2.3 Functions

The word *function*, used casually, expresses the notion of dependence.

- For example, a person might say that election results are a function of the economy, meaning that the winner of an election is determined by how the economy is doing.
- Another person may claim that car sales are a function of the weather, meaning that the number of cars sold on a given day is affected by the weather.

In nearly every physical phenomenon we observe that one quantity depends on another. For example, your height depends on your age, the temperature depends on the date, the cost of mailing a package depends on its weight. We use the term function to describe this dependence of one quantity on another. That is, we say:

- Height is a function of age.
- Temperature is a function of date.
- Cost of mailing a package is a function of weight.

Definition of Function

A function is a rule. To talk about a function , we need to give it a name. We will use letters such as *f, g, h,…* to represent functions.
For examples, we can use the letter *f* to represent a rule as follows:
"*f*" is the rule "square the number"
When we write $f(2)$, we mean "apply the rule *f* to the number 2". Applying the rule gives
$f(2)=2^2=4$. Similarly, $f(3)=3^2=9$, and in general $f(x)=x^2$.

<u>Definition 1</u>

> **A function** is a rule that assigns to each element x in a set A exactly one element , called $f(x)$, in a set B.

<u>Notes</u>:

- We usually consider functions for which the sets A and B are sets of real numbers.
- The symbol $f(x)$ is read " f of x" or "f at x" and is called the **value of *f* at *x***, or the **image of *x* under *f*.**
- The set **A** is called the **domain** of the function.
- The **range** of a function *f* is the set of all possible values of $f(x)$ as x varies throughout the domain.

Definition 2

A function is a relationship between two variables: **independent variable (input)** and **dependent variable (output)** that assigns to each independent variable a unique value of the dependent variable.

Domain and Range

If $y = f(x)$, then

- the **domain** of f is the set of values for the independent variable, x

$$D_f = \{x \mid f(x) \in \mathbb{R}\}$$

- the **range** of f is the set of values for the dependent variable, y

$$R_f = \{y \mid y = f(x),\ x \in D_f\}$$

Functions Defined by Tables

Exercise 1 a) Is C a function of M? Explain.

b) Is M a function of C? Explain.

c) If $C = f(M)$, find $f(3)$. What does it mean?

d) Solve $f(M) = 7.95$. What does it mean?

TABLE 2

Cost of merchandise (M)	**Shipping Charges (C)**
\$0.01 – 10.00	\$2.50
10.01 – 20.00	3.75
20.01 – 30.00	4.85
30.01 – 50.00	5.95
50.01 – 75.00	7.95
Over 75.00	8.95

Exercise 2 Tables 2a, 2b, and 2c represent the relationship between the button number, *N*, which you push, and the snack, *S*, delivered by three different vending machines.

Table 2a
Vending Machine #1

N	***S***
1	m&ms
2	pretzels
3	dried fruit
4	Hersheys
5	fat free cookies
6	Snickers

Table 2b
Vending Machine #2

N	***S***
1	m&ms or dried fruit
2	Pretzels or Hersheys
3	Snickers or fat free cookies

Table 2c
Vending Machine #3

N	***S***
1	m&ms
2	m&ms
3	dried fruit
4	Hersheys
5	Hersheys
6	fat free cookies
7	Snickers
8	Snickers

One of these vending machines is not a good one to use, because *S* is not a function of *N*. Which one? Explain why this makes it a bad machine to use.

Functions Defined By Graphs

How to Tell if a Graph Represents a Function: the Vertical Line Test

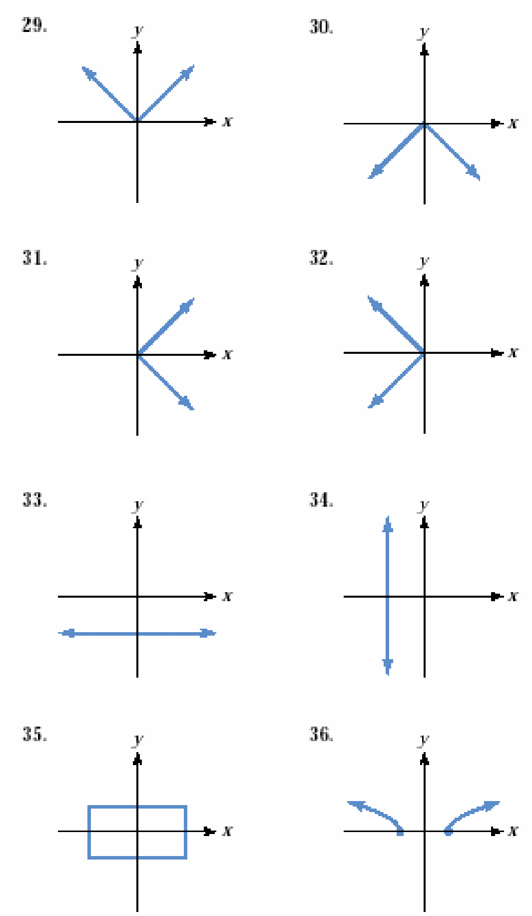
In general, for y to be a function of x, each value of x must be associated with exactly one value of y. Let us think of what this requirement means graphically. In order for a graph to represent a function, each x-value must correspond to exactly one y-value. This means that the graph must not intersect any vertical line at more than one point. Otherwise, the curve would contain two points with different y-values but the same x-value.

The vertical line test:

A graph represents a function if and only if any vertical line intersects the graph in at most one point.

Exercise 3

Use the vertical line test to identify the graphs in which y is a function of x. For those graph, identify the domain and range.



Exercise 4

The graph shows the daily megawatts of electricity used on a record-breaking summer day in Sacramento.

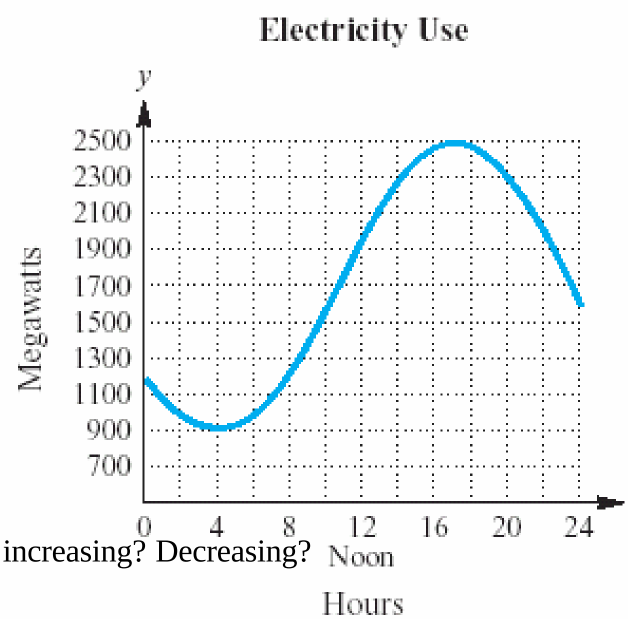
a) Is this the graph of a function?

b) What is the domain? What is the range?

c) Estimate the number of megawatts used at 8am.

d) At what time was the most electricity used? the least?

e) Call this function *f*. What is $f(12)$?
What does it mean?

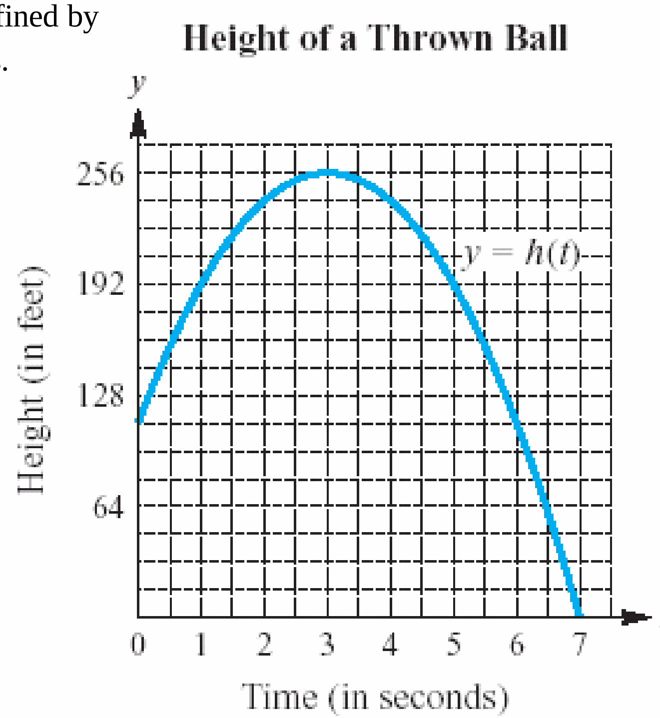


f) During what time intervals is electricity usage increasing? Decreasing?

Source: Sacramento Municipal Utility District.

Exercise 5

A ball is thrown straight up into the air. The function defined by $y = h(t)$ gives the height of the ball (in feet) at t seconds.



a) What is the height of the ball after 2 seconds?

b) When will the height be 192 feet?

c) During what time intervals is the ball going up? Down?

d) How high does the ball go, and when does the ball reach
its maximum height?

e) At how many seconds does the ball hit the ground?

Increasing, Decreasing, and Constant Functions

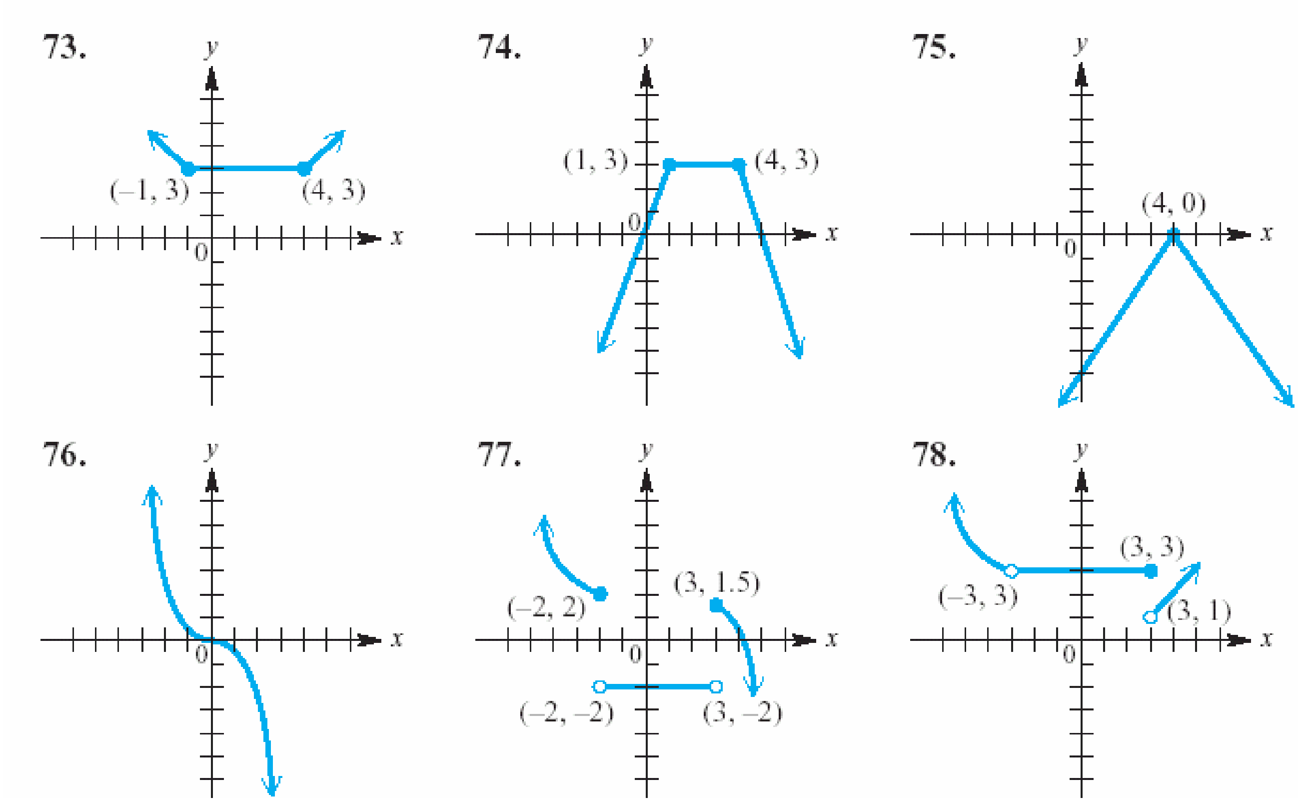
Suppose that a function f is defined over an interval I. If x_1 and x_2 are in I,

a) f **increases** on I if, whenever $x_1 < x_2,\ f(x_1) < f(x_2)$;
b) f **decreases** on I if, whenever $x_1 < x_2,\ f(x_1) > f(x_2)$;
c) f **is constant** on I if, for every x_1 and x_2, $f(x_1) = f(x_2)$.

Exercise 6

a) Which graph represents a function? Explain.
b) Find the domain and range of each relation.
c) Determine the intervals of the domain for which each function is increasing, decreasing, and constant.
d) Identify the intercepts of each graph.
e) Assuming each graph represents $y = f(x)$, solve the following inequalities for each function:

$f(x) \geq 0,\ f(x) < 0$



Exercise 7 a) Label the axes for a sketch of a problem, which says, “Sketch a graph of the cost of manufacturing q items…”

b) Label the axes for a problem, which says, “Graph the pressure, p, of a gas as a function of its volume, v, where p is in pounds per square inch and v is in cubic inches.”

d) Label the axes for a problem which asks you to “Graph D in terms of y…”

Functions Defined by Equations

Exercise 8 Recall from geometry that if we know the radius of a circle, we can find its area. If we let $A = q(r)$ represent the area of a circle as a function of its radius, then a formula for $q(r)$ is

$$A = q(r) = p r^2.$$

Use the above formula, where r is in cm, to evaluate $q(10)$ and $q(20)$. Explain what your results tell you about circles.

Exercise 9 Let $f(x) = \frac{-7}{x-13}$. Answer the following:

a) Is y a function of x? Why?

b) Find the domain and the range.

c) Find $f(0)$, $f(2)$, $f(-x)$, $f(a+h)$

Exercise 10 Suppose $v(t) = t^2 - 2t$ gives the velocity, in ft/sec, of an object at time t, in seconds.

a) Is v a function of t? Explain.

b) Which variable is independent and which one is dependent?

c) What is $v(0)$ and what does it represent?

d) What is $v(3)$ and what does it represent?

Exercise 11

$y=\sqrt{4x+2}$ $\qquad$ $y=-\sqrt{x}$ $\qquad$ $y=-6x+8$ $\qquad$ $x+y<4$

Answer the following:

a) Decide whether each relation defines y as a function of x.

b) Give the domain and the range.

c) Rewrite the equations (when possible) using function notation.

d) Find $f(-x)$ and $f(2x)$ for each function.

Average Rate of Change of a Function

<u>Definition 3</u>

For a function *f* that is smooth and continuous on an interval containing *a* and *b*, the **average rate of change between *a* and *b*** is

$$\frac{f(b)-f(a)}{b-a}\text{, where } a\neq b.$$

<u>Definition 4</u>

For a function $f(x)$ that is smooth and continuous on the interval containing x and $x+h$ where $h\neq 0$ (constant), the **difference quotient for *f*** is defined as

$$\frac{f(x+h)-f(x)}{h}$$

Exercise 12 Compute and simplify the difference quotient for each function given.

a) $f(x)=2x-3$

b) $g(x)=x^2+3$

c) $h(x)=\dfrac{2}{x}$

Exercise 13 If an arrow is shot vertically from a bow with an initial speed of 192 ft/sec, the height of the arrow can be modeled by the function $h(t)=-16t^2+192t$, where $h(t)$ represents the height of the arrow after t sec (assume the arrow was shot from the ground level).

a) What is the arrow's height at $t=1$ sec?

b) What is the arrow's height at $t=2$ sec?

c) What is the average rate of change from $t=1$ to $t=2$?

d) What is the average rate of change from $t=10$ to $t=11$?