QUIZ #3 @ 25 points

Write neatly. Show all work. **Write all responses on separate paper.** **Clearly label the exercises.**

1. Let $f(x)=\begin{cases} 1-x, & x<2 \\ 2, & x=2 \\ \dfrac{x}{3}, & x>2 \end{cases}$.

Find the following limits (if possible). If a limit does not exist, explain why.

a) $\lim_{x\to 2^+} f(x)$

b) $\lim_{x\to 2^-} f(x)$

c) $\lim_{x\to 0} f(x)$

d) $\lim_{x\to 2} f(x)$

2. Find the following limits. Show all work. Do not just write an answer. No work, no credit given.

a) $\lim_{x\to 0^-} \dfrac{|x|}{x}$

b) $\lim_{x\to 0^+} \sqrt{x}\sin\dfrac{1}{x}$

c) $\lim_{x\to 0} \dfrac{\tan 3x}{x}$

d) $\lim_{x\to -\infty} \dfrac{-9x^4+5x^2-1}{2x^4-3x+5}$

e) $\lim_{h\to 0^+} \dfrac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}$

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Quiz 3 - Solutions

① $f(x)=\begin{cases} 1-x, & x<2 \\ 2, & x=2 \\ \frac{x}{3}, & x>2 \end{cases}$

(a) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x}{3} = \boxed{\frac{2}{3}}$

(b) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(1-x) = 1-2 = \boxed{-1}$

(c) $\lim_{x\to 0} f(x) = \lim_{x\to 0}(1-x) = \boxed{1}$

(d) $\lim_{x\to 2} f(x)$ $\boxed{\text{doesn't exist}}$

because $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

② (a) $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x}$

$= \lim_{x\to 0^-}(-1) = \boxed{-1}$

(b) $\lim_{x\to 0^+} \sqrt{x}\sin\frac{1}{x}$

We know $-1 \le \sin\frac{1}{x} \le 1$ for any $x \neq 0$ / $\cdot\sqrt{x}>0$

(when $x\to 0^+$, $x>0$, so $\sqrt{x}>0$)

$-\sqrt{x} \le \sqrt{x}\sin\frac{1}{x} \le \sqrt{x}$

$\rightarrow 0 \leftarrow$ when $x\to 0^+$

So, by the squeeze theorem,

$\boxed{\lim_{x\to 0^+} \sqrt{x}\sin\frac{1}{x} = 0}$

(c) $\lim_{x\to 0} \frac{\tan 3x}{x} = \lim_{x\to 0} \frac{\sin 3x}{x\cos 3x}$

$= \lim_{x\to 0} \frac{\sin 3x}{3x}\cdot\frac{3}{\cos 3x}$

$= \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \lim_{x\to 0}\frac{3}{\cos 3x}$

$= 1\cdot\frac{3}{1} = \boxed{3}$

(d) $\lim_{x\to -\infty} \frac{-9x^4+5x^2-1}{2x^4-3x+5}$

$= \lim_{x\to -\infty} \frac{\frac{-9x^4}{x^4}+\frac{5x^2}{x^4}-\frac{1}{x^4}}{\frac{2x^4}{x^4}-\frac{3x}{x^4}+\frac{5}{x^4}}$

$= \lim_{x\to -\infty} \frac{-9+\frac{5}{x^2}-\frac{1}{x^4}}{2-\frac{3}{x^3}+\frac{5}{x^4}} = \frac{-9+0-0}{2-0+0}$

$= \boxed{\frac{-9}{2}}$

(e) $\lim_{h\to 0^+} \frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h} = \left(\frac{0}{0}\right)$

$= \lim_{h\to 0^+} \frac{(\sqrt{h^2+4h+5}-\sqrt{5})(\sqrt{h^2+4h+5}+\sqrt{5})}{h(\sqrt{h^2+4h+5}+\sqrt{5})}$

$= \lim_{h\to 0^+} \frac{h^2+4h+\cancel{5}-\cancel{5}}{h(\sqrt{h^2+4h+5}+\sqrt{5})}$

$= \lim_{h\to 0^+} \frac{\cancel{h}(h+4)}{\cancel{h}(\sqrt{h^2+4h+5}+\sqrt{5})}$

$= \frac{4}{\sqrt{5}+\sqrt{5}}$

$= \boxed{\frac{4}{2\sqrt{5}}}$