REVIEW TEST 2
Chapter 4 (4.1 – 4.5, 4.7, 4.9), Section 7.1

To prepare for the test, you should:

- study **<u>all quizzes</u>** and all **<u>examples done in class</u>**, as well as your **<u>homework</u>** from the listed sections.
- know how to prove formally the following theorems or properties:
    - Section 4.2
        - Theorem 5 page 290 (Functions with zero derivatives are constant)
        - Corollary 7 page 291 (Functions with the same derivative differ by a constant)

- Handout 4.1, 4.2 – all examples and exercises
- Handout 4.3, 4.5 – all exercises
- Handout 4.7– all exercises
- know the following:
    - the differentiation rules for polynomials, exponential, , products, and quotients
    - the differentiation rules for trigonometric functions and their inverses, as well as for exponential and logarithmic functions
    - The Chain Rule
    - derivatives of higher order
    - logarithmic differentiation
    - definition of a critical number
    - The Closed Interval Method
    - The First Derivative Test
    - The Second Derivative Test
    - Concavity test
    - definition of an inflection point
    - Local and global extremes
    - The Extreme Value Theorem
    - Rolle's Theorem
    - The Mean Value Theorem and its corollaries
    - The Concavity Test
    - The Increasing/Decreasing Test
    - how to graph a function (the same way it was done in class)
    - indeterminate forms and L'Hopital's rule
    - antiderivatives of a function
    - Integration by parts
    - Optimization applications

- **More practice Chapter 4:**

<u>4.1 – 4.3, 4.5 Finding Critical Numbers. Finding Absolute Minimum and Maximum Values of a Function and Graphing a Function</u>

<u>Definition</u> A critical number of a function f is a number c in the domain of f such that either $f'(c)=0$ or $f'(c)$ does not exist.

<u>The Closed Interval Method</u>

To find the absolute minimum and maximum values of a continuous function f on a closed interval [a,b]:

1. Find the critical numbers of *f*.
2. Find the values of *f* at the critical numbers and at the endpoints of the interval.
3. The largest of the values is the absolute maximum value; the smallest of the values is the absolute minimum value.

<u>Exercise 1</u> Find the critical numbers of each function:

a) $f(x)=x^{\frac{3}{5}}(4-x)$ c) $f(z)=\dfrac{z+1}{z^2+z+1}$ d) $f(x)=x^{\frac{4}{5}}(x-4)^2$ h) $f(x)=x\ln x$

b) $f(r)=\dfrac{r}{r^2+1}$ e) $F(x)=\sqrt[3]{x^2-x}$ f) $f(q)=\sin^2(2q)$ g) $g(q)=q+\sin q$

<u>Exercise 2</u> Find the absolute minimum and maximum values of each function on the given interval:

a) $f(x)=x-2\sin x,\ \ x\in[0,2p]$ d) $f(x)=\sin x+\cos x,\ \ x\in\left[0,\dfrac{p}{3}\right]$

b) $f(x)=\sqrt{9-x^2},\ \ x\in[-1,2]$ e) $f(x)=x-2\cos x,\ \ x\in[-p,p]$

c) $f(x)=x^2+\dfrac{2}{x},\ \ x\in\left[\dfrac{1}{2},2\right]$ f) $f(x)=x-2\sin x, x\in[0,3p]$

<u>Exercise 3</u> Graph each function (as we did in class):

b) $f(x)=\dfrac{x}{(1+x)^2}$ d) $f(x)=2\cos x+\sin^2 x,\ \ x\in[-p,p]$

c) $f(x)=\dfrac{\ln x}{\sqrt{x}}$ e) $f(x)=\dfrac{1+x^2}{1-x^2}$

<u>4.4 L'Hopital's Rule</u> Suppose *f* and *g* are differentiable and $g'(x)\neq 0$ near *a* (except possibly at *a*). Suppose that

$$\lim_{x\to a} f(x)=0 \qquad \text{and} \qquad \lim_{x\to a} g(x)=0$$

or that

$$\lim_{x\to a} f(x)=\pm\infty \qquad \text{and} \qquad \lim_{x\to a} g(x)=\pm\infty$$

(In other words, we have an indeterminate form of type $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$). Then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).

Exercise 4 — Find each limit. Use l'Hopital 's Rule where appropriate. (i)
If there is a more elementary method, consider it. (ii)
If l'Hopital's Rule does not apply, explain why. (iii)

a) $\lim_{x\to -1}\frac{x^2-1}{x+1}$ f) $\lim_{x\to 0}\frac{\sin^{-1}x}{x}$ g) $\lim_{x\to 0^+} x^{\sin x}$ m) $\lim_{x\to\infty}\left(xe^{\frac{1}{x}}-x\right)$

b) $\lim_{x\to 1}\frac{x^a-1}{x^b-1}$ l) $\lim_{x\to\infty} x^3e^{-x^2}$ h) $\lim_{x\to 0}\frac{\cos mx-\cos nx}{x^2}$ n) $\lim_{x\to 0}(1-2x)^{\frac{1}{x}}$

c) $\lim_{x\to 0^+}\sqrt{x}\ln x$ s) $\lim_{x\to 0^+} x^2\ln x$ i) $\lim_{x\to\infty} e^{-x}\ln x$ o) $\lim_{x\to 1^+}(x-1)\tan\left(\frac{p\,x}{2}\right)$

d) $\lim_{x\to 0}\frac{\tan px}{\tan qx}$ j) $\lim_{x\to\infty} x^{\frac{\ln 2}{1+\ln x}}$ p) $\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$

e) $\lim_{x\to -\infty} x^2e^x$ k) $\lim_{x\to 0}\frac{1-e^{-2x}}{\sec x}$ r) $\lim_{x\to 0^+}(-\ln x)^x$

Answers Exercise 1: a) $0, 3/2$; b) ± 1; c) 0, -2; d) 0, 8/7, 4; e) 0, 1/2,1; f) $kp/4$, k integer; g) $(2k+1)p$, k integer; h) 1/e; i) Exercise 2: a) abs. min: $f\left(\frac{p}{3}\right)=\frac{p}{3}-\sqrt{3}$, abs. max: $f\left(\frac{5p}{3}\right)=\frac{5p}{3}+\sqrt{3}$; b) abs. max: $f(0)=3$, abs. min: $f(2)=\sqrt{5}$; c) abs. max: $f(2)=5$, abs. min: $f(1)=3$; d) abs. max: $f\left(\frac{p}{4}\right)=\sqrt{2}$, abs. min: $f(0)=1$; e) abs. max: $f(p)=p+2$, abs. min: $f\left(-\frac{p}{6}\right)=-\frac{p}{6}-\sqrt{3}$; f) abs. min: $f\left(\frac{p}{3}\right)=\frac{p}{3}-\sqrt{3}\approx -0.68$, abs. max: $f(3p)=3p$ Exercise 4 a) ii -2; b) i a/b; c) i 0; d) i p/q; e) i 0; f)i 1; g) 1; h) i $\frac{n^2-m^2}{2}$; i) i 0; j) 2; k) iii 0; l) i 0; m) 1; n) e^{-2}; o) i $-2/p$; p) i 1/2 ; r) 1; s) i 0.

4.9 Antiderivatives/ Indefinite Integrals ; 7.1 Integration by Parts

Exercise 5 — Find:

a) $\int\frac{\csc q\cot q}{2}dq$ b) $\int\left(4\sec x\tan x-2\sec^2 x\right)dx$ c) $\int x\sin\frac{x}{2}dx$ j) $\int x(\ln x)^2dx$

d) $\int\left(\frac{1}{x}-\frac{5}{x^2+1}\right)dx$ e) $\int xe^{3x}dx$ f) $\int\cot^2 y\,dy$ k) $\int\sin 2t\cos 4t\,dt$

g) $\int e^{2x}\cos 3x\,dx$ h) $\int\frac{\csc q}{\csc q-\sin q}dq$ i) $\int x^{\sqrt{2}-1}dx$

Textbook – Chapter 4 Review page 359

Exercises 1 – 14, 19, 20, 26, 27, 29, 32, 33, 34, 45, 65 - 74