QUIZ #8 @ 25 points

Write neatly, Show all work. Use only information covered up to this point.
Write all responses on separate paper. **Clearly label the exercises.**

1. Analyze and graph the following function:

$$y = x^4 + 2x^3.$$

Show domain, intercepts, end-behavior, intervals on which the function increases and decreases, as well as concavity. Show all work and organize all data about f', f, and f'' in a table.

2. Solve the following limits. Clearly justify your steps (if applying l'Hopital, show why the rule can be applied and what case it is specifically).

a) $\lim\limits_{x\to 0} \dfrac{7x^2}{\cos x - 1}$

b) $\lim\limits_{x\to -\infty} \left(x^2 - 5\right)e^x$

c) $\lim\limits_{x\to \infty} x^2 e^{-x}$

Math 180 Quiz # 8 - SOLUTIONS

① $f(x) = x^4 + 2x^3$

x	$-\infty$		-2		$-\frac{3}{2}$		-1		0		∞
f'	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$+$ $+$ $+$ $+$	$+$
f	∞	$\searrow$	0	$\searrow$	$-\frac{27}{16}$	$\nearrow$	-1	$\nearrow$	0	$\nearrow$	∞
f''	$+$	$+$ $+$ $+$	$+$	$+$	$+$	$+$	0	$-$	0	$+$ $+$ $+$ $+$	$+$

(f) Domain: $x \in \mathbb{R}$

x-n: $x^4 + 2x^3 = 0$

$x^3(x+2) = 0$ $\Big\langle$ $x = 0$ $(0,0)$; $x = -2$ $(-2,0)$

y-n: $(0,0)$

$\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{\pm\infty} x^4\left(1 + \frac{2}{x}\right) = \infty(1+0) = \infty$

(f') $f'(x) = 4x^3 + 6x^2$

$f'(x) = 0 \Rightarrow$

$4x^3 + 6x^2 = 0$

$2x^2(2x+3) = 0$ $\Big\langle$ $x = 0$; $x = -\frac{3}{2}$ critical points

The sign of f' is given by $2x+3$ $\quad - \quad -\frac{3}{2} \quad + \quad > x$

$(2x^2 > 0, \ \forall x \neq 0)$

$f\left(-\frac{3}{2}\right) = \left(\frac{-3}{2}\right)^4 + 2\left(\frac{-3}{2}\right)^3 = \frac{81}{16} - \frac{54}{8} = \frac{-27}{16} =$

(f'') $f''(x) = 12x^2 + 12x$

$f''(x) = 0 \Rightarrow$

$12x^2 + 12x = 0$

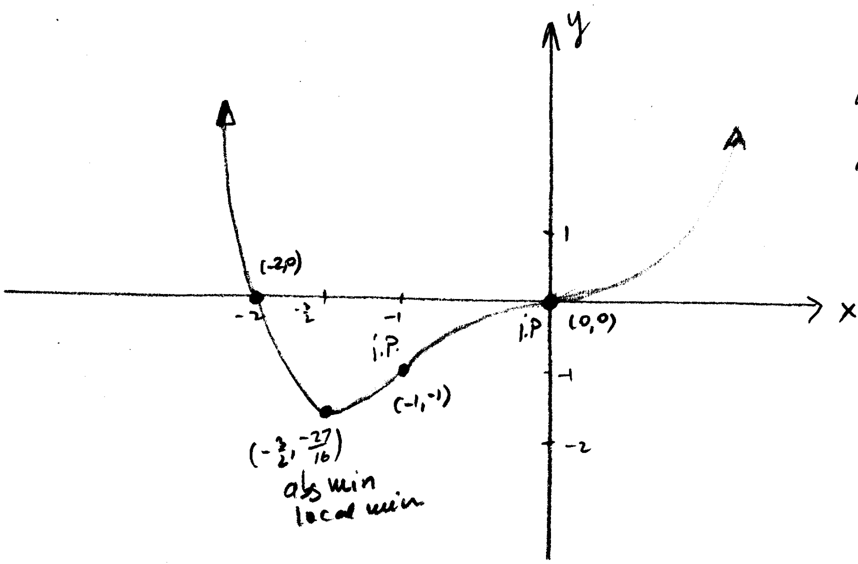
$12x(x+1) = 0$ $\Big\langle$ $x = 0$; $x = -1$

The Sign of f'': $+ \quad -1 \quad - \quad 0 \quad + \quad > x$

$y = 12x^2 + 12x$

parabola opening upword

$f(-1) = (-1)^4 + 2(-1)^3 = 1 - 2 = -1$



abs. min / local $\left(\frac{-3}{2}, \frac{-27}{16}\right)$

inflection points $(-1,1)$ and $(0,0)$

(2a) $\lim\limits_{x\to 0} \dfrac{7x^2}{\cos x - 1} = \dfrac{0}{0}$ (l'Hopital)

$= \lim\limits_{x\to 0} \dfrac{14x}{-\sin x} = \dfrac{0}{0}$ (l'Hopital)

$= \lim\limits_{x\to 0} \dfrac{14}{-\cos x} = \dfrac{14}{-1} = -14$

(2b) $\lim\limits_{x\to -\infty} (x^2-5)e^{x} = \infty \cdot 0$

$= \lim\limits_{x\to -\infty} \dfrac{x^2-5}{e^{-x}} = \dfrac{\infty}{\infty}$ (l'Hopital)

$= \lim\limits_{x\to -\infty} \dfrac{2x}{-e^{-x}} = \dfrac{\infty}{\infty}$ (l'Hopital)

$= \lim\limits_{-\infty} \dfrac{2}{e^{-x}} = \dfrac{2}{e^{\infty}} = \dfrac{2}{\infty} = 0$

(2c) $\lim\limits_{x\to \infty} x^2 e^{-x} = \infty \cdot 0$

$= \lim\limits_{\infty} \dfrac{x^2}{e^x} = \dfrac{\infty}{\infty}$ (l'Hopital)

$= \lim\limits_{\infty} \dfrac{2x}{e^x} = \dfrac{\infty}{\infty}$ (l'Hopital)

$= \lim\limits_{\infty} \dfrac{2}{e^x} = \dfrac{2}{\infty} = 0$