QUIZ #4 @ 25 points

Write neatly. Show all work. Use only information covered up to this point.
Write all responses on separate paper. **Clearly label the exercises.**

1) Find the following limits. If a limit doesn't exist, clearly show why. Do not just an answer. No work, no credit given.

a) $\lim\limits_{x\to 2^+} \dfrac{x^2-3x+2}{x^3-4x}$

b) $\lim\limits_{x\to 2} \dfrac{x-3}{x^2-4}$

c) $\lim\limits_{x\to 0^-} \dfrac{-1}{2x}$

d) $\lim\limits_{x\to 0^+} \left(1+\csc x\right)$

2) Find an equation for the tangent to the curve $f\left(x\right)=3-x^2$ at the point $\left(2,-1\right)$.

MATH180 | Quiz 4 - SOLUTIONS |

(1a) $\lim\limits_{x\to 2^+} \dfrac{x^2-3x+2}{x^3-4x} =$

$= \lim\limits_{x\to 2^+} \dfrac{(x-2)(x-1)}{x(x^2-4)}$

$= \lim\limits_{x\to 2^+} \dfrac{\cancel{(x-2)}(x-1)}{x\cancel{(x-2)}(x+2)}$

$= \lim\limits_{x\to 2^+} \dfrac{x-1}{x(x+2)} = \dfrac{2-1}{2(2+2)} = \boxed{\dfrac{1}{8}}$

(b) $\lim\limits_{x\to 2} \dfrac{x-3}{x^2-4} = \lim\limits_{x\to 2} \dfrac{x-3}{(x-2)(x+2)}$

Note that if $x>2$, $x-2>0$
if $x<2$, $x-2<0$

$\lim\limits_{x\to 2^-} \dfrac{x-3}{(x-2)(x+2)} = \dfrac{2-3}{0^-(2+2)} = \dfrac{-1}{0^-} = \infty$

$\lim\limits_{x\to 2^+} \dfrac{x-3}{(x-2)(x+2)} = \dfrac{2-3}{0^+(4)} = \dfrac{-1}{0^+} = -\infty$

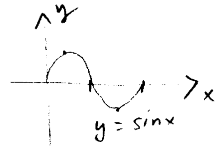
Therefore $\boxed{\lim\limits_{x\to 2} \dfrac{x-3}{x^2-4} \text{ doesn't exist}}$

(c) $\lim\limits_{x\to 0^-} \dfrac{-1}{2x} = \dfrac{-1}{0^-} = \boxed{\infty}$

(d) $\lim\limits_{x\to 0^+} (1+\csc x) =$

$= \lim\limits_{x\to 0^+} \left(1+\dfrac{1}{\sin x}\right)$

(when $x\to 0^+$, $\sin x>0$



$= 1+\dfrac{1}{0^+} = \boxed{\infty}$

(2) $f(x) = 3-x^2 \quad (2,-1)$

For the equation of the tangent we need
{ a point $(2,-1)$ - given
{ slope m

$m = \lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$

$= \lim\limits_{h\to 0} \dfrac{(3-(2+h)^2)-(3-2^2)}{h}$

$= \lim\limits_{h\to 0} \dfrac{3-4-4h-h^2+1}{h}$

$= \lim\limits_{h\to 0} \dfrac{-\cancel{h}(4+h)}{\cancel{h}}$

$= -\lim\limits_{h\to 0}(4+h) = -4$

so $m=-4$

Therefore,

$y-(-1) = -4(x-2)$

$y+1 = -4x+8$

$\boxed{y = -4x+7}$

The equation of the tangent line at $(2,-1)$