Review Test #2
Chapter 4 (4.1 – 4.5), Chapter 5 (5.1 – 5.5),Chapter 6 (6.1 – 6.4), Chapter 7 (7.1 – 7.4), and 10.7

To prepare for the test, you should study all exercises and examples done in class, plus:

- Quiz #2 (#1, 3, 4, 5)
- Homework assigned from the listed sections
- Handout Partial Fraction (more practice)
- Handout 4.3 – 4.6 (#1, 5, 6, 8, 9,11)

Trigonometry

You should know the following:

- how to graph the basic functions sin, cos, tan, cot, sec, csc
- domain, range, period, amplitude (when defined) and vertical asymptotes (when applicable) for the basic functions
- how to graph transformations of trigonometric functions (vertical translations, vertical stretching and compression, horizontal stretching and compression, horizontal shifting
- how to graph the inverse sine, inverse cosine, and inverse tangent functions
- domain and range for the inverse functions (all six inverse functions)
- evaluate the inverse sine, cosine, and tangent functions
- compose trigonometric functions and their inverses
- prove trigonometric identities
- solve trigonometric equations
- IMPORTANT FORMULAS

  - $\tan x = \dfrac{\sin x}{\cos x}$
  - $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$
  - $\sec x = \dfrac{1}{\cos x}$
  - $\csc x = \dfrac{1}{\sin x}$
  - $\sin^2 x + \cos^2 x = 1$
  - sine and cosine functions have period $2p$

    $\sin(x + 2kp) = \sin x$

    $\cos(x + 2kp) = \cos x$
  - tangent function has period p

    $\tan(x + kp) = \tan x$
  - $\cos(a+b) = \cos a \cos b - \sin a \sin b$
  - $\cos(a-b) = \cos a \cos b + \sin a \sin b$
  - $\sin(a+b) = \sin a \cos b + \sin b \cos a$
  - $\sin(a-b) = \sin a \cos b - \sin b \cos a$

OTHER FORMULAS

$\sin(x+p) = -\sin x$

$\cos(x+p) = -\cos x$

$\cos(-x) = \cos x$ cosine is an even function

$\sin(-x) = -\sin x$ sine is an odd function

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

$\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$

- $\cos 2a = \cos^2 a - \sin^2 a$ $\qquad \cos a = \pm\sqrt{\frac{1+\cos 2a}{2}}$
- $\cos 2a = 2\cos^2 a - 1$
- $\cos 2a = 1 - 2\sin^2 a$ $\qquad \sin a = \pm\sqrt{\frac{1-\cos 2a}{2}}$
- $\sin 2a = 2\sin a \cos a$

More practice:

1. Solve the following equations:

a) $10^{x+3} = 5e^{7-x}$ b) $2e^{3x} = 4e^{5x}$ c) $2x-1 = e^{\ln x^2}$ d) $5^x = 3^{2x-1}$

e) $2\log(x-1) = \frac{5}{2}\log x^5 - \log\sqrt{x}$ f) $\log_2 x + \log_3 x = 1$ g) $x^{\log x + 2} = 1000$

2. Let $f(t) = 1 + \ln t$.

a) Graph the function. b) State the domain, range, and vertical asymptote.

c) Find the exact x- and y-intercepts (if any).

d) Does the function have an inverse> Explain. Find $f^{-1}(t)$.

e) Graph the inverse $f^{-1}(t)$ showing the symmetry through $y = t$.

f) State the domain, range, and horizontal asymptote for the inverse function $f^{-1}(t)$.

g) Find the exact x- and y-intercepts of the inverse function $f^{-1}(t)$ (if any).

3. If $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$ is a one-to-one function, find $f^{-1}(x)$.

4 Let $f(t) = 1 + 2\ln(t-1)$.

a) Graph the function. b) State the domain, range, and vertical asymptote.

c) Find the exact x- and y-intercepts (if any).

d) Does the function have an inverse> Explain. Find $f^{-1}(x)$.

e) Graph the inverse $f^{-1}(x)$ showing the symmetry through $y = x$.

f) State the domain, range, and horizontal asymptote for the inverse function $f^{-1}(x)$.

g) Find the exact x- and y-intercepts of the inverse function $f^{-1}(x)$ (if any).

5. Let $f(x) = \log_x\left(x^2 - 3x + 9\right) - 2$

a) Find the domain of this function. b) Solve the equation $f(x) = 0$.

6. Find a formula for $\sin(x+y+z)$ in terms of sine and cosine of x, y and z.

7. Find the period and two vertical asymptotes for the function $y = \tan(px)$.

8. Show that tangent is an odd function.

9. Graph the following functions on graphing paper. In each case, identify the amplitude (when defined) and the period and label the axes accurately. Clearly label the problems. Explain in words what and how you are graphing.

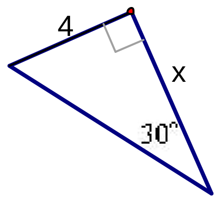
a) $y = 1 + \sin x$ from -2π to 4π b) $y = 4\sin\frac{1}{3}x$ over one period

b) $y = -2\cos x$ over one period d) $y = \tan 2x$ over one period e) $y = 2\sin\left(x - \frac{\pi}{3}\right)$

10. Find all real numbers *x* that satisfy each equation. You may show all work on this sheet of paper. Justify your answers.

a) $\cos x = 0$ b) $\sin x = 0$ c) $\tan x = 0$ d) $\cot x = 0$

11. Find the side labeled *x*.



12. Sketch a right triangle that has one acute angle θ, and find the other five trigonometric ratios of θ

$\sin\theta = \frac{3}{7}$

13. Find the exact value of each expression.

a) $\sin\frac{\pi}{6} + \cos\frac{5\pi}{3}$ b) $\tan\frac{3\pi}{4}\left(\sin\frac{2\pi}{3} - \sec\frac{5\pi}{4}\right)$ c) $\sin\left(-\frac{3\pi}{4}\right)$

d) $\tan\left(\frac{13\pi}{6}\right)$ e) $\cos\left(\frac{5\pi}{6}\right)$

14. Use the unit circle to find all the values of θ between 0 and 2π for which

a) $\sin\theta = \frac{1}{2}$ b) $\cos\theta = -\frac{\sqrt{2}}{2}$

15. Prove the following trigonometric identities:

a) $\csc\theta + \sin(-\theta) = \frac{\cos^2\theta}{\sin\theta}$ b) $\frac{\cos\alpha}{1+\sin\alpha} + \frac{1+\sin\alpha}{\cos\alpha} = 2\sec\alpha$

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16. Graph the function $y = \cos x$. Show the graph over two periods. Answer the following questions:

a) What is the domain?
b) What is the range?
c) What is the period?
d) What is the amplitude?
e) What are the x-intercepts?
f) What is the y-intercept?
g) Is the function even or odd? How is that shown in the graph?

17. Graph $f\left(x\right)=\sin x$ and $f^{-1}\left(x\right)=\sin^{-1}\left(x\right)$ on the same coordinate system, showing the relation between the two graphs (symmetry about the line $y = x$). Answer the following questions:

a) What is the domain and range of $f\left(x\right)=\sin x$?

b) What is the domain and range of $f^{-1}\left(x\right)=\sin^{-1}\left(x\right)$?

18. Evaluate the following. Give exact answers whenever possible.

a) $\sin^{-1}\left(\frac{1}{2}\right)$ b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ c) $\tan^{-1}\left(-1\right)$ d) $\cos\left(\sin^{-1}\frac{3}{5}\right)$

e) $\sin^{-1}\left(\sin\frac{5p}{8}\right)$ f) $\cos^{-1}\left(\cos\frac{2p}{7}\right)$ g) $\tan\left(\tan^{-1}100.23\right)$ h) $\cos\frac{p}{12}$

19. Prove the following identities:

a) $\dfrac{\sin x+1}{\cos x+\cot x}=\tan x$ b) $\dfrac{\cos t}{1+\sin t}=\dfrac{1-\sin t}{\cos t}$ c) $\tan\left(a+b\right)=\dfrac{\tan a+\tan b}{1-\tan a\tan b}$

20. Solve the following equations. When appropriate, show EXACT answers.
ONLY when NO exact answer is possible, express solutions rounded to two decimal places.

a) Find ALL solutions: $2\sin x-1=0$ b) Solve on $[0,2p)$: $\sin(2x)=1$

c) Solve on $[0^\circ,360^\circ)$: $2\tan q+2=0$ d) Solve on $[0,2p)$: $\cos q=-0.8$

e) Solve on $[0,2p)$: $2\sin^2 x-\sin x-1=0$ f) Find ALL solutions: $\sin 2q-\cos q=0$

g) Solve on $[0,2p)$: $2\sin^2 q-2\cos q-1=0$

21.

a) Write $\cos^2 x$ in terms of $\cos 2x$.

b) Write $\sin^2\frac{x}{3}$ in terms of a trig function of power 1.

c) Write $\cos^4 x$ and $\sin^4 x$ in terms of cosine and/or sine functions with power 1.

d) Write $\tan^2 x$ in terms of $\sec^2 x$.

e) Write $\cot^2 x$ in terms of $\cos^2 x$.