2.2 Functions

The word *function*, used casually, expresses the notion of dependence. For example, a person might say that election results are a function of the economy, meaning that the winner of an election is determined by how the economy is doing. Another person may claim that car sales are a function of the weather, meaning that the number of cars sold on a given day is affected by the weather.

In mathematics, the meaning of the word *function* is more precise, but the basic idea is the same.

Definition 1

A function is a relationship between two quantities. If the value of one quantity uniquely determines the value of the second quantity, we say the second quantity is a function of the first.

Example 1 In the early 1980s, the recording industry introduced the compact disc (CD) as an alternative to vinyl long playing records (LPs). Table 1 gives the number of units (in millions) of CDs sold for the years 1982 through 1987. The year uniquely determines the number of CDs sold. Thus, we say that the number of CDs sold is a function of the year. We may also say that the number of CDs sold depends on the year.

TABLE 1
Millions of CDs sold, by year

Year	**Sales (millions)**
1982	0
1983	0.8
1984	5.8
1985	23
1986	53
1987	102

Note: The quantities described by a function are called *variables.*

Definition 2

A function is a relationship between two variables: **independent variable (input)** and **dependent variable (output)** that assigns to each independent variable a unique value of the dependent variable.

Function Notation

There is a convenient notation we use when discussing functions. First, we choose a name for the particular function, let's say f. That is, f is the name of the relationship between the two variables. Let's say t (the year) and S (the CD's sold) from Example 1. Then we can write

$$S = f(t)$$

which means "S is a function of t, and f is the name of the function."

Domain and Range

If $y = f(x)$, then

- the **domain** of f is the set of values for the independent variable, x

$$D_f = \{x \mid f(x) \in \mathbb{R}\}$$

- the **range** of f is the set of values for the dependent variable, y

$$R_f = \{y \mid y = f(x),\ x \in D_f\}$$

Functions Defined by Tables

Exercise 1 a) Is C a function of M? Explain.

b) Is M a function of C? Explain.

c) If $C = f(M)$, find $f(3)$. What does it mean?

d) Solve $f(M) = 6.95$. What does it mean?

TABLE 2
Millions of CDs sold, by year

Cost of merchandise (M)	**Shipping Charges (C)**
\$0.01 – 10.00	\$2.50
10.01 – 20.00	3.75
20.01 – 30.00	4.85
30.01 – 50.00	5.95
50.01 – 75.00	7.95
Over 75.00	8.95

Exercise 2 Tables 2a, 2b, and 2c represent the relationship between the button number, N, which you push, and the snack, S, delivered by three different vending machines.

Table 2a
Vending Machine #1

N	***S***
1	m&ms
2	pretzels
3	dried fruit
4	Hersheys
5	fat free cookies
6	Snickers

Table 2b
Vending Machine #2

N	***S***
1	m&ms or dried fruit
2	Pretzels or Hersheys
3	Snickers or fat free cookies

Table 2c
Vending Machine #3

N	***S***
1	m&ms
2	m&ms
3	dried fruit
4	Hersheys
5	Hersheys
6	fat free cookies
7	Snickers
8	Snickers

One of these vending machines is not a good one to use, because S is not a function of N. Which one? Explain why this makes it a bad machine to use.

Functions Defined By Graphs

How to Tell if a Graph Represents a Function: the Vertical Line Test

In general, for y to be a function of x, each value of x must be associated with exactly one value of y. Let us think of what this requirement means graphically. In order for a graph to represent a function, each x-value must correspond to exactly one y-value. This means that the graph must not intersect any vertical line at more than one point. Otherwise, the curve would contain two points with different y-values but the same x-value.

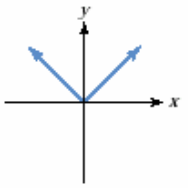
The vertical line test:

If any vertical line intersects a graph in more than one point, then the graph does not represent a function.

Exercise 3

Use the vertical line test to identify the graphs in which y is a function of x. For those graph, identify the domain and range.

29.



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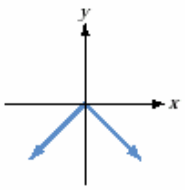

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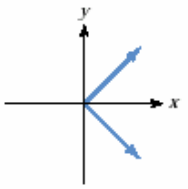

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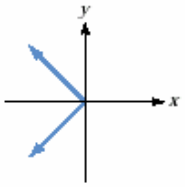

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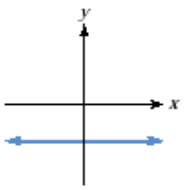

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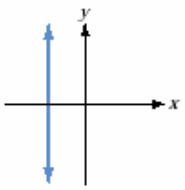

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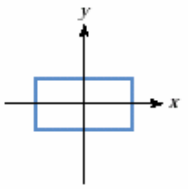

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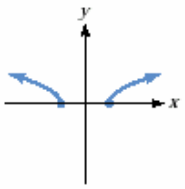
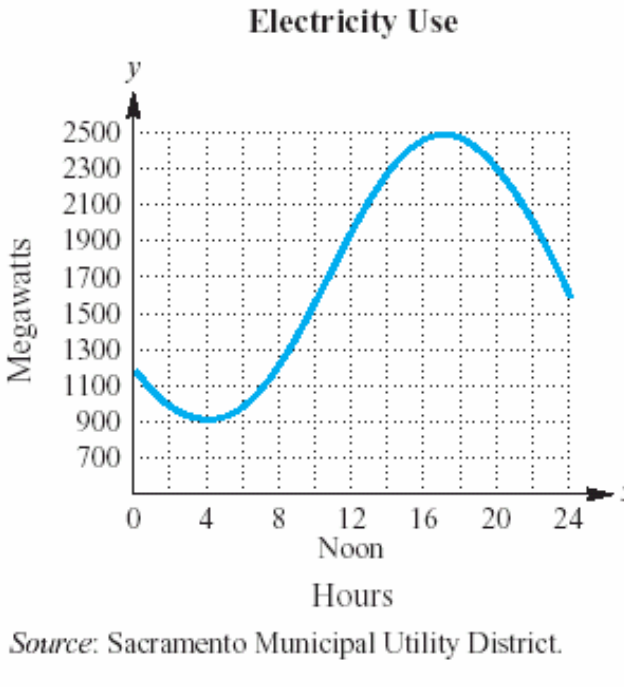


Exercise 4 (textbook 2.2 # 79)

The graph shows the daily megawatts of electricity used on a record-breaking summer day in Sacramento.



a) Is this the graph of a function?

b) What is the domain? What is the range?

c) Estimate the number of megawatts used at 8am.

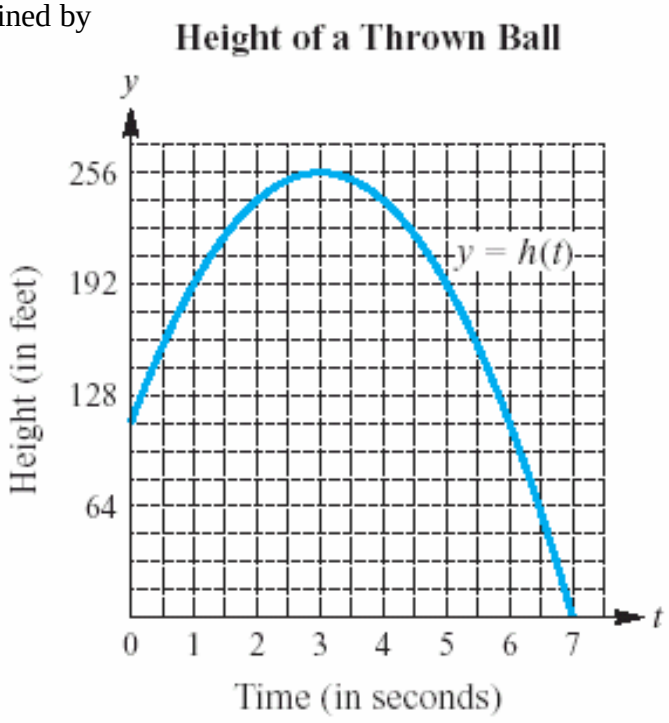
d) At what time was the most electricity used? the least?

e) Call this function *f*. What is $f(12)$? What does it mean?

f) During what time intervals is electricity usage increasing? Decreasing?

Exercise 5 (textbook 2.2 #80)

A ball is thrown straight up into the air. The function defined by $y = h(t)$ gives the height of the ball (in feet) at t seconds.



a) What is the height of the ball after 2 seconds?

b) When will the height be 192 feet?

c) During what time intervals is the ball going up? Down?

d) How high does the ball go, and when does the ball reach its maximum height?

e) At how many seconds does the ball hit the ground?

Increasing, Decreasing, and Constant Functions

Suppose that a function f is defined over an interval I. If x_1 and x_2 are in I,

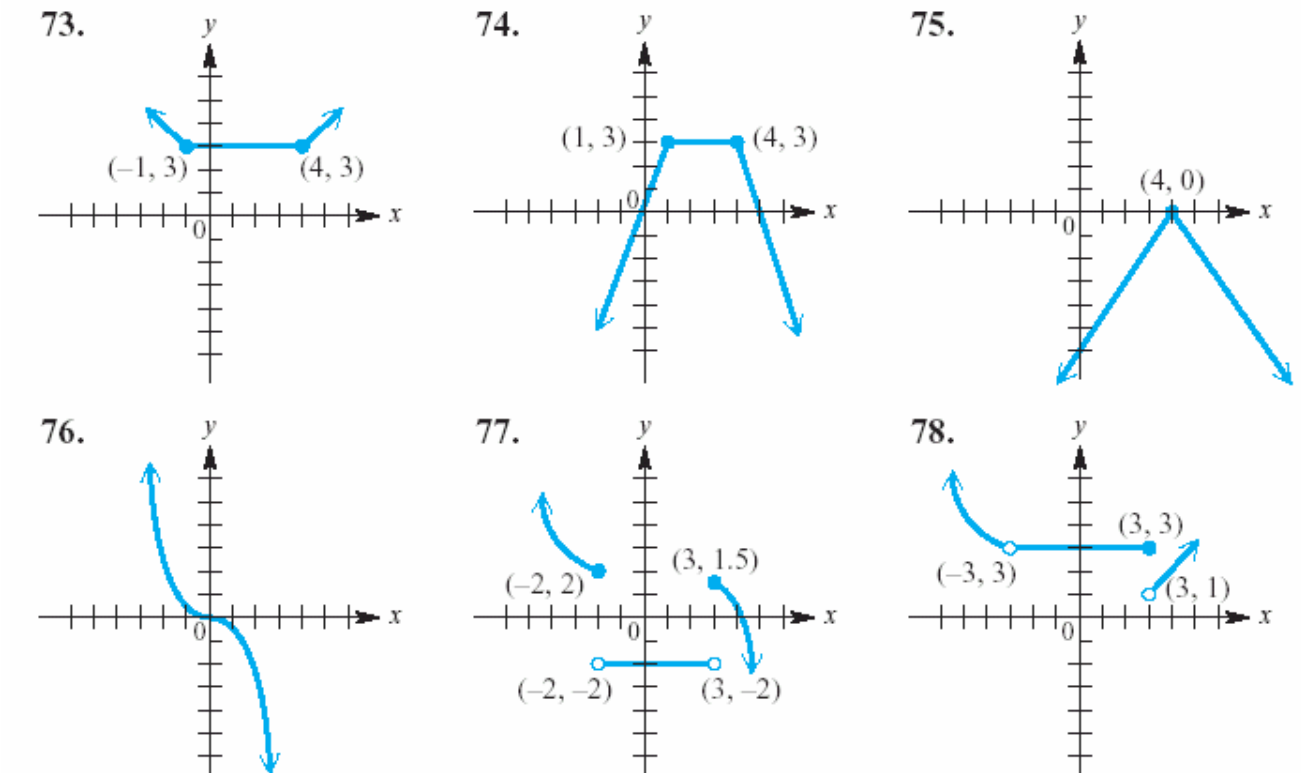
a) f **increases** on I if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$;

b) f **decreases** on I if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$;

c) f **is constant** on I if, for every x_1 and x_2, $f(x_1) = f(x_2)$.

Exercise 6 (textbook 2.2)

Determine the intervals of the domain for which each function is a) increasing, (b) decreasing, and (c) constant.



Exercise 7 a) Label the axes for a sketch of a problem, which says, "Sketch a graph of the cost of manufacturing q items…"

b) Label the axes for a problem, which says, "Graph the pressure, p, of a gas as a function of its volume, v, where p is in pounds per square inch and v is in cubic inches."

c) Label the axes for a problem which asks you to "Graph D in terms of y…"

Functions Defined by Equations

Exercise 8 Recall from geometry that if we know the radius of a circle, we can find its area. If we let $A = q(r)$ represent the area of a circle as a function of its radius, then a formula for $q(r)$ is

$$A = q(r) = p r^2 .$$

Use the above formula, where r is in cm, to evaluate $q(10)$ and $q(20)$. Explain what your results tell you about circles.

Exercise 9 Let $f(x) = \dfrac{-7}{x-13}$. Answer the following:

a) Is y a function of x? Why?

b) Find the domain and the range.

c) Find $f(0), f(2), f(-x), f(a+h)$

Exercise 10 Suppose $v(t) = t^2 - 2t$ gives the velocity, in ft/sec, of an object at time *t*, in seconds.

a) Is *v* a function of *t*? Explain.

b) Which variable is independent and which one is dependent?

c) What is $v(0)$ and what does it represent?

d) What is $v(3)$ and what does it represent?

Exercise 11

$y = \sqrt{4x+2}$ $y = -\sqrt{x}$ $y = -6x + 8$ $x + y < 4$

Answer the following:

a) Decide whether each relation defines *y* as a function of *x*.

b) Give the domain and the range.

c) Rewrite the equations (when possible) using function notation.

d) Find $f(-x)$ and $f(2x)$ for each function.