ASSIGNMENT

SECTION 5.5

(4) $\int \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\, dt = I$

let $1-\cos\frac{t}{2} = u$

$\frac{1}{2}\cdot \sin\frac{t}{2}\, dt = du$

$\Rightarrow \sin\frac{t}{2}\, dt = 2\,du$

$I = \int 2u^2\, du = 2\cdot\frac{u^3}{3} + C$

$\boxed{I = \frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C}$

(16) $\int \frac{3\,dx}{(2-x)^2}$

let $2-x = u$

$-dx = du \Rightarrow dx = -du$

$\int \frac{3\,dx}{(2-x)^2} = -\int \frac{3\,du}{u^2}$

$= -3\int u^{-2}\,du$

$= -3\,\frac{u^{-2+1}}{-1} + C$

$= 3(2-x)^{-1} + C$

$= \boxed{\frac{3}{2-x} + C}$

(22) $\int \tan^2 x \sec^2 x\, dx = I$

let $\tan x = u$

$\sec^2 x\, dx = du$

$I = \int u^2\, du = \frac{u^3}{3} + C$

$\boxed{I = \frac{1}{3}\tan^3 x + C}$

(34) $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\,\sin^2\sqrt{\theta}}\, d\theta = I$

let $\sin\sqrt{\theta} = u$

$(\cos\sqrt{\theta})\cdot\frac{1}{2\sqrt{\theta}}\, d\theta = du$

$\Rightarrow \frac{\cos\sqrt{\theta}}{\sqrt{\theta}}\, d\theta = 2\,du$

$I = \int \frac{2\,du}{u^2} = 2\int u^{-2}\,du$

$= 2\left(-u^{-2+1}\right) + C$

$= -2(\sin\sqrt{\theta})^{-1} + C$

$\boxed{I = \frac{-2}{\sin\sqrt{\theta}} + C}$

(40) $\int (\sin 2\theta)\, e^{\sin^2\theta}\, d\theta = I$

let $\sin^2\theta = u$

$2\sin\theta\cos\theta\, d\theta = du$

$\sin 2\theta\, d\theta = du$

$I = \int e^u\, du = e^u + C$

$$\boxed{I = e^{\sin^2\theta} + C}$$

(52) $\int \frac{\sqrt{\tan^{-1}x}}{1+x^2}\, dx = I$

let $\tan^{-1}x = u$

$\frac{1}{1+x^2}\, dx = du$

$I = \int \sqrt{u}\, du = \int u^{\frac{1}{2}}\, du$

$I = \frac{u^{\frac{1}{2}+1}}{\frac{3}{2}} + C$

$I = \frac{2}{3}(\tan^{-1}x)^{\frac{3}{2}} + C$

$$\boxed{I = \frac{2}{3}\sqrt{(\tan^{-1}x)^3} + C}$$

SECTION 5.6

(28) $I = \int_0^{\pi/3} \frac{4\sin\theta}{1-4\cos\theta}\, d\theta$

let $1-4\cos\theta = u$

$4\sin\theta\, d\theta = du$

when $\theta = 0,\ u = 1-4 = -3$

$\theta = \frac{\pi}{3},\ u = 1 - 4\cdot\frac{1}{2} = -1$

$I = \int_{-3}^{-1} \frac{du}{u} = \ln|u| \Big]_{-3}^{-1}$

$I = \ln|-1| - \ln|-3| = \ln 1 - \ln 3$

$$\boxed{I = -\ln 3 = \ln \tfrac{1}{3}}$$

(40) $\int_1^{e^{\pi/4}} \frac{4\, dt}{t(1+\ln^2 t)} = I$

let $\ln t = u$

$\frac{1}{t}\, dt = du$

when $t = 1,\ u = 0$

$t = e^{\pi/4},\ u = \ln e^{\frac{\pi}{4}} = \frac{\pi}{4}$

$I = 4\int_0^{\pi/4} \frac{du}{1+u^2} = 4\tan^{-1}u \Big]_0^{\pi/4}$

$I = 4(\tan^{-1}\frac{\pi}{4} - \tan^{-1}0)$

$I = 4(\tan^{-1}\frac{\pi}{4} - 0)$

$$\boxed{I = 4\tan^{-1}\frac{\pi}{4}}$$

(46) $$\int_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}} \frac{dy}{y\sqrt{9y^2-1}} = I$$

let $3y = u \Rightarrow y = \frac{u}{3}$

$3dy = du \Rightarrow dy = \frac{1}{3}du$

when $y = \frac{-2}{3}$, $u = 3\left(\frac{-2}{3}\right) = -2$

$y = \frac{-\sqrt{2}}{3}$, $u = 3\left(\frac{-\sqrt{2}}{3}\right) = -\sqrt{2}$

$$I = \int_{-2}^{-\sqrt{2}} \frac{1}{3} \cdot \frac{du}{\frac{u}{3}\sqrt{u^2-1}}$$

$$I = \int_{-2}^{-\sqrt{2}} \frac{3}{3} \frac{du}{u\sqrt{u^2-1}}$$

Recall that

$(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$ if $x > 1$

$(\sec^{-1} x)' = \frac{1}{-x\sqrt{x^2-1}}$ if $x < -1$

Therefore,

$$I = -\sec^{-1} x \Big]_{-2}^{-\sqrt{2}}$$

$$I = -\left(\sec^{-1}(-\sqrt{2}) - \sec^{-1}(-2)\right)$$

$\sec^{-1}(-\sqrt{2}) = x \in [0, \pi]$ iff

$\sec x = -\sqrt{2}$ iff

$\frac{1}{\cos x} = -\sqrt{2}$ iff

$\cos x = \frac{-\sqrt{2}}{2}$ iff

$x = \frac{3\pi}{4}$

so $\sec^{-1}(-\sqrt{2}) = \frac{3\pi}{4}$

similarly, $\sec^{-1}(-2) = \frac{2\pi}{3}$

Therefore,

$$I = -\left(\frac{3\pi}{4} - \frac{2\pi}{3}\right)$$

$$\boxed{I = -\frac{\pi}{12}}$$