TEST #2 @ 165 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

PART I – Solve all problems:

1. Simplify the following g expression. Write the final answer using only positive exponents (if necessary).

$$\left(\frac{mn^{-2}p^{3}}{m^{-1}np^{-4}}\right)^{-2}\left(\frac{m^{3}np^{-2}}{m^{-2}n^{2}p^{3}}\right)^{-1}$$

2. Let $P(x)=2x^{5}+3x^{4}-\frac{1}{2}x^{3}-10x-2$ be a polynomial in x. Answer the following questions:

a) How many terms does it have?

b) What is the degree of the polynomial?

c) What is the constant term?

d) Find $P(0)$, $P(-1)$, and $P(a^{2})$ and simplify.

3. Use long division to find:

$$\frac{2x^{3}-5x^{2}-6x+15}{x-3}$$

4. Factor each expression as completely as possible. If prime, state so.

a) $26a^{4}b-34a^{3}c^{2}+28a^{2}b^{3}$

b) $49-m^{2}$

c) $a^{2}-4a-ab+4b$

d) $64x^{3}-27$

e) $25x^{2}+100$

f) $2x+x^{2}-15$

g) $3x^{2}-11x-20$

h) $1-16m^{4}$

i) $9y^{2}+12y-15$

j) $3t^{2}-15t+16$

k) $c^{2}+16$

l) $3a^{2}+24a+48$

m) $8+n^{3}$

n) $6a^{2}+40a+24$

5. Do the following operations:

a) $3^{-2}+\left(\frac{1}{2}\right)^{-1}+(-2)^{-1}+5^{0}$

b) $\left(x^{3}+4\right)^{2}$

c) $(3a-2c)^{2}$

d) $\frac{x+5}{x+10}\div\left(\frac{x^{2}+10x+25}{x^{2}+10x}\cdot\frac{10x}{x^{2}+15x+50}\right)$

e) $\frac{\frac{1}{x^{2}}+\frac{1}{y^{2}}}{\frac{1}{x}-\frac{1}{y}}$

f) $\frac{8}{a^{2}+6a}-\frac{3}{a^{2}+4a-12}$

g) $\frac{4y}{y^{2}-1}-\frac{5}{y^{2}+2y+1}$

h) $\frac{m}{m^{2}-1}+\frac{m-1}{m^{2}+2m+1}$

6. Solve the following equations by factoring.

a) $x(2x-3)=-1$

b) $3m^{2}+6m=0$

c) $(a-2)(a-4)=15$

d) $t^{3}+35t=12t^{2}$

7. Solve each equation. Write conditions whenever necessary.

a) $5x\left(x-\frac{3}{2}\right)\left(5x-\frac{2}{3}\right)\left(3-\frac{x}{4}\right)=0$

b) $\frac{3x-1}{x-2}=\frac{5}{x-2}+1$

c) $\frac{x+4}{x^{2}-3x+2}-\frac{5}{x^{2}-4x+3}=\frac{x-4}{x^{2}-5x+6}$

d) $\frac{1}{x-1}+\frac{1}{2}=\frac{2}{x^{2}-1}$

e) $I=\frac{E}{R+r}$ solve for r.

f) $m=\frac{Ry}{t}$ solve for t.

8. A stone is thrown upward from the top of a building. The height of the stone above the ground t seconds later is given by the formula

$$h=-16t^{2}+32t+240$$

where h is in feet.

a) Find the initial height of the stone.

b) Find the height of the stone after 1 second.

c) When will the stone be 256 feet above the ground?

d) How long will it take to hit the ground?

PART II of the test on next page.

PART II – Choose TWO problems. You can solve one more for extra credit.

1. Hooke's law for an elastic spring states that the distance a spring stretches varies directly with the force applied. If a force of 75 lb stretches a certain spring 15 inches, how much will a force of 150 lb stretch the spring?

2. A ladder is leaning against a building. The distance from the bottom of the ladder to the building is 4 ft less than the length of the ladder. How high up the side of the building is the top of the ladder if that distance is 2 ft less than the length of the ladder?

3. A boat goes 7 mph in still water. It takes as long to go 20 mi upstream as 50 mi downstream. Find the speed of the current.

4. Ella can paint a room, working alone, in 5 hours. Frank can do the job in 4 hours. How long will it take them to paint the room if they work together?

TEST 2 - SOLUTIONS

① $\left(\frac{mn^{-2}p^3}{m^{-1}np^{-4}}\right)^{-2}\left(\frac{m^3np^{-2}}{m^{-2}n^2p^3}\right)^{-1} =$

$\left(m^{1+1}n^{-2-1}p^{3+4}\right)^{-2}\left(m^{3+2}n^{1-2}p^{-2-3}\right)^{-1} =$

$\left(m^2n^{-3}p^7\right)^{-2}\left(m^5n^{-1}p^{-5}\right)^{-1} =$

$m^{-4}n^6p^{-14}m^{-5}np^5 =$

$m^{-4-5}n^{6+1}p^{-14+5} =$

$m^{-9}n^7p^{-9} = \boxed{\frac{n^7}{m^9p^9}}$

② $P(x) = 2x^5 + 3x^4 - \frac{1}{2}x^3 - 10x - 2$

a) 5 terms

b) degree 5

c) -2

d) $\boxed{P(0) = -2}$

$P(-1) = 2(-1) + 3(1) - \frac{1}{2}(-1) - 10(-1) - 2$

$P(-1) = -2 + 3 + \frac{1}{2} + 10 - 2$

$= 9 + \frac{1}{2} = \frac{19}{2}$

$\boxed{P(-1) = \frac{19}{2}}$

$P(a^2) = 2(a^2)^5 + 3(a^2)^4 - \frac{1}{2}(a^2)^3 - 10(a^2) - 2$

$\boxed{P(a^2) = 2a^{10} + 3a^8 - \frac{1}{2}a^6 - 10a^2 - 2}$

③

$$x-3 \;\overline{)\; 2x^3 - 5x^2 - 6x + 15}$$

$2x^2 + x - 3$

$-2x^3 + 6x^2$

$x^2 - 6x + 15$

$-x^2 + 3x$

$-3x + 15$

$+3x - 9$

6

Therefore,

$\boxed{\frac{2x^3 - 5x^2 - 6x + 15}{x-3} = 2x^2 + x - 3 + \frac{6}{x-3}}$

④

a) $26a^4b - 34a^3c^2 + 28a^2b^3 =$

$\boxed{2a^2\left(13a^2b - 17ac^2 + 14b^3\right)}$

b) $49 - m^2 = 7^2 - m^2$

$\boxed{(7-m)(7+m)}$

c) $a^2 - 4a - ab + 4b =$

$a(a-4) - b(a-4) =$

$\boxed{(a-4)(a-b)}$

d) $64x^3 - 27 = (4x)^3 - 3^3$

$= (4x-3)\left((4x)^2 + 4x(3) + 3^2\right)$

$= \boxed{(4x-3)(16x^2 + 12x + 9)}$

e) $25x^2 + 100 =$

$\boxed{25(x^2 + 4)}$

f) $2x + x^2 - 15 =$

$x^2 + 2x - 15 =$

$\boxed{(x+5)(x-3)}$

$10 = +5 \times -3$ (+5, −3)

$S = 2$

$15 = 5 - 3$

g) $3x^2 - 11x - 20 =$

$3x^2 + 4x - 15x - 20 =$

$x(3x+4) - 5(3x+4) =$

$\boxed{(3x+4)(x-5)}$

$p = -60 \times -15$ (+4, −15)

$S = -11$

$60 - 15 \cdot 4$

h) $1-16m^4 = 1^2-(4m^2)^2$
$=(1-4m^2)(1+4m^2)$
$=(1^2-(2m)^2)(1+4m^2)$
$=\boxed{(1-2m)(1+2m)(1+4m^2)}$

i) $9y^2+12y-15=$
$\boxed{3(3y^2+4y-5)}$

$p=-15 \swarrow^{+}_{-}$
$S=4$
$15=5\cdot 3$
$=15\cdot 1$

j) $\boxed{3t^2-15t+16 = \text{prime}}$

$p=48 \swarrow^{-}$
$S=-15$
$48=2\cdot 24$
$=4\cdot 12$
$=8\cdot 6$
$=16\cdot 3$
$=48\cdot 1$

k) $\boxed{c^2+16 \text{ - prime}}$

l) $3a^2+24a+48=$
$3(a^2+8a+16)=$
$\boxed{3(a+4)^2}$

m) $8+n^3 = 2^3+n^3$
$=(2+n)(2^2-2n+n^2)$
$=\boxed{(2+n)(4-2n+n^2)}$

n) $6a^2+40a+24=$
$2(3a^2+20a+12)=$
$2(3a^2+18a+2a+12)=$
$2(3a(a+6)+2(a+6))=$
$\boxed{2(a+6)(3a+2)}$

$p=36 \swarrow^{+18}_{+2}$
$S=20$
$36=18\cdot 2$

(5)

a) $3^{-2}+\left(\frac{1}{2}\right)^{-1}+(-2)^{-1}+5^0=$
$\frac{1}{3^2}+\frac{1}{\left(\frac{1}{2}\right)^1}+\frac{1}{(-2)^1}+1=$
$\frac{1}{9}+\frac{18/2}{1}-\frac{9/1}{2}+\frac{18/1}{1}=$

$LCD=18$
$\frac{2+36-9+18}{18}=\boxed{\frac{47}{18}}$

b) $(x^3+4)^2=(x^3)^2+2(x^3)4+4^2$
$=\boxed{x^6+8x^3+16}$

c) $(3a-2c)^2=(3a)^2-2(3a)(2c)+(2c)^2$
$=\boxed{9a^2-12ac+4c^2}$

d) $\frac{x+5}{x+10}\div\left(\frac{x^2+10x+25}{x^2+10x}\cdot\frac{10x}{x^2+15x+50}\right)=$

$\frac{x+5}{x+10}\div\left(\frac{(x+5)^2}{x(x+10)}\cdot\frac{10x}{(x+10)(x+5)}\right)=$

$\frac{x+5}{x+10}\div\frac{10(x+5)}{(x+10)^2}=$

$\frac{x+5}{x+10}\cdot\frac{(x+10)^2}{10(x+5)}=\boxed{\frac{x+10}{10}}$

e) $\frac{\frac{1^2}{x^2}+\frac{x^2}{y^2}}{\frac{y}{x}-\frac{x}{y}}=\frac{\frac{y^2+x^2}{x^2y^2}}{\frac{y-x}{xy}}$

$=\frac{x^2+y^2}{x^2y^2}\cdot\frac{xy}{y-x}$

$=\boxed{\frac{x^2+y^2}{xy(y-x)}}$

f) $\frac{8}{a^2+6a} - \frac{3}{a^2+4a-12} =$

$\frac{8}{a(a+6)} - \frac{3}{(a+6)(a-2)} =$

$LCD = a(a+6)(a-2)$

$\frac{8(a-2) - 3a}{a(a+6)(a-2)} =$

$\frac{8a-16-3a}{a(a+6)(a-2)} = \boxed{\frac{5a-16}{a(a+6)(a-2)}}$

g) $\frac{4y}{y^2-1} - \frac{5}{y^2+2y+1} =$

$\frac{4y}{(y-1)(y+1)} - \frac{5}{(y+1)^2} =$

$LCD = (y+1)^2(y-1)$

$\frac{4y(y+1) - 5(y-1)}{(y-1)(y+1)^2} =$

$\frac{4y^2+4y-5y+5}{(y-1)(y+1)^2} = \boxed{\frac{4y^2-y+5}{(y-1)(y+1)^2}}$

h) $\frac{m}{m^2-1} + \frac{m-1}{m^2+2m+1} =$

$\frac{m}{(m+1)(m-1)} + \frac{m-1}{(m+1)^2} =$

$LCD = (m-1)(m+1)^2$

$\frac{m(m+1) + (m-1)^2}{(m-1)(m+1)^2} =$

$\frac{m^2+m+m^2-2m+1}{(m-1)(m+1)^2} =$

$\boxed{\frac{2m^2-m+1}{(m-1)(m+1)^2}}$

(6) a) $x(2x-3) = 1$

$2x^2-3x+1=0$

$P = 2 \quad -1$
$S = -3 \quad -2$

$2 = 1 \cdot 2$

$2x^2-x-2x+1=0$

$x(2x-1)-(2x-1)=0$

$(2x-1)(x-1)=0$

$2x-1=0$ OR $x-1=0$

$x=\frac{1}{2}$ $\quad x=1$

$\boxed{x \in \{\frac{1}{2}, 1\}}$

b) $3m^2+6m=0$

$3m(m+2)=0$

$m=0$ OR $m+2=0$

$m=-2$

$\boxed{m \in \{0, -2\}}$

c) $(a-2)(a-4)=15$

$a^2-6a+8-15=0$

$a^2-6a-7=0$

$(a-7)(a+1)=0$

$a-7=0$ OR $a+1=0$

$a=7$ $\quad a=-1$

$\boxed{a \in \{7, -1\}}$

d) $t^3+35t=12t^2$

$t^3-12t^2+35t=0$

$t(t^2-12t+35)=0$

$t(t-7)(t-5)=0$

$t=0$ OR $t-7=0$ OR $t-5=0$

$t=7$ $t=5$

$\boxed{t\in\{0,7,5\}}$

(7) a) $5x(x-\frac{3}{2})(5x-\frac{2}{3})(3-\frac{x}{4})=0$

$x=0$

OR

$x-\frac{3}{2}=0 \Rightarrow x=\frac{3}{2}$ OR

$5x-\frac{2}{3}=0 \Rightarrow 5x=\frac{2}{3} \Rightarrow x=\frac{2}{15}$ OR

$3-\frac{x}{4}=0 \Rightarrow 3=\frac{x}{4} \Rightarrow x=12$

$\boxed{x\in\{0,\frac{3}{2},\frac{2}{15},12\}}$

b) $\frac{3x-1}{x-2}=\frac{5}{x-2}+1$

Condition: $x\neq 2$

$\frac{3x-1}{x-2}-\frac{5}{x-2}=1$

$\frac{3x-1-5}{x-2}=1$

$\frac{3x-6}{x-2}=1 \Rightarrow$

$3x-6=x-2$

$3x-x=-2+6$

$2x=4$

$x=2$ not possible

Therefore, no solutions

$\boxed{x\in\emptyset}$

c) $\frac{x+4}{x^2-3x+2}-\frac{5}{x^2-4x+3}=\frac{x-4}{x^2-5x+6}$

$\frac{\frac{x-3}{x+4}}{(x-1)(x-2)}-\frac{\frac{x-2}{5}}{(x-1)(x-3)}=\frac{\frac{x-1}{x-4}}{(x-2)(x-3)}$

Conditions: $\begin{cases}x\neq 1\\ x\neq 2\\ x\neq 3\end{cases}$

$LCD=(x-1)(x-2)(x-3)$

$(x-3)(x+4)-5(x-2)=(x-1)(x-4)$

$x^2+x-12-5x+10=x^2-5x+4$

$-4x-2=-5x+4$

$-4x+5x=4+2$

$x=6$

$\boxed{x\in\{6\}}$

d) $\frac{1}{x-1}+\frac{1}{2}=\frac{2}{x^2-1}$

$\frac{2(x+1)}{x-1}+\frac{x^2-1}{2}=\frac{2}{(x-1)(x+1)}$

Conditions: $\begin{cases}x\neq 1\\ x\neq -1\end{cases}$

$LCD=2(x-1)(x+1)$

$2(x+1)+x^2-1=4$

$2x+2+x^2-1-4=0$

$x^2+2x-3=0$

$(x-1)(x+3)=0$

$x-1=0$ OR $x+3=0$

$x=1$ $x=-3$

not possible

$\boxed{x \in \{-3\}}$

e) $i=\frac{E}{R+r}$, $r=?$

$i(R+r)=E$

$R+r=\frac{E}{i}$

$\boxed{r=\frac{E}{i}-R}$ or $r=\frac{E-Ri}{i}$

f) $m=\frac{Ry}{t}$, $t=?$

$mt=Ry$

$\boxed{t=\frac{Ry}{m}}$

(8) $h=-16t^2+32t+240$

a) $t=0$, $h=240$ ft

b) $t=1$, $h=-16+32+240$

$h=256$ ft

c) $t=?$ if $h=256$

$256=-16t^2+32t+240$

$16t^2-32t-240+256=0$

$16t^2-32t+16=0$

$16(t^2-2t+1)=0$

$16(t-1)^2=0$

$t-1=0$

$t=1$ second

The stone will be 256 ft above the ground after 1 second

d) $t=?$ if $h=0$

$0=-16t^2+32t+240$ / $\cdot(-1)$

$16t^2-32t-240=0$

$16(t^2-2t-15)=0$

$16(t-5)(t+3)=0$

$t-5=0$ OR $t+3=0$

$t=5$ $t=-3$ not possible

So, the stone will hit the ground after 5 seconds

PART II

(1) let d = distance (in)
F = force (lb)

then $d = kF$, k = constant of variation

if $F = 75$ lb, then $d = 15$ in

$\Rightarrow$

$15 \text{ in} = k \cdot 75 \text{ lb}$

$k = \frac{15}{75}$ in/lb

$k = \frac{1}{5}$ in/lb

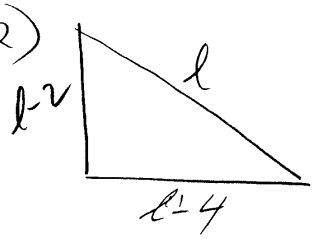
Therefore, $\boxed{d = \frac{1}{5}F}$

if $F = 150$ lb, find d

$d = \frac{1}{5} \cdot 150 = 30$

$\boxed{d = 30 \text{ in}}$

(2)



let l = length of ladder

Pythagorean theorem:

$(l-2)^2 + (l-4)^2 = l^2$

$l^2 - 4l + 4 + l^2 - 8l + 16 = l^2$

$l^2 - 12l + 20 = 0$

$(l-2)(l-10) = 0$

$l - 2 = 0$ or $l - 10 = 0$

$l = 2$ not possible $(l-2=0, l-4<0)$

$l = 10$

Therefore, the top of the ladder is $l - 2 = 8$ ft high up the side of the building

(3) let x = speed of current

	distance	rate	time
upstream	20 mi	$7-x$	same
downstream	50 mi	$7+x$	

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

$\frac{20}{7-x} = \frac{50}{7+x}$

$\frac{2}{7-x} = \frac{5}{7+x}$

$2(7+x) = 5(7-x)$

$14 + 2x = 35 - 5x$

$2x + 5x = 35 - 14$

$7x = 21 \Rightarrow \boxed{x = 3 \text{ mph}}$

the speed of current

(4) let x = time to paint it together

	Time to finish job	Part of job done per hour
Ella	5	$\frac{1}{5}$
Frank	4	$\frac{1}{4}$
together	x	$\frac{1}{x}$

$\frac{4x}{1} \quad \frac{5x}{1} \quad \frac{20}{1}$

$\frac{1}{5} + \frac{1}{4} = \frac{1}{x}$

$LCD = 20x$

$4x + 5x = 20$

$9x = 20$

$x = \frac{20}{9}$

$\boxed{x = 2\frac{2}{9} \text{ hours}}$ to finish together