Quiz #6

Find the derivative of each function:

① $y = \dfrac{\cos x}{1 - \sin x}$

② $y = \tan(3 - \sin(4x))$

③ $y = \sqrt{2x - x^2}$

④ $y = \sin(x^3 e^{2x})$

⑤ Find y'' if $y = \csc x$

M180

Solutions - Quiz #6

(1) $y = \dfrac{\cos x}{1-\sin x}$

$$y' = \frac{dy}{dx} = \frac{(\cos x)'(1-\sin x) - \cos x(1-\sin x)'}{(1-\sin x)^2}$$

$$= \frac{-\sin x(1-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$$

$$= \frac{1-\sin x}{(1-\sin x)^2} = \boxed{\frac{1}{1-\sin x}}$$

(2) $y = \tan(3-\sin(4x))$

$\left((\tan u)' = \dfrac{1}{\cos^2 u} \cdot u' \quad \text{with } u = 3-\sin(4x) \right)$

$$y' = \frac{dy}{dx} = \frac{1}{\cos^2(3-\sin 4x)} \cdot (3-\sin(4x))'$$

$\left((\sin u)' = \cos u \cdot u' \quad \text{with } u = 4x \right)$

$$= \frac{-\cos(4x)\cdot(4x)'}{\cos^2(3-\sin(4x))}$$

$$= \boxed{\frac{-4\cos(4x)}{\cos^2(3-\sin(4x))}}$$

(3) $y = \sqrt{2x-x^2}$

$\left((\sqrt{u})' = \dfrac{1}{2\sqrt{u}} u' ; \ u = 2x-x^2 \right)$

$$y' = \frac{dy}{dx} = \frac{1}{2\sqrt{2x-x^2}} \cdot (2x-x^2)'$$

$$= \frac{2-2x}{2\sqrt{2x-x^2}}$$

$$= \frac{2(1-x)}{2\sqrt{2x-x^2}} = \boxed{\frac{1-x}{\sqrt{2x-x^2}}}$$

(4) $y = \sin(x^3 e^{2x})$

$\left((\sin u)' = \cos u \cdot u' \quad \text{with } u = x^3 e^{2x} \right)$

$$y' = \cos(x^3 e^{2x}) \cdot (x^3 e^{2x})'$$

$$= \cos(x^3 e^{2x}) \cdot (3x^2 e^{2x} + x^3 (e^{2x})')$$

$\left((e^u)' = e^u \cdot u' \quad \text{with } u = 2x \right)$

$$= \cos(x^3 e^{2x}) \cdot (3x^2 e^{2x} + x^3 \cdot e^{2x} \cdot (2x)')$$

$$= \cos(x^3 e^{2x}) \cdot (3x^2 e^{2x} + 2x^3 e^{2x})$$

$$= \boxed{x^2 e^{2x}(3+2x)\cos(x^3 e^{2x})}$$

(5) $y = \csc x$

Method I

$$y = \frac{1}{\sin x}$$

$$y' = \frac{(1)' \cdot \sin x - 1 \cdot (\sin x)'}{(\sin x)^2}$$

$$y' = \frac{0 - \cos x}{\sin^2 x}$$

$$y' = \frac{-\cos x}{\sin^2 x}$$

$$y'' = \frac{(-\cos x)' \sin^2 x - (-\cos x)(\sin^2 x)'}{(\sin^2 x)^2}$$

$\left((u^2)' = 2u \cdot u' \right.$ with $\left. u = \sin x \right)$

$$y'' = \frac{-(-\sin x) \sin^2 x + \cos x \cdot \left(2 \sin x \cdot (\sin x)'\right)}{\sin^4 x}$$

$$= \frac{\sin^3 x + 2 \sin x \cos x \cdot \cos x}{\sin^4 x}$$

$$= \frac{\sin x (\sin^2 x + 2\cos^2 x)}{\sin^4 x}$$

$$= \frac{\sin^2 x + 2\cos^2 x}{\sin^3 x}$$

$$= \boxed{\frac{1 + \cos^2 x}{\sin^3 x}}$$

Method II

$y = \csc x$

$y' = -\csc x \cot x$

$y'' = -(\csc x)' \cot x + (-\csc x)(\cot x)'$

$y'' = -(-\csc x \cot x) \cot x - \csc x \dfrac{-1}{\sin^2 x}$

$$\boxed{y'' = \csc x \cot^2 x + \frac{\csc x}{\sin^2 x}}$$

$$y'' = \frac{1}{\sin x} \cdot \frac{\cos^2 x}{\sin^2 x} + \frac{1}{\sin^3 x}$$

$$\boxed{y'' = \frac{1 + \cos^2 x}{\sin^3 x}}$$