REVIEW
Chapter 1
Solving Equations and Inequalities

<u>Definition</u> The standard form of a quadratic or second degree equation in one variable is $ax^2 + bx + c = 0$ where $a, b, c \in \mathbb{R}; a \neq 0$.

<u>Solving quadratic equations</u>

(1) THE FACTORING METHOD – used to solve equations of the form $ax^2 + bx + c = 0$ that are factorable (see factoring methods on page 2)

<u>Zero-Factor Property</u>: The product of two factors equals zero if and only if one of the factors (or both) is zero.

$$AB = 0 \Leftrightarrow A = 0 \;\; or \;\; B = 0$$

(2) EXTRACTION OF ROOTS – used to solve equations of the form

$$x^2 = k \qquad \text{or} \qquad (x-p)^2 = k.$$

$$\sqrt{x^2} = \sqrt{k} \qquad\qquad \sqrt{(x-p)^2} = \sqrt{k}$$

$$x = \pm\sqrt{k} \qquad\qquad x - p = \pm\sqrt{k}$$

$$x = p \pm \sqrt{k}$$

(3) COMPLETING THE SQUARE $ax^2 + bx + c = 0$

Step 1: Coefficient of x^2 equal to 1.

Step 2: Constant isolated.

Step 3: Complete the square by adding $\left(\frac{1}{2} \cdot coefficient\ of\ x\right)^2$ to both sides of the equation and solve by the extraction of roots method.

(4) QUADRATIC FORMULA If $ax^2 + bx + c = 0$, then the solutions are given by:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<u>Definition</u> The discriminant of a quadratic equation is $\Delta = b^2 - 4ac$

<u>Properties</u> **(1)** If $a, b, c \in \mathbb{R}$, then:

If $\Delta > 0$, the equation has two distinct real solutions.

If $\Delta = 0$, the equation has one real (rational) solution.

If $\Delta < 0$, the equation has two complex (nonreal) solutions.

(2) If $a, b, c \in \mathbb{Q}$, then:

If Δ is a perfect square, the equation has **rational solutions**.

If Δ is not a perfect square, then the equation has **irrational solutions.**

<u>Factoring a polynomial</u>

1. GCF Factor out the greatest common factor (if any).

2. Special products

<u>Two terms</u>	<u>Three terms</u>
$a^2 - b^2 = (a-b)(a+b)$	$a^2 + 2ab + b^2 = (a+b)^2$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	$a^2 - 2ab + b^2 = (a-b)^2$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	

3. Factoring technique to factor out a trinomial $ax^2 + bx + c$

<u>$a = 1$</u>

$$x^2 + bx + c = (x + \square)(x + \square)$$

product = c
sum = b

<u>$a \neq 1$</u>

split the middle term bx

$$ax^2 + bx + c = ax^2 + \square x + \square x + c$$

product = ac
sum = b
then factor by grouping

4. If more than four term, factor by grouping.

Exercise #1 Solve the following linear equations:

a) $5(x+3)+4x-3=-(2x-4)+2$

b) $\frac{5}{6}x-2x+\frac{4}{3}=\frac{5}{3}$

c) $-8(3x+4)+6x=4(x-8)+4x$

Exercise #2 Solve by **factoring** (the zero-factor property):

a) $x^2+2x-8=0$

b) $5x^2-3x=2$

c) $-6x^2+7x=-10$

d) $3x^2-75=0$

Exercise #3 Solve the equation in the set of complex numbers by **extracting roots** (the square root property).

a) $\frac{2x^2}{3}=4$

b) $\left(t-\frac{1}{2}\right)^2=\frac{3}{4}$

c) $1-3(x-1)^2=28$

d) $(-2x+5)^2=-8$

Exercise #4 Solve the equations in the set of complex numbers by **completing the square**.

a) $x^2-\frac{5}{3}x=1$

b) $2p^2+p-10=0$

c) $-4x^2+8x=7$

Exercise #5 Solve the equations in the set of complex numbers by the **quadratic formula.**

a) $x^2-\frac{x}{2}+1=0$

b) $\frac{1}{2}a^2-3=-\frac{1}{4}a$

c) $3-\frac{4}{x}-\frac{2}{x^2}=0$

d) $2x^2+x-5=0$

Exercise #6 Solve the following equations in the set of complex numbers.

a) $x^3+27=0$ b) $2x^7-128x=0$ c) $x^4-16=0$

Exercise #7 a) Write (in standard form) a quadratic equation with rational coefficients that has $2+\sqrt{3}$ as a solution.

b) Write (in standard form) a quadratic equation with integer coefficients that has 2 and $-\frac{1}{2}$ as solutions.

c) Write (in standard form) a quadratic equation with real coefficients that has 1+i as a solution.

Exercise #8 a) Determine k such that the solutions of $3x^2+4x=k$ are nonreal complex numbers.

b) Find the value(s) of k that will make the solutions of the following equation equal:
$$(k-1)x^2+(k-1)x+1=0$$

Exercise #9 Solve each equation for the indicated variable:

a) $3x^2+xy+y^2=2$, for y;

b) $A=2w^2+4lw$, for w;

c) $a^2+b^2=c^2$, for b

Exercise #10 Solve the following equations:

a) $2x^4-7x^2+5=0$

d) $\sqrt{y+9}=y+3$

b) $(x+5)^{\frac{4}{3}}+(x+5)^{\frac{2}{3}}-20=0$

c) $7x^{-2}-10x^{-1}-8=0$

Exercise #11 Solve the following equations:

a) $x^2+\sqrt{3}x-\dfrac{1}{4}=0$

d) $100-2(3x-1)^2=0$

b) $3x(x+1)=2x+2$

e) $40-12m^2=0$

c) $3x^2\left(x+\dfrac{1}{2}\right)\left(2x-\dfrac{1}{3}\right)\left(5x-\dfrac{1}{2}\right)=0$

f) $5-(x-1)^2=(x-2)^2$

Exercise #12 Solve the following equations:

a) $\dfrac{4x+3}{4}-\dfrac{2x}{x+1}=x$

c) $\dfrac{4}{x^2+x-6}-\dfrac{1}{x^2-4}=\dfrac{2}{x^2+5x+6}$

b) $\dfrac{x}{x-3}=\dfrac{3}{x-3}+3$

d) $\dfrac{2x-5}{x}=\dfrac{x-2}{3}$

Exercise #13 Solve the following equations:

a) $\sqrt{4x+5}-2=2x-7$

b) $\sqrt{x}-\sqrt{x-12}=2$

c) $(2x+5)^{\frac{1}{3}}-(6x-1)^{\frac{1}{3}}=0$

Exercise #14 Solve each inequality. Write each solution set in interval notation and graph it.

a) $\dfrac{2x-5}{-8}\le 1-x$

d) $x^2+4x>-1$

b) $-3\le\dfrac{x-4}{-5}<4$

e) $x^3+4x^2-9x-36\ge 0$

c) $x^2\le 9$

f) $(x-5)^2(x+1)<0$

Exercise #15 Solve each rational inequality. Write each solution set in interval notation.

a) $\dfrac{x+1}{x-4} > 0$ b) $\dfrac{x+3}{x-5} \le 1$

Answers:

#1 a) -6/11; b) -2/7; c) 0.

#2a) -4,2; b) -2/5, 1; c)-5/6, 2.

#3 a) $\pm\sqrt{6}$; b) $\frac{1}{2}\pm\frac{\sqrt{3}}{2}$; c) $1\pm 3i$; d) $\frac{5}{2}\pm\sqrt{2}i$.

#4 a) $\frac{5\pm\sqrt{61}}{6}$;b) -5/2,2; c) $\frac{2\pm\sqrt{3}i}{2}$.

#5 a) $\frac{1\pm\sqrt{15}i}{4}$; b) $\frac{-1\pm\sqrt{97}}{4}$; c) $\frac{2\pm\sqrt{10}}{3}$; d) $\frac{-1\pm\sqrt{41}}{4}$.

#6 a) -3, $\frac{3\pm 3\sqrt{3}i}{2}$; b) 0, $\pm 2, 1\pm\sqrt{3}i, -1\pm\sqrt{3}i$.

#7 a) $x^2-4x+1=0$; b) $2x^2-3x-2=0$; c) $x^2-2x+2=0$.

#8) a) $k<-\frac{4}{3}$; b) 5

#9 a) $\frac{-x\pm\sqrt{8-11x^2}}{2}$; b) $\frac{-2l\pm\sqrt{4l^2+2A}}{2}$; c) $\pm\sqrt{c^2-a^2}$.

#10 a) $\pm 1, \pm\frac{\sqrt{10}}{2}$; b) -13, 3, $-5\pm 5\sqrt{5}i$ c) -7/4,1/2; d) 9.

#11 a) $\frac{-\sqrt{3}\pm 2}{2}$; b) $\frac{2}{3}, -1$; c) $0, -\frac{1}{2}, \frac{1}{6}, \frac{1}{10}$; d) $\frac{1\pm 5\sqrt{2}}{3}$; e) $\pm\frac{\sqrt{30}}{3}$; f) $0, 3$.

#12 a)3/5; b) $\varnothing$; c) -9; d) 3,5.

#13 a) 5; b)16; c) 3/2.

#14 a) $x\le\frac{1}{2}$; b) $x\in(-16,19]$;c) $[-3,3]$ d) $x<-2-\sqrt{3}$ or $x>-2+\sqrt{3}$; e) $x\in[-4,-3]\cup[3,\infty)$; f) $x<-1$.

#15 a) $x<-1$ or $x>4$; b) x<5.