Math 180 Fall 2008 Name:

QUIZ #5 @ 25 points

Write neatly. Show all work. Use only information covered up to this point.
Write all responses on separate paper. Clearly label the exercises.

1. Find the derivative of each function:

a) $y = \sqrt[3]{x^2}$

b) $f(t) = \dfrac{t^2 - 1}{t^2 + t - 2}$

c) $y = x^4 e^x + \dfrac{1}{x}$

d) $f(x) = \dfrac{1}{x}\left(x^2 + e^x\right)$

2. Find the first and second derivatives of the following function:

$$y = \frac{x^3 + 7}{x}$$

Quiz 5

(1) (a) $y = \sqrt[3]{x^2} = x^{\frac{2}{3}}$

$\frac{dy}{dx} = y' = (x^{\frac{2}{3}})'$

$= \frac{2}{3} x^{\frac{2}{3}-1}$

$= \frac{2}{3} x^{-\frac{1}{3}}$

$$\boxed{y' = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}}$$

(b) $f(t) = \frac{t^2-1}{t^2+t-2}$

$f(t) = \frac{(t-1)(t+1)}{(t+2)(t-1)}$

$f(t) = \frac{t+1}{t+2}$

$f'(t) = \frac{df}{dt} = \frac{(t+1)'(t+2) - (t+1)(t+2)'}{(t+2)^2}$

$= \frac{1\cdot(t+2) - (t+1)\cdot 1}{(t+2)^2}$

$= \frac{t+2-t-1}{(t+2)^2}$

$$\boxed{f'(t) = \frac{1}{(t+2)^2}}$$

(c) $y = x^4 e^x + \frac{1}{x}$

$\frac{dy}{dx} = y' = (x^4)'e^x + x^4(e^x)' + (\frac{1}{x})'$

$$\boxed{y' = 4x^3 e^x + x^4 e^x - \frac{1}{x^2}}$$

(d) $f(x) = \frac{1}{x}(x^2 + e^x)$

Method I: Product rule

$f'(x) = (\frac{1}{x})'(x^2+e^x) + \frac{1}{x}(x^2+e^x)'$

$= \frac{-1}{x^2}(x^2+e^x) + \frac{1}{x}(2x+e^x)$

$= -1 - \frac{e^x}{x^2} + 2 + \frac{e^x}{x}$

$$\boxed{f'(x) = 1 - \frac{e^x}{x^2} + \frac{e^x}{x}}$$

Method II: Quotient rule

$f(x) = \frac{x^2+e^x}{x}$

$f'(x) = \frac{(x^2+e^x)'x - (x^2+e^x)(x)'}{x^2}$

$= \frac{(2x+e^x)x - (x^2+e^x)\cdot 1}{x^2}$

$= \frac{2x^2 + xe^x - x^2 - e^x}{x^2}$

$$\boxed{f'(x) = \frac{x^2 + xe^x - e^x}{x^2}}$$

Method III

$$f(x) = x + \frac{e^x}{x}$$

$$f'(x) = 1 + \frac{(e^x)'x - e^x \cdot x'}{x^2}$$

$$f'(x) = 1 + \frac{e^x x - e^x}{x^2}$$

$$\boxed{f'(x) = \frac{x^2 + xe^x - e^x}{x^2}}$$

(2) $$y = \frac{x^3+7}{x} = x^2 + \frac{7}{x}$$

$$y' = \frac{dy}{dx} = 2x + 7 \cdot \frac{-1}{x^2}$$

$$\boxed{y' = 2x - \frac{7}{x^2}}$$

$$y' = 2x - 7x^{-2}$$

$$y'' = \frac{d^2y}{dx^2} = 2 - 7(-2)x^{-3}$$

$$y'' = 2 + 14x^{-3}$$

$$\boxed{y'' = 2 + \frac{14}{x^3}}$$