -f(x)$

d) $y = f(-x)$



Exercise #5 Suppose the point (8,12) is on the graph of $y = f(x)$. Find a point on the graph of each function.

a) $y = f(x+4)$

c) $y = \frac{1}{4}f(x)$

b) $y = f(x)+4$

d) $y = 4f(x)$

Exercise #6 Graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all steps (equations and meaning).

a) $f(x) = 2(x-2)^2 - 4$

b) $g(x) = -|x+3| + 2$

c) $h(x) = 3\sqrt{-x+2} - 1$

More piecewise-defined functions (2.7)

Exercise #7 Let $f(x) = \begin{cases} \sqrt{x+4}, & \text{if } -4 \le x \le 0 \\ |x-2|, & \text{if } 0 < x \le 7 \\ 1, & \text{if } x > 7 \end{cases}$ a piecewise-defined function

a. Graph the function.
b. Identify the domain and range.
c. Identify the intervals on which the function is increasing, decreasing, constant.

Exercise #8 Let $f(x) = \begin{cases} -(x-1)^2 + 5, & \text{if } -1 < x \le 4 \\ 2x - 12, & \text{if } x > 4 \end{cases}$ a piecewise-defined function

d. Graph the function.
e. Identify the domain and range.
f. Identify the intervals on which the function is increasing, decreasing, constant.