Name:

QUIZ #1 @ 25 points

Write neatly. Show all work. **Write all responses on separate paper.** **Clearly label the exercises.**

1. Let $f(x) = 2x - 3$. Answer the following:
   a) Graph the equation showing the x- and y-intercepts.
   b) What is the domain and the range of the function?
   c) Find and simplify $\dfrac{f(x+h) - f(x)}{h}$ (if $h \neq 0$).
   d) Find a formula to shift the graph to the right 1 unit.

2. Let $f(x) = \sqrt{25 - x^2}$. Answer the following:
   a) Draw a graph of the given function.
   b) State its domain and range.
   c) State the intervals on which the function is increasing, decreasing, or constant.
   d) Is the function even, odd, or neither? Justify your answer (algebraically or graphically).

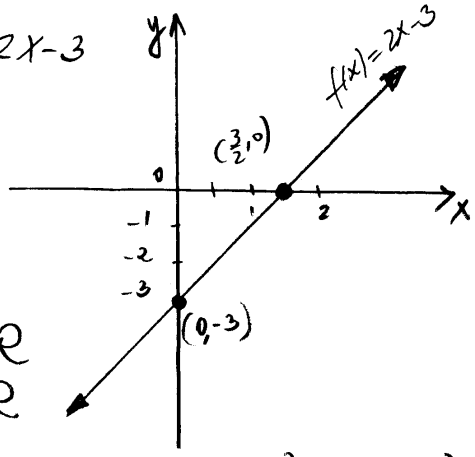
3. Let $f(x) = \cos 2x - \dfrac{1}{2}$. Answer the following questions:
   a) Graph the function over one period.
   b) Find the exact x-intercepts from the graph shown.

Quiz #1 - SOLUTIONS

(1) $f(x) = 2x - 3$

a)

x	y
0	-3
$\frac{3}{2}$	0



b) Domain = $\mathbb{R}$
Range = $\mathbb{R}$

c) $\frac{f(x+h) - f(x)}{h} = \frac{(2(x+h)-3) - (2x-3)}{h}$

$= \frac{2x + 2h - 3 - 2x + 3}{h} = \frac{2h}{h} = 2$

d) $y = f(x-1)$
$= 2(x-1) - 3$
$y = 2x - 5$

(2) $f(x) = \sqrt{25 - x^2}$

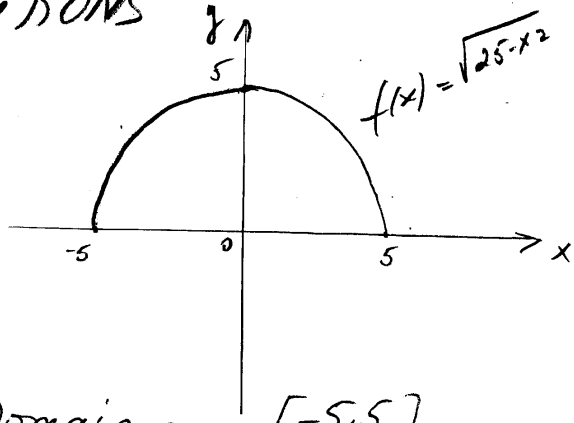
a) $y = \sqrt{25 - x^2}$

Domain: $25 - x^2 \geq 0$
$x^2 - 25 \leq 0$
$x \in [-5, 5]$

Note that $y \geq 0$ for any $x \in [-5, 5]$

$y^2 = 25 - x^2$
$x^2 + y^2 = 25$ - circle of center $(0,0)$ and radius 5

because $y \geq 0$, we will consider only the upper half of the circle $x^2 + y^2 = 25$



b) Domain = $[-5, 5]$
Range = $[0, 5]$

c) f - increasing on $[-5, 0]$
f - decreasing on $[0, 5]$

d) f = even
because its graph is symmetric about the y-axis

OR

Algebraic proof

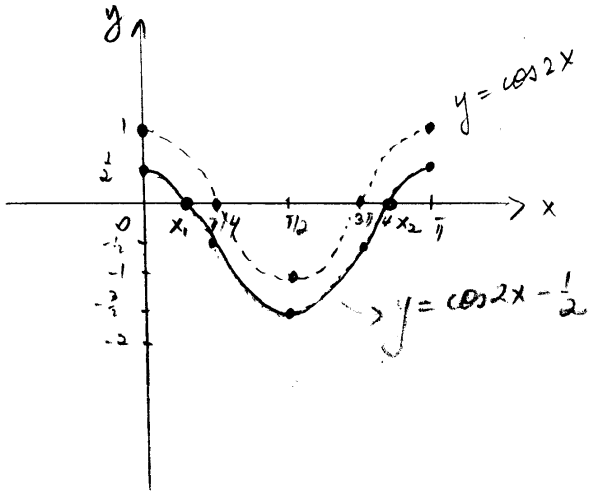
$f(-x) = \sqrt{25 - (-x)^2}$
$= \sqrt{25 - x^2}$
$= f(x)$

so f - even

(3) $f(x) = \cos 2x - \frac{1}{2}$

a) period $T = \frac{2\pi}{2} = \pi$
amplitude $A = 1$

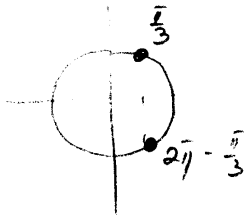
- Take $[0, \pi]$, divide it into 4 equal parts;
- sketch a cosine curve of amplitude 1
- shift down $\frac{1}{2}$ unit.



b) x-int: let $y = 0$

$\cos 2x - \frac{1}{2} = 0$

$\cos 2x = \frac{1}{2}$



$$\begin{cases} 2x = \frac{\pi}{3} + 2\pi k \\ \text{or} \\ 2x = \frac{5\pi}{3} + 2\pi k \end{cases}, \quad k \in \mathbb{Z}$$

$$\begin{cases} x = \frac{\pi}{6} + \pi k \\ \text{or} \\ x = \frac{5\pi}{6} + \pi k \end{cases} \quad k \in \mathbb{Z}$$

if $k = 0$, $x_1 = \frac{\pi}{6}$

$x_2 = \frac{5\pi}{6}$

x-int: $(\frac{\pi}{6}, 0)$ and $(\frac{5\pi}{6}, 0)$