REVIEW TEST 1
Chapter 2 (2.1 – 2.3, 2.5 – 2.8) Chapter 3 (3.1 – 3.4)

After completing these sections, you should:

- be able to find the rate of change of a function over an interval
- know the definition of the limit of a function at a point
- be able to find limits given a graph
- be able to calculate limits using The Limit Laws
- know and be able to apply The Squeeze Theorem in finding limits
- be able to calculate one-sided limits and infinite limits
- know the definition of a function continuous at a point
- prove a function is continuous at a point using the definition
- be able to recognize points of discontinuity
- be able to find the slope of the tangent to a curve at a given point
- be able to find an equation of the tangent line to a graph at a point
- be able to find vertical and horizontal tangents to a graph
- be able to recognize the indeterminate forms $\frac{0}{0}, 0\cdot\infty, \infty-\infty, \frac{\infty}{\infty}$
- know how to find indeterminate limits
- be able to find the derivative of a function using the definition
- know the differentiation rules for polynomials, exponential, products, and quotients
- be able to find higher-order derivatives
- know the differentiation rules for the trigonometric functions
- know how to calculate derivatives of composite functions using the chain rule

Know how to prove formally the following theorems or properties:

- Section 2.8
  - Theorem (Differentiability implies continuity)

- Sections 3.1 - 3.3
  - Rule (Derivative of a constant function)
  - Rule (Constant multiple rule)
  - Rule (Derivative sum rule)
  - Rule (Derivative of the sine function)
  - Rule (Derivative of the cosine function)
  - Rules (Derivatives of the other trigonometric functions: tangent, cotangent, cosecant, secant)

Know how to prove the following special limits:

- $\lim_{x\to\infty}\frac{\sin x}{x}=0$ and $\lim_{x\to\infty}\frac{\cos x}{x}=0$
- $\lim_{x\to 0}x^n\sin\frac{1}{x}=0$ and $\lim_{x\to 0}x^n\sin\frac{1}{x}=0$ for $\forall n\in\mathbb{N}$.
- $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ (3.3, page 192)

To prepare for the test, you should study **Quiz 1** and all **examples done in class**, as well as your **homework** and all **handouts** .

More practice:

Textbook Chapter 2 Review pages 166 – 168 : 1, 3 – 20, 23, 24, 29, 33, 35 – 39, 41, 47

Textbook Chapter 3 Review pages 267 – 268: 1 – 8, 10, 11, 13, 16, 18 – 20, 22 – 24, 27, 31, 32, 34, 35, 37, 39, 51, 57, 58, 59, 61, 65

1. Let $f(x)=\begin{cases}\sqrt{-x}, x<0\\ 3-x, 0\le x<3.\\ (x-3)^2, x\ge 3\end{cases}$

a) Evaluate each limit, if it exists. (i) $\lim_{x\to 0^+} f(x)$ (ii) $\lim_{x\to 0^-} f(x)$ (iii) $\lim_{x\to 0} f(x)$ (iv) $\lim_{x\to 3^-} f(x)$ (v) $\lim_{x\to 3^+} f(x)$ (vi) $\lim_{x\to 3} f(x)$

b) Where is f discontinuous?

2. Find an equation of the tangent line to the curve $y=x^3-2x$ at the point $(2,4)$ in two ways: a) using the definition; b) using the differentiation rules.

3. Let $f(x)=\begin{cases}x+1, x\le a\\ x^2, x>a\end{cases}$ Find a such that f is continuous everywhere.

4-11. Find the limits.

4. $\lim_{x\to 1} e^{x^3-x}$

5. $\lim_{x\to -3}\frac{x^2-9}{x^2+2x-3}$

6. $\lim_{h\to 0}\frac{(h-1)^3+1}{h}$

7. $\lim_{r\to 9}\frac{\sqrt{r}}{(r-9)^4}$

8. $\lim_{v\to 4^+}\frac{4-v}{|4-v|}$

9. $\lim_{x\to\infty} e^{-3x}$

10. $\lim_{x\to 0}\frac{1-\sqrt{1-x^2}}{x}$

11. $\lim_{x\to\infty}\frac{\sqrt{3x^2-1}}{x-1}$

12. Find $\lim_{x\to 0}\sqrt{x^3+x^2}\sin\frac{p}{x}$.

13. Prove that $\lim_{x\to 0} x^4\cos\frac{2}{x}=0$

14. Prove that $\lim_{x\to 0^+}\sqrt{x}e^{\sin(p/x)}=0$

15-16. Find the limit, if it exists. If the limit does not exist, explain way.

15. $\lim_{x\to -4}|x+4|$

16. $\lim_{x\to 2}\frac{|x-2|}{x-2}$

17 – 25 Find the limits.

17 $\lim_{x\to 5^-}\frac{e^x}{(x-5)^3}$

18. $\lim_{x\to\infty}\frac{x+2}{\sqrt{9x^2+1}}$

19. $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

20. $\lim_{x\to\infty}\frac{\sin^2 x}{x^2}$

21. $\lim_{x\to\infty}\cos x$

22. $\lim_{x\to -\infty} x^3-5x^2$

23. $\lim_{x\to 0}\frac{\sin^2 x}{x}$

24. $\lim_{x\to -1}\frac{\sin(x^2-x-2)}{x+1}$

25. $\lim_{x\to 1}\frac{\sin(1-\sqrt{x})}{x-1}$

26. $\lim_{x\to 0}\frac{\tan 4x}{x}$

27. $\lim_{q\to 0}\frac{\sin q}{q+\tan q}$

28. Find $\lim_{x\to\infty} f(x)$ if $\dfrac{4x-1}{x} < f(x) < \dfrac{4x^2+3x}{x^2}$ for all $x > 5$.

29 - 34 Differentiate the functions.

29. $y = ae^v + \dfrac{b}{v} + \dfrac{c}{v^2}$, find $\dfrac{dy}{dv}$

30. $y = \dfrac{x^2+4x+3}{\sqrt{x}}$, find $\dfrac{dy}{dx}$

31. $y = \dfrac{t^2}{3t^2-2t+1}$, find $\dfrac{dy}{dt}$

32. $f(x) = \dfrac{x}{x+\dfrac{c}{x}}$

33. $y = \dfrac{1}{s+ke^s}$, find $\dfrac{dy}{ds}$

34. $P = \dfrac{nRT}{V-nb} - \dfrac{an^2}{V^2}$, find $\dfrac{dP}{dV}$

35. Find the first and second derivatives of the function $G(r) = \sqrt{r} + \sqrt[3]{r}$.

36. Find equations of both lines through the point $(2,-3)$ that are tangent to the parabola $y = x^2 + x$.

37. Find a second-degree polynomial P such that $P(2) = 5$, $P'(2) = 3$, and $P''(2) = 2$.

38. Find an equation of the normal line to the curve $y = \sqrt{x}$ that is parallel to the line $2x + y = 1$.

39. If f is a differentiable function, find an expression for the derivative of each of the following functions:

a) $y = x^2 f(x)$ b) $y = \dfrac{f(x)}{x^2}$ c) $y = \dfrac{1+xf(x)}{\sqrt{x}}$

40. Find equations of the tangent lines to the curve $y = \dfrac{x-1}{x+1}$ that are parallel to the line $x - 2y = 2$.

41 - 42 Differentiate the functions

41. $h(t) = \csc t + e^t \cot t$

42. $y = \dfrac{\tan x}{x}$

43 – 45 Find an equation of the tangent line to the curve at the given point

43. $y = e^x \cos x$, $(0,1)$

44. $y = \sec x - 2\cos x$, $\left(\dfrac{p}{3}, 1\right)$

45. $y = \sin(\sin x)$, $(p, 0)$

46. For what values of x does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent?

47. Find the points with $x \in [0, 2p]$ that are on the curve $y = \dfrac{\cos x}{2+\sin x}$ at which the tangent is horizontal.

48 – 49 Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

48. $\dfrac{d^{99}}{dx^{99}}(\sin x)$

49. $\dfrac{d^{35}}{dx^{35}}(x \sin x)$

50. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle q with the plane, then the magnitude of the force is

$$F = \frac{mW}{m\sin q + \cos q}$$

where m is a constant called the coefficient of friction.

a) Find the rate of change of F with respect to q.
b) When is this rate of change equal to 0?

51 – 58 Find the derivative of each function.

50. $f(x) = \sin\left(cos\left(\tan x\right)\right)$ 51. $y = e^{\sqrt{x}}$ 52. $y = \sin\left(e^{x}\right)$ 53. $g(t) = \dfrac{1}{\left(t^4+1\right)^3}$

54. $y = xe^{-x^2}$ 55. $f(y) = \left(\dfrac{y-6}{y+7}\right)^3$ 56. $y = \dfrac{r}{\sqrt{r^2+1}}$ 57. $y = \dfrac{e^{2u}}{e^{u}+e^{-u}}$

58. $y = \tan^4(3x)$ 59. $y = \sqrt{\cos t}$ 60. $y = e^{\sin 2t}$ 61. $y = \sin(5x) + \cos\left(\sqrt{x}\right)$

62. $y = \tan(\sin x)$ 63. $y = \sqrt{4e^x + x^6 - 7}$ 64. $y = \tan\left(5 - \sin 2t\right)$ 65. $y = \left(5x^3 - x^4\right)^7$

66. $y = \dfrac{1}{3x-2}$ 67. $r = -(\sec q + \tan q)^{-1}$ 68. $y = \left(\dfrac{\sin t}{1+\cos t}\right)^2$ 69. $y = \left(e^{\sin\frac{t}{2}}\right)^3$

70. a) For what x is $f(x) = \left|x^2 - 9\right|$ differentiable? Find $f'(x)$.
b) Graph f, f'.

71. If $f(2) = 10$, $f'(x) = x^2 f(x)$ for any x, find $f''(2)$.

72. Let $y = 4 + \cot x - 2\csc x$.

a) Find the equation of the tangent line at $(\frac{p}{2}, 2)$.
b) Find the equation of the horizontal tangent (in the first quadrant).

73. Find r such that $y = e^{rx}$ satisfies the differential equation $y'' - 4y' + y = 0$

74 – 75 Differentiate:

74. $y = 2^{t^3}$ 75. $y = 3^{\frac{z}{z-1}}$

76. Use the chain rule to find the derivative of $y = |x|$

77. If n is a positive integer, find the derivative of $f(x) = \sin^n x \cos nx$ and simplify as much as possible.

Answers to selected exercises on next page

Answers

1) a) (i) 3 (ii) 0 (iii) DNE (iv) 0 (v) 0 (vi) 0 ; b) 0, -3

2) $y-4=10(x-2)$; 3) $\frac{1\pm\sqrt{5}}{2}$; 4) 1; 5) $\frac{3}{2}$; 6) 3; 7) ∞; 8) -1; 9) 0; 10) 0; 11) $\sqrt{3}$; 12) 0; 13); 14); 15) 0

16) DNE; 17) $-\infty$; 18) $\frac{1}{3}$; 19) $\frac{1}{6}$; 20) 0; 21) DNE; 22) $-\infty$; 23) 0; 24) -3; 25) $-\frac{1}{2}$; 26) 4; 27) ½; 28) 4;

29) $y'=ae^{v}-\frac{b}{v^2}-\frac{2c}{v^3}$; 30) $y'=\frac{3}{2}\sqrt{x}+\frac{2}{\sqrt{x}}-\frac{3}{2x\sqrt{x}}$; 31) $y'=\frac{2t(1-t)}{\left(3t^2-2t+1\right)^2}$; 32) $f'(x)=\frac{2cx}{\left(x^2+c\right)^2}$;

33) $-\frac{1+ke^{s}}{\left(s+ke^{s}\right)^2}$; 34) $3\frac{-nRT}{(V-nb)^2}+\frac{2an^2}{V^3}$; 35) $G'(r)=\frac{1}{2\sqrt{r}}+\frac{1}{3\sqrt[3]{r^2}}$, $G''(r)=-\frac{1}{4\sqrt{r^3}}-\frac{2}{9\sqrt[3]{r^5}}$; 36) $y=-x-1$

and $y=11x-25$; 37) $P(x)=x^2-x+3$; 38) y= - 2x+3; 39) a) $y'=x^2f'(x)+2xf(x)$, b) $y'=\frac{xf'(x)-2f(x)}{x^3}$,

c) $y'=\frac{xf(x)+2x^2f'(x)-1}{2x^{3/2}}$; 40) $y=\frac{1}{2}x-\frac{1}{2}$ and $y=\frac{1}{2}x+\frac{7}{2}$; 41) $-\csc t\cot t+e^{t}\left(\cot t-\csc^2 t\right)$;

42) $\frac{x\sec^2 x-\tan x}{x^2}$; 43) $y=x+1$; 44) $y=3\sqrt{3}x+1-p\sqrt{3}$; 45) $y=-x+p$;

46) $\frac{2p}{3}+2pk$ or $\frac{4p}{3}+2pk, k\in\mathbb{Z}$; 47) $\left(\frac{11p}{6},\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{7p}{6},-\frac{1}{\sqrt{3}}\right)$; 48) $-\cos x$; 49) $-35\sin x-x\cos x$;

50) a) $\frac{mW(\sin q-m\cos q)}{(m\sin q+\cos q)^2}$; b) $q=\tan^{-1}m$; 51) $-\cos\left(cos\left(\tan x\right)\right)\sin\left(\tan x\right)\sec^2 x$; 52) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$; 53) $e^{x}\cos e^{x}$;

54) $\frac{-12t^3}{\left(t^4+1\right)^4}$; 55) $e^{-x^2}\left(1-2x^2\right)$; 56) $\frac{39(y-6)^2}{(y+7)^4}$; 57) $\frac{1}{\left(r^2+1\right)^{3/2}}$; 58) $\frac{e^{3u}\left(e^{2u}+3\right)}{\left(e^{2u}+1\right)^2}$; 61) $5\cos 5x-\frac{\sin\sqrt{x}}{2\sqrt{x}}$;

62) $\frac{\cos x}{\cos^2(\sin x)}$; 63) $\frac{2e^{x}+3x^5}{\sqrt{4e^{x}+x^6-7}}$; 64) $\frac{-\cos 2t}{\cos^2(5-\sin 2t)}$; 65) $7\left(5x^3-x^4\right)^6\left(15x^2-4x^3\right)$; 66) $\frac{-3}{(3x-2)^2}$;

67) $\frac{1}{1+\sin q}$; 68) $\frac{2\sin q}{(1+\cos q)^2}$; 69: $\frac{3}{2}e^{3\sin\frac{t}{2}}\cos\frac{t}{2}$; 70) all real numbers except 3, -3; 71) 200;

72) $y=-x+\frac{p}{2}+2$, $y=4-\sqrt{3}$; 73) $2\pm\sqrt{3}$; 77) $n\sin^{n-1}x\cos(n+1)x$