Math 130 Fall 2008 Name:

TEST #2 @ 150 points

Write neatly. Show all work. **Write all responses on separate paper.** **Clearly label the exercises.**

1. Consider the polynomial function $f(x) = x^4 + 6x^3 + 9x^2 - 4x - 12$.
Questions *a – e* below relate to this polynomial function.

a) Describe the long-term behavior of this function; that is, the end-behavior. Give reasons for your answer.

b) Using Descartes' rule of signs, determine the number of positive real zeros and the number of negative real zeros for $f(x)$.

c) Find all the real zeros of $f(x)$ and factor $f(x)$ completely.

d) What are the intercepts of the graph of $f(x)$? Write each intercept as an ordered pair.

e) Sketch a graph of $f(x)$ showing how it passes through its intercepts. Plot additional points (if necessary) to get the shape of the graph. Clearly label all the points.

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2. Consider $f(x) = \dfrac{x^2 - 2x - 15}{x^2 - 4}$.
Questions *a – g* below relate to this polynomial function.

a) Factor the numerator and the denominator.

b) What is the domain of the function?

c) What are the vertical asymptotes?

d) What is the horizontal asymptote?

e) What are the intercepts for this function? Write them as ordered pairs.

f) What are the common points (if any) of the graph with the horizontal asymptote?

g) Plot additional points (if necessary) to get the shape of this function and sketch a graph.

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3. Write a function of minimal degree with real coefficients whose zeros are 3, -5, and 2 - i.

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4. Do the following:

a) Write the expression as a single logarithm with coefficient 1. Assume all variables are positive real numbers: $3\log_5 x - 2\log_5 y + \log_5(z+2)$

b) Write the expression as a single logarithm with coefficient 1. Assume all variables are positive real numbers:

$$\log(x^2 - 9) - 2[\log(x+3) + 3\log x]$$

c) Expand the expression as much as possible. Simplify the result if possible. All variables are positive real numbers:

$$\log_2 \sqrt[4]{\frac{16x^3}{y^5}}$$

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5. Let $f(x) = 2\ln(x-3)+1$.

a) Graph the function using transformations. Clearly show how you're obtaining the graph, that is, show all equations, their meaning, and the corresponding graphs.

b) State the domain, range, and asymptote.

c) Find the exact x- and y-intercepts (if any).

d) Does the function have an inverse? Explain.

e) Find $f^{-1}(x)$.

f) State the domain, range, and asymptote for the inverse function $f^{-1}(x)$.

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6. Solve the following equations. Give exact answer(s) as well as approximations (when appropriate). Write conditions (if any).

a) $\log_2(2x-3)+\log_2(x+1)=1$

b) $4^x = 3^{2x+1}$

c) $\ln 5x - \ln(2x-1) = \ln 4$

d) $e^{x-1} = \left(\dfrac{1}{e^4}\right)^{x+1}$

e) Solve for *t:* $P = P_0 e^{kt}$

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7. Find the domain of each function:

a) $f(x) = 300(2.7)^{x-1}$

b) $g(x) = \log_4(7-2x)$

8. State whether each statement is TRUE or FALSE. DO NOT prove.

a) $\log(a+b) = \log a + \log b$

b) $\log\left(\frac{a}{b}\right) \neq \frac{\log a}{\log b}$

c) $\log 5a^3 = 3\log 5a$

d) $\log(xy) = (\log x)(\log y)$

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9. If the size of a bacteria colony triples in 4 hours, how long will it take for the number of bacteria to double?

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10. At the World Championship races held at Rome's Olympic Stadium in 1987, American sprinter Carl Lewis ran the 100-m race in 9.86 sec. His speed in meters per second after t seconds is closely modeled by the function defined by

$$f(t) = 11.65\left(1 - e^{-\frac{t}{1.27}}\right).$$

After how many seconds was he running at the rate of 10 m per sec?

Extra credit @ 5 points

A group of agricultural scientists has been studying how the growth of a particular type of bacteria is affected by the acidity level of the soil. One colony of the bacteria is placed in a soil that is slightly acidic. A second colony of the same size is placed in a neutral soil. Suppose that after analyzing the data, the scientists determine that the size of each population over time can be modeled by the following functions.

Colony of neutral soil: $y = \frac{2t+1}{t+1}, t \geq 0$

Colony of acidic soil: $y = \frac{4t+3}{t^2+3}, t \geq 0$

In both cases, y represents the population, in thousands, after t hours.

a) What is the initial population for each colony?

b) Determine the long-term behavior of each colony.

Math 130

TEST # 2

(1) $f(x) = x^4 + 6x^3 + 9x^2 - 4x - 12$

(a) The end behavior is given by the leading term x^4

when $x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to +\infty$

(b) $f(x)$ has one variation in sign, so there is one positive zero

$f(-x) = x^4 - 6x^3 + 9x^2 + 4x - 12$

1 2 3

$f(-x)$ has 3 variations in sign, so there could be 3 or 1 negative zeros

(c) Positive rational zeros:

$\frac{p}{q} = \frac{\text{factors of } 12}{\text{factors of } 1} = \frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1}$

$\frac{p}{q} \in \{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$

check $x = 1$ note $f(1) = 0$

	1	6	9	-4	-12
1	1	7	16	12	0

$f(x) = (x-1)(x^3 + 7x^2 + 16x + 12)$

note that there are no positive zeros

	1	7	16	12
-2	1	5	6	0

$f(x) = (x-1)(x+2)(x^2+5x+6)$

$f(x) = (x-1)(x+2)(x+2)(x+3)$

$f(x) = (x+2)^2(x+3)(x-1)$

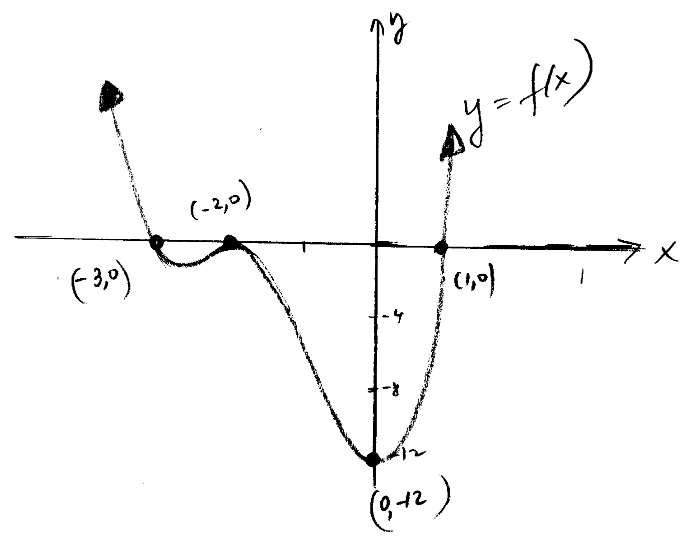
All the zeros: $\begin{cases} x = -2 & m = 2 \\ x = -3 & m = 1 \\ x = 1 & m = 1 \end{cases}$

(d) x-int: $(-2,0)$, $(-3,0)$, $(1,0)$

y-int: $(0,-12)$

(e)

x	$-\infty$	-3	-2	0	1	∞
$f(x)$	∞	0	0	-12	0	∞
		$m=1$	$m=2$		$m=1$	



(2) $f(x) = \dfrac{x^2 - 2x - 15}{x^2 - 4}$

(a) $f(x) = \dfrac{(x-5)(x+3)}{(x-2)(x+2)}$

(b) $x \in \mathbb{R} \setminus \{2, -2\}$

(c) V.A. $x = 2$, $x = -2$

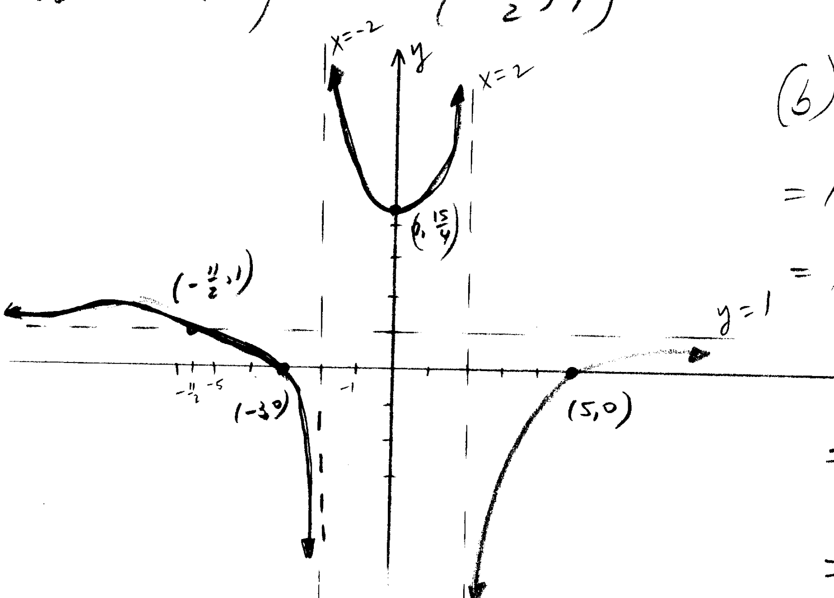
(d) H.A. $y = 1$

(e) x-n: $y = 0$ iff $(x-5)(x+3) = 0$
$x = 5$, $x = -3$

y-n: $x = 0$, $y = \frac{15}{4}$

(f) $f(x) = 1$ iff
$\dfrac{x^2 - 2x - 15}{x^2 - 4} = 1$ iff
$x^2 - 2x - 15 = x^2 - 4$ iff
$-2x - 15 = -4$
$2x = -11$, $x = -\frac{11}{2}$

Common point $(-\frac{11}{2}, 1)$



TP: $x = -10$, $y = \frac{(-)(-)}{(-)(-)} = (+)$

(3) $x = 3$
$x = -5$
$x = 2 - i$ then, $x = 2 + i$ is also a solution

$f(x) = (x-3)(x+5)(x-(2-i))(x-(2+i))$
$f(x) = (x-3)(x+5)(x-2+i)(x-2-i)$
$f(x) = (x-3)(x+5)((x-2)^2 - i^2)$
$f(x) = (x-3)(x+5)(x^2 - 4x + 4 - (-1))$
$f(x) = (x-3)(x+5)(x^2 - 4x + 5)$

(4) (a)
$3\log_5 x - 2\log_5 y + \log_5 (z+2) =$
$= \log_5 x^3 - \log_5 y^2 + \log_5 (z+2)$
$= \log_5 \frac{x^3}{y^2} + \log_5 (z+2)$
$= \log_5 \dfrac{x^3(z+2)}{y^2}$

(b) $\log(x^2-9) - 2[\log(x+3) + 3\log x] =$
$= \log(x^2-9) - 2(\log(x+3) + \log x^3)$
$= \log(x^2-9) - 2(\log x^3(x+3))$
$= \log(x^2-9) - \log x^6(x+3)^2$
$= \log \dfrac{x^2-9}{x^6(x+3)^2} = \log \dfrac{(x-3)(x+3)}{x^6(x+3)^2}$
$= \log \dfrac{x-3}{x^6(x+3)}$

(c) $\log_2 \sqrt[4]{\frac{16x^3}{y^5}} = \log_2\left(\frac{16x^3}{y^5}\right)^{\frac{1}{4}}$

$= \frac{1}{4}\log_2 \frac{16x^3}{y^5} = \frac{1}{4}\left(\log_2(16x^3) - \log_2 y^5\right)$

$= \frac{1}{4}\left(\log_2 16 + \log_2 x^3 - 5\log_2 y\right)$

$= \frac{1}{4}\left(4 + 3\log_2 x - 5\log_2 y\right)$

$= 1 + \frac{3}{4}\log_2 x - \frac{5}{4}\log_2 y$

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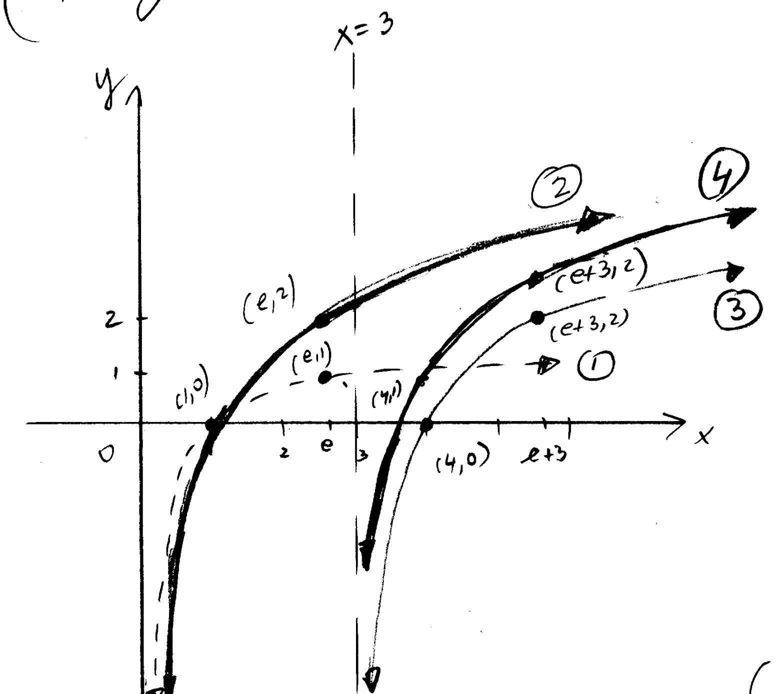
(5) $f(x) = 2\ln(x-3) + 1$

(a) 1st $y = \ln x$
2nd $y = 2\ln x$ vertical stretch by a factor of 2
3rd $y = 2\ln(x-3)$ shift right 3
4th $y = 2\ln(x-3) + 1$ shift up 1



$y = \ln x$ $\quad x=1, y=0 \quad (1,0)$
$\quad x=e, y=1 \quad (e,1)$

(b) Domain: $x \in (3, \infty)$
Range: $y \in \mathbb{R}$
VA: $x = 3$

(c) x-int: $y = 0$ [illegible]

$2\ln(x-3) + 1 = 0$

$\ln(x-3) = \frac{-1}{2}$

$e^{-\frac{1}{2}} = x - 3$

$x = 3 + e^{-\frac{1}{2}}$

x-int: $\left(3 + \frac{1}{\sqrt{e}}, 0\right)$

y-int: none $\quad (x \neq 0)$

(d) f is a one-to-one fct therefore it has an inverse

(e) 1st $y = 2\ln(x-3) + 1$
2nd solve for x

$y - 1 = 2\ln(x-3)$

$\ln(x-3) = \frac{y-1}{2}$

$e^{\frac{y-1}{2}} = x - 3$

$x = 3 + e^{\frac{y-1}{2}}$

3rd $x \longleftrightarrow y$

$y = 3 + e^{\frac{x-1}{2}}$

$f^{-1}(x) = 3 + e^{\frac{x-1}{2}}$

(f) Domain: $x \in \mathbb{R}$
Range $y \in (3, \infty)$
H.A. $y = 3$

(6)

(a) $\log_2(2x-3)+\log_2(x+1)=1$

Conditions:

$$\begin{cases} 2x-3>0 \\ x+1>0 \end{cases} \iff \begin{cases} x>\frac{3}{2} \\ x>-1 \end{cases} \iff \boxed{x>\frac{3}{2}}$$

$\log_2(2x-3)(x+1)=1$

$2^1=(2x-3)(x+1)$

$2x^2-x-5=0$

$$x_{1,2}=\frac{1\pm\sqrt{1+40}}{4}=\frac{1\pm\sqrt{41}}{4}$$

$x_1=\frac{1-\sqrt{41}}{4}<0 \notin (\frac{3}{2},\infty)$

$x_2=\frac{1+\sqrt{41}}{4}\approx\frac{1+6.4}{4}>1.5$

So, $\boxed{x=\frac{1+\sqrt{41}}{4}}$

(b) $4^x=3^{2x+1}$ $\quad /\ln$

$\ln 4^x=\ln 3^{2x+1}$

$x\ln 4=(2x+1)\ln 3$

$x\ln 4=2x\ln 3+\ln 3$

$x\ln 4-2x\ln 3=\ln 3$

$x(\ln 4-2\ln 3)=\ln 3$

$\boxed{x=\frac{\ln 3}{\ln 4-2\ln 3}}$ or $x=\frac{\ln 3}{\ln\frac{4}{9}}$

$x\approx-1.35$

(c) $\ln 5x-\ln(2x-1)=\ln 4$

Conditions

$$\begin{cases} 5x>0 \\ 2x-1>0 \end{cases} \iff \begin{cases} x>0 \\ x>\frac{1}{2} \end{cases} \iff \boxed{x>\frac{1}{2}}$$

$\ln\frac{5x}{2x-1}=\ln 4$

The natural log function is one-to-one $\Rightarrow$

$\frac{5x}{2x-1}=4$

$5x=4(2x-1)$

$5x=8x-4$

$4=3x,\quad x=\frac{4}{3}>\frac{1}{2}$

So. $\boxed{x=\frac{4}{3}}$

(d) $e^{x-1}=\left(\frac{1}{e^4}\right)^{x+1}$

$e^{x-1}=(e^{-4})^{x+1}$

$e^{x-1}=e^{-4(x+1)}$

The exponential fct. is one-to-one $\Rightarrow$

$x-1=-4(x+1)$

$x-1=-4x-4$

$5x=-3$

$\boxed{x=\frac{-3}{5}}$

-5

(e) $P = P_0 e^{kt}$

$e^{kt} = \frac{P}{P_0}$ / ln

$\ln e^{kt} = \ln \frac{P}{P_0}$

$kt = \ln \frac{P}{P_0}$

$\boxed{t = \frac{1}{k} \ln \frac{P}{P_0}}$

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(7) (a) $x \in \mathbb{R}$

(b) condition: $7 - 2x > 0$

$7 > 2x$

$x < \frac{7}{2}$

Domain: $x \in (-\infty, \frac{7}{2})$

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(8) (a) False

$\log a + \log b = \log(ab)$

$\neq \log(a+b)$

(b) True

$\log(\frac{a}{b}) = \log a - \log b$

$\neq \frac{\log a}{\log b}$

(c) False

$\log 5a^3 = \log 5 + 3 \log a$

$\neq 3 \log 5a$

(d) False

$\log(xy) = \log x + \log y$

$\neq (\log x)(\log y)$

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(9) let t = time (in hours)

N = number of bacteria

N_0 = initial number

t	N
0	N_0
4	$3N_0$
8	$3^2 N_0$
12	$3^3 N_0$
t	$3^{t/4} N_0$

So $N = N_0 (3)^{\frac{t}{4}}$

$t = ?$, $N = 2N_0$

$2\cancel{N_0} = \cancel{N_0} (3)^{\frac{t}{4}}$

$3^{(\frac{t}{4})} = 2$ / ln

$\ln 3^{t/4} = \ln 2$

$\frac{t}{4} \ln 3 = \ln 2$

$t \ln 3 = 4 \ln 2$

$t = \frac{4 \ln 2}{\ln 3} \approx 2.5$ hours

The colony will double in about 2.5 hours

(10) $f(t) = 11.65\left(1 - e^{\frac{-t}{1.27}}\right)$

t = time (in seconds)

$f(t)$ = speed (in m/sec)

$t = ?$ if $f(t) = 10$

$10 = 11.65\left(1 - e^{-t/1.27}\right)$

$1 - e^{\frac{-t}{1.27}} = \frac{10}{11.65}$

$e^{-\frac{t}{1.27}} = 1 - \frac{10}{11.65}$

$e^{-\frac{t}{1.27}} = \frac{1.65}{11.65} = \frac{165}{1165}$

$e^{-\frac{t}{1.27}} = \frac{165}{1165} \quad // \ln$

$\ln e^{-\frac{t}{1.27}} = \ln \frac{165}{1165}$

$\frac{-t}{1.27} = \ln \frac{165}{1165}$

$t = -1.27 \ln \frac{165}{1165}$

$t \simeq 2.4823$ seconds.

He was running at 10 m/sec after about 2.4823 sec.