TEST #1 @ 165 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. For an exercise to be complete there needs to be a detailed solution to the problem. No proof, no credit given! Clearly label each exercise.

PART ONE – SOLVE ALL

1. Simplify the following expression:

$$x\left[-x^2+x\left(2x-(5-x)\right)\right]$$

2. Solve the following equations:

a) $2(8y+1)-3(2-5y)+2(1-14y)=0$

b) $\frac{2}{5}x+\frac{1}{10}x-18=\frac{1}{20}x$

c) $11x-5(x+2)=6x+5$

d) $\frac{5a+1}{6}=\frac{2-a}{3}$

e) $3(6-4b)=2(-6b+9)$

f) $\frac{1}{6}m=\frac{3}{4}m$

g) $y=mx+b$ solve for m.

h) $C=\frac{5}{9}(F-32)$ solve for F.

3. For each inequality do the following:

i) Solve the inequality;
ii) Graph the solution set on the number line;
iii) Write the solution set using interval notation.

a) $-3(2x-1)\le 4$

b) $-12\le \frac{1}{2}t+1<4$

4. Graph the following inequality in a rectangular coordinate system. Label the boundary line and all the points used. Clearly show how you obtain the graph. Show all work.

$$x-5y<10$$

5. Answer each question:

a) Write the standard form of a linear equation in two variables (in general). Then give an example.

b) Write the slope-intercept form of a line (in general). Then give an example and specify the slope and *y*-intercept.

c) Write the point-slope form of a line (in general). Then give an example and specify the point and slope.

d) Explain the relationship between two parallel lines and their slopes. Then give an example of two lines that are parallel; that is, write their equations. Specify how you know they are parallel and distinct.

e) Explain the relationship between two perpendicular lines and their slopes. Then give an example of two lines that are perpendicular; that is, write their equations. Specify how you know they are perpendicular.

f) Write a mathematical formula for the slope of a line (in general).

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6. Graph each line on a separate coordinate system showing the intercepts (whenever appropriate). Label the axes and all the points used.

a) $2x - 3y = 0$

b) $2x - 5 = 0$

c) $y + 4 = 0$

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7. Let $3x + 4y = 36$ be a linear equation in two variables. Answer the following questions:

a) What is the slope of the given line?

b) What is the slope of a line parallel to the given line?

c) What is the slope of a line perpendicular to the given line?

d) What are the coordinates of the *x*- and *y*-intercepts?

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8. Do the following:

a) Write an equation of the line with slope 3 and *y*-intercept $(0, -2)$.

b) Write an equation of the line with slope $\frac{1}{2}$ passing through the point $(-1, 3)$. Then put the equation in slope-intercept form and standard form.

c) Write an equation of the line passing through the points $(3, -1)$ and $(2, 5)$.

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9. If one added to three times a number is three less than four times the number, find the number. Clearly define your variable and write an equation to represent the problem.

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10. The perimeter of a rectangle is 36 yd. The width is 18 yd less than twice the length. Find the length and the width of the rectangle. Clearly define the variable(s), translate the problem mathematically and solve the equation(s).

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11. Let

$$\begin{cases} 2x - y = 4 \\ 2x + 3y = 12 \end{cases}$$ be a system of two linear equation in two variables.

Do the following:

a) Solve the system graphically. Clearly show how you are graphing. Identify the solution on the graph.

b) Solve the system using the substitution method.

c) Solve the system using the elimination method.

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12. The table shows online retail spending R in billions of dollars t years after 1998.

t	R
1	15
3	35

a) Which variable is independent and which one is dependent?

b) What does the ordered pair $(1,15)$ mean in the context of this problem?

c) Assuming that the online retail spending grows at a linear rate, find the slope and its meaning in this context.

PART II – Choose TWO of the following problems. You may solve one other problem for extra credit.

For each problem, define the variable(s) clearly, then translate the problem mathematically and solve the equation(s).

1. How many liters of a 4% acid solution should be mixed with 50 liters of a 1% solution to obtain a solution that is 2% acid?

2. Mark is saving money for college. He deposited some money in a savings account paying 4% and $3000 more than that amount in a second account paying 6%. The two accounts produced a total of $780 interest in 1 year. How much did he invest at each rate?

3. A movie theater has two ticket prices: $8 for adults and $5 for children. If the box office took in $4116 from the sale of 600 tickets, how many tickets of each kind were sold?

4. Mary counted the money in her piggy bank. She found that she had only quarters and dimes. When she added up her money, she found that she had 39 coins worth a total of $7.50. How many coins of each kind did she have?

TEST 1 - SOLUTIONS

① $x[-x^2+x(2x-(5-x))] =$

$= x[-x^2+x(2x-5+x)]$

$= x[-x^2+x(3x-5)]$

$= x(-x^2+3x^2-5x)$

$= x(2x^2-5x)$

$= \boxed{2x^3-5x^2}$

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②

ⓐ $2(8y+1)-3(2-5y)+2(1-14y)=0$

$16y+2-6+15y+2-28y=0$

$3y-2=0$

$3y=2$

$\boxed{y=\frac{2}{3}}$

ⓑ $\overset{4}{\frac{2}{5}}x+\overset{2}{\frac{1}{10}}x-\overset{20}{18}=\frac{1}{20}x$

LCD = 20

$8x+2x-360=x$

$10x-360=x$

$10x-x=360$

$9x=360$

$x=\frac{360}{9}$

$\boxed{x=40}$

ⓒ $11x-5(x+2)=6x+5$

$11x-5x-10=6x+5$

$6x-10=6x+5$

contradiction

There are no solutions.

$\boxed{x\in\emptyset}$

ⓓ $\frac{5a+1}{6}=\frac{2-a}{3}$

cross-product property $\Rightarrow$

$3(5a+1)=6(2-a)$

$15a+3=12-6a$

$15a+6a=12-3$

$21a=9$

$a=\frac{9}{21}=\frac{3}{7}$

$\boxed{a=\frac{3}{7}}$

ⓔ $3(6-4b)=2(-6b+9)$

$18-12b=-12b+18$

identity

Any real number is a solution

$\boxed{b\in\mathbb{R}}$

(f) $\frac{1}{6}m = \frac{3}{4}m$

$\frac{m}{6} = \frac{3m}{4}$

cross-product property

$4m = 6 \cdot 3m$

$4m = 18m$

$0 = 18m - 4m$

$0 = 14m$

$\boxed{m = 0}$

(g) $y = mx + b$, $m = ?$

$y - b = mx$

$\boxed{m = \frac{y-b}{x}}$

(h) $C = \frac{5}{9}(F - 32)$, $F = ?$

$9C = 5(F - 32)$ $\quad | \div 5$

$\frac{9C}{5} = F - 32$

$\frac{9C}{5} + 32 = F$

$\boxed{F = \frac{9}{5}C + 32}$

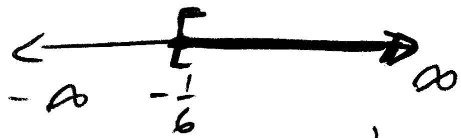
(3) (a) $-3(2x-1) \leq 4$

$-6x + 3 \leq 4$

$-6x \leq 4 - 3$ $\quad | \div -6$

$-6x \leq 1$

$\boxed{x \geq \frac{-1}{6}}$



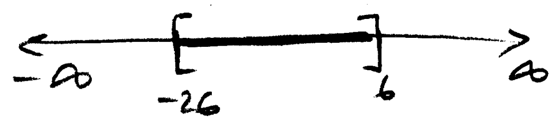
$x \in [-\frac{1}{6}, \infty)$

(b) $-12 \leq \frac{1}{2}t + 1 < 4$

$-12 - 1 \leq \frac{1}{2}t < 4 - 1$ $\quad | \cdot 2$

$-13 \leq \frac{1}{2}t < 3$

$\boxed{-26 \leq t < 6}$



$t \in [-26, 6]$

(4) $x-5y<10$ half plane

Boundary line

$x-5y=10$

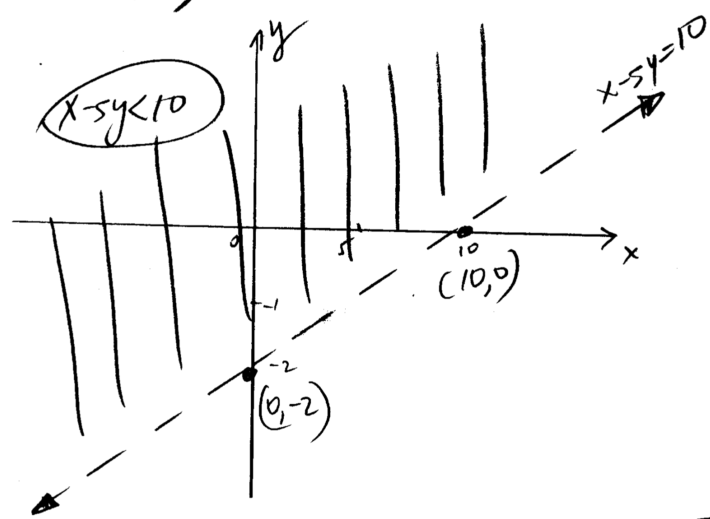
x	y
0	−2
10	0

Test point $(0,0) \notin$ line

$0-5(0)<10$

$0<10$ true

so $(0,0)$ = solution



(5) (a) $ax+by=c$

$2x-7y=5$

(b) $y=mx+b$

$y=3x+4$ $\begin{cases} m=3 \\ (0,4)=y\text{-}I \end{cases}$

(c) $y-y_1=m(x-x_1)$

$y-3=-\frac{1}{2}(x+5)$

$\begin{cases} m=\frac{-1}{2} \\ (-5,3) \text{ - point on the line} \end{cases}$

(d) Two lines are parallel if and only if they have the same slope.

$y=2x+5$

$y=2x-1$

$m_1=m_2=2$

$\left.\begin{matrix} b_1=5 \\ b_2=-1 \end{matrix}\right\}$ so the lines are distinct

The two lines are distinct and they have the same slope, so they are parallel

(e) Two lines are perpendicular if and only if the product of their slopes is −1

$y=3x+7$

$y=\frac{-1}{3}x+2$

$m_1=3$

$m_2=\frac{-1}{3}$

and $m_1 \cdot m_2=-1$

so the lines are $\perp$

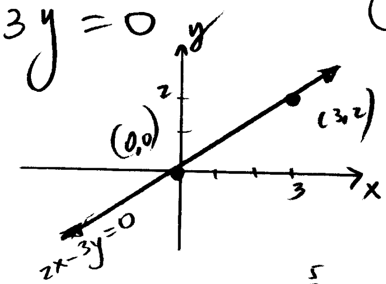
(f) $m=\frac{\Delta y}{\Delta x}$

$m=\frac{y_1-y_2}{x_1-x_2}$

where (x_1,y_1) and (x_2,y_2) $\in$ line

(6) (a) $2x - 3y = 0$

x	y
0	0
3	2

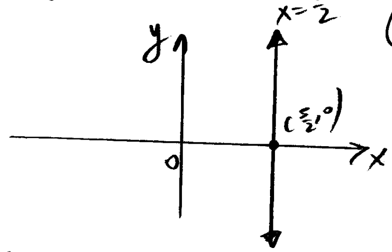


(b) $2x - 5 = 0$

$2x = 5$

$x = \frac{5}{2}$

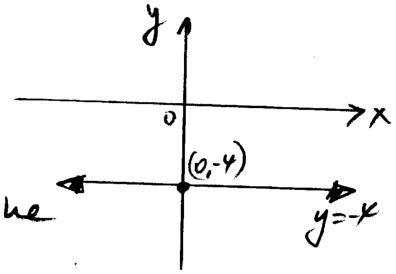
vertical line



(c) $y + 4 = 0$

$y = -4$

horizontal line



(7) $3x + 4y = 36$

(a) $4y = -3x + 36 \quad | \div 4$

$y = \frac{-3}{4}x + \frac{36}{4}$

$y = \frac{-3}{4}x + 9$, so $\boxed{m = \frac{-3}{4}}$

(b) $l_1 \parallel l_2$ iff $m_1 = m_2$

so $\boxed{m_{\parallel} = \frac{-3}{4}}$

(c) $l_1 \perp l_2$ iff $m_1 = \frac{-1}{m_2}$

so $\boxed{m_{\perp} = \frac{4}{3}}$

(d)

x	y
0	9
12	0

$(0, 9) = y\text{-}\cap$

$(12, 0) = x\text{-}\cap$

(8) (a) $m = 3$

$(0, -2) = y\text{-}\cap$

$y = mx + b$

$\boxed{y = 3x - 2}$

(b) $m = \frac{1}{2}$

$(-1, 3)$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{2}(x - (-1))$

$\boxed{y - 3 = \frac{1}{2}(x + 1)}$

$y - 3 = \frac{1}{2}x + \frac{1}{2}$

$y = \frac{1}{2}x + \frac{1}{2} + 3$

$\boxed{y = \frac{1}{2}x + \frac{7}{2}}$ slope-∩ form

$\boxed{-\frac{1}{2}x + y = \frac{7}{2}}$ standard form

(c) $(3, -1)$ and $(2, 5)$

$m = \frac{\Delta y}{\Delta x} = \frac{5 - (-1)}{2 - 3} = \frac{6}{-1}$

$m = -6$

use $m = -6$ and $(2, 5)$

$y - y_1 = m(x - x_1)$

$\boxed{y - 5 = -6(x - 2)}$

(9) let x = the number

$$3x+1 = 4x-3$$

$$1+3 = 4x-3x$$

$$4 = x$$

so $\boxed{x=4}$

The number with the given property is 4.

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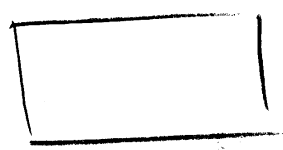
(10) perimeter = 36 yd



Method I

let l = length

then $2l-18$ = width

Perimeter = 36

$$2l + 2(2l-18) = 36$$

$$2l + 4l - 36 = 36$$

$$6l = 72$$

$$l = \frac{72}{6} = 12$$

then $2l-18 = 2(12)-18 = 6$

The length is 12 yd

The width is 6 yd

Method II

let l = length

w = width

$$\begin{cases} w = 2l-18 \\ 2l+2w = 36 \end{cases} \Big/ \div 2$$

$$\begin{cases} w = 2l-18 \\ l+w = 18 \end{cases}$$

Substitution method $\Rightarrow$

$$l+(2l-18) = 18$$

$$3l = 36 \Rightarrow l = 12$$

then $w = 2(12)-18 = 6$

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(11) $\begin{cases} 2x-y=4 \\ 2x+3y=12 \end{cases}$

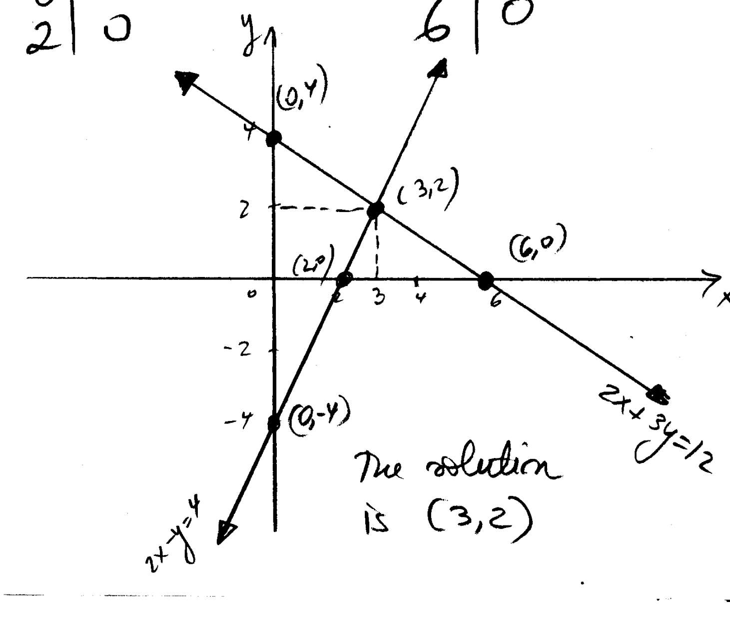
(a) Graphical method.

$2x-y=4$

x	y
0	-4
2	0

$2x+3y=12$

x	y
0	4
6	0



(b) Substitution method

$$\begin{cases} 2x - y = 4 & \text{eq. (1)} \\ 2x + 3y = 12 & \text{eq. (2)} \end{cases}$$

eq. (1): $2x - y = 4$

$2x - 4 = y$

$y = 2x - 4$

eq. (2): $2x + 3(2x-4) = 12$

$2x + 6x - 12 = 12$

$8x = 24$

$x = 3$

Then $y = 2x - 4$

$y = 2(3) - 4 = 2$

The solution is $(3, 2)$.

(c) Elimination method

$$\begin{cases} 2x - y = 4 \\ 2x + 3y = 12 \end{cases} \Big| \cdot(-1)$$

$$\begin{cases} -2x + y = -4 \\ 2x + 3y = 12 \end{cases}$$

(+) $4y = 8 \Rightarrow y = 2$

Then, $2x - y = 4$

$2x - 2 = 4$

$2x = 6 \Rightarrow x = 3$

Solution $= (3, 2)$.

(2)

t	R
1	15
3	35

t = number of years after 1998

R = online retail spending (in billions of \$)

(a) t = independent

R = dependent variable

(b) $(1, 15)$: $t = 1$, $R = 15$

1 year after 1998 (in 1999) the online retail spending was 15 billion \$

(c) $m = \dfrac{\Delta R}{\Delta t} = \dfrac{35 - 15}{3 - 1} = \dfrac{20}{2} = 10$

$m = 10$ billion \$/year

The slope shows the rate of increase of the online retail spending per year

The online retail spending increases at a rate of 10 billion \$ per year.

PART II

(1) 4% 1% 2%

$$\boxed{\begin{matrix} x \\ \text{liters} \end{matrix}} + \boxed{\begin{matrix} 50 \\ \text{liters} \end{matrix}} = \boxed{\begin{matrix} x+50 \\ \text{liters} \end{matrix}}$$

let x = the number of liters of the 4% solution

$$4\% x + 1\% (50) = 2\% (x+50)$$
$$\frac{4}{100}x + \frac{1}{100} \cdot 50 = \frac{2}{100}(x+50) \quad \Big| \cdot 100$$
$$4x + 50 = 2(x+50)$$
$$4x + 50 = 2x + 100$$
$$4x - 2x = 100 - 50$$
$$2x = 50 \implies \boxed{x = 25 \text{ liters of } 4\% \text{ solution}}$$

(2) Accounts 780$ interest
- I at 4% x \$
- II at 6% $(x+3000)$ \$

Method I

let x = amount at 4%

then $x+3000$ = amount at 6%

Then, Total interest = 780

$$4\% x + 6\% (x+3000) = 780$$
$$\frac{4}{100}x + \frac{6}{100}(x+3000) = 780 \quad \Big| \cdot 100$$
$$4x + 6(x+3000) = 78{,}000$$
$$4x + 6x + 18{,}000 = 78{,}000$$
$$10x = 60{,}000 \implies x = 6000 \$$$

He invested 6000 \$ at 4% and 9000 \$ at 6%.

Method II let x = \$ at 4%
y = \$ at 6%

$$\begin{cases} y = x + 3000 \\ 4\% x + 6\% y = 780 \end{cases}$$

and solve the system using substitution (for example)

(3) let a = number of adult tickets
c = number of children tickets

$$\begin{cases} a + c = 600 \quad \Big| \cdot (-5) \\ 8a + 5c = 4116 \end{cases}$$

use Elimination method

$$\begin{cases} -5a - 5c = -3000 \\ 8a + 5c = 4116 \end{cases}$$

$$3a = 1116$$
$$a = \frac{1116}{3} = 372 \text{ children tickets}$$

$$a + c = 600$$
$$372 + c = 600$$
$$c = 600 - 372 = 228 \text{ adult tickets}$$

They sold 228 adult tickets and 372 children tickets

(4) Let x = the number of quarters
y = the number of dimes

$$\begin{cases} x+y=39 \\ 25x+10y=750 \end{cases}$$

- total number of coins is 39
- total value is 7.50 \$ = 750 cents

Use elimination method

$$\begin{cases} -10x-10y=-390 \\ 25x+10y=750 \end{cases}$$

$15x = 360$

$x = \frac{360}{15} = 24$ quarters

$x+y=39$

$24+y=39$

$y=39-24=15$ dimes

She had 24 quarters and 15 dimes