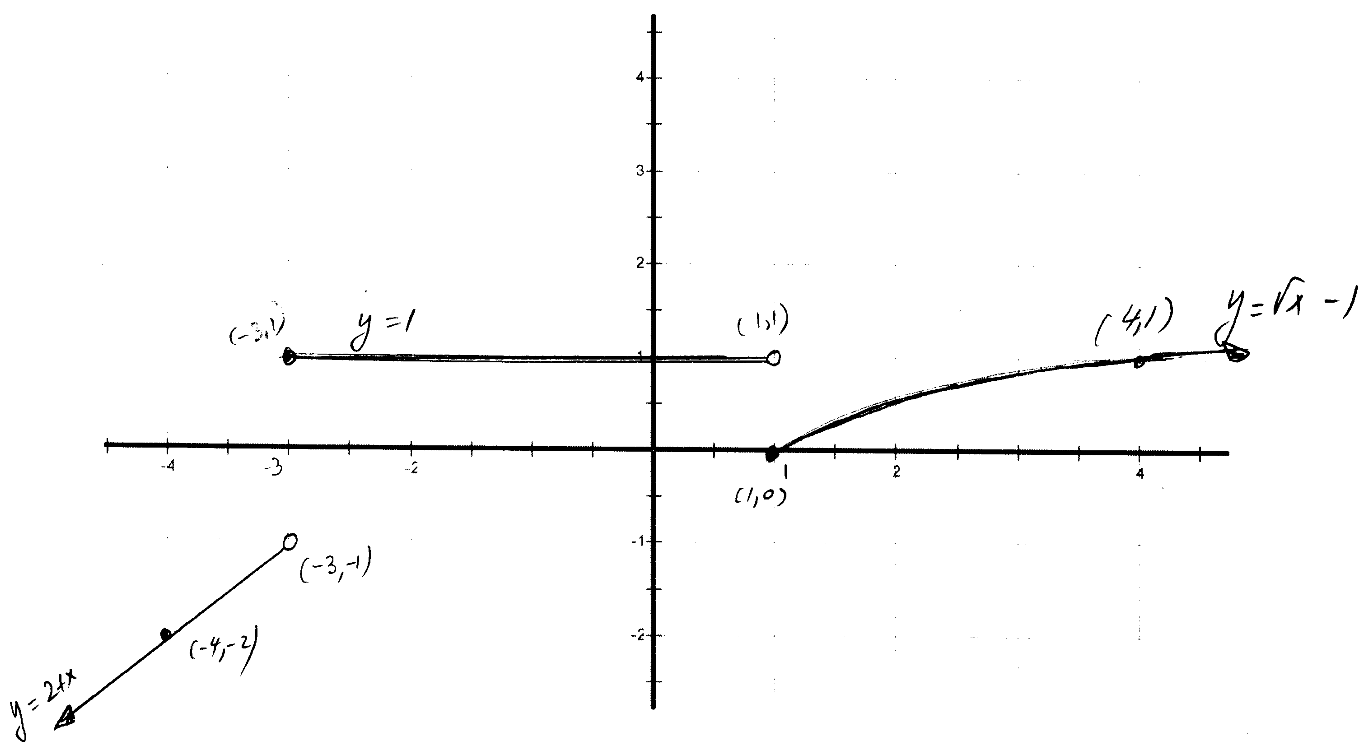
TEST #1 @ 150 points

Write neatly. Show all work. Write all responses on separate paper. Clearly label the exercises.

1. A piecewise-defined function is given.

$$f(x)=\begin{cases} 2+x & \text{if } x<-3 \\ 1 & \text{if } -3\le x<1 \\ \sqrt{x}-1 & \text{if } x\ge 1 \end{cases}$$



You may use the above grid to graph. Write all the answers and show ALL your work on separate paper.

a) Sketch a graph for the function. Clearly show how you obtain the points you are using for the graph. Label the axes and all points used.

b) State its domain and range in interval notation.

c) On what interval(s) is the function increasing, decreasing, constant?

d) Find $f(-3)$, $f\left(\sqrt{10}\right)$, and $f(3)$.

2. Let $4x^2+4y^2+4x-4y-7=0$

a) Decide whether the equation represents a circle or not? If it does, give the exact center and radius.

b) Does the equation from (a) represent y as a function of x? Explain.

c) Find the exact x- and y-intercepts (if any).

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3. Solve the following equations in the set of complex numbers:

a) $(3a+1)^2+\frac{1}{9}=0$

b) $\frac{2}{3}t^2-4=-\frac{1}{4}t$

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4. Let

$$f(x)=x^2-3x+2$$
$$g(x)=3x-7$$
$$F(x)=\sqrt{10-x}$$
$$l(x)=\frac{x+2}{5x-2}$$

be four functions. Do the following.

a) Find the domain of each function.

b) Find $g(3x)$

c) Find $\frac{g(x+h)-g(x)}{h}$

d) Find $f\left(\sqrt{x}+1\right)$.

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5. Let $3x+5y=15$ be a linear equation in two variables. Do the following:

a) Graph the equation using the intercepts method. Clearly label the axes and the intercepts.

b) Find the slope of the line.

c) Find an equation for the line that is perpendicular to the given line and passes through $(2,-3)$.

6. Harry has a taco stand. He has found that his daily costs are approximated by $C(x) = x^2 - 40x + 610$, where $C(x)$ is the cost, in dollars, to sell x units of tacos. Find the number of units of tacos she should sell to minimize her costs. What is the minimum costs?

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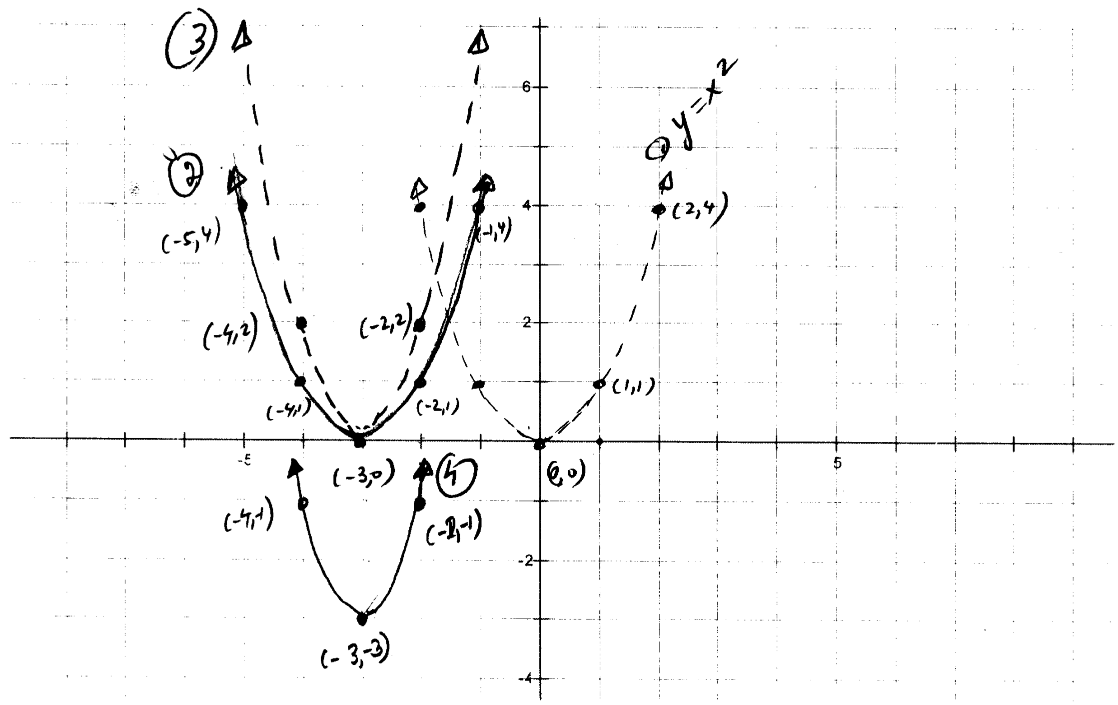
7. Let $f(x) = 2(x+3)^2 - 3$. Answer the following questions:

a) Identify the equation – that is, what kind of equation is it and what does its graph represent.
b) Identify the vertex.
c) Graph the function using transformations. You may use the grid to graph. Clearly show all the steps: the equations and their meaning on separate paper. Graph all steps.
d) Find the domain and the range.
e) Solve the inequality: $f(x) \geq 0$.



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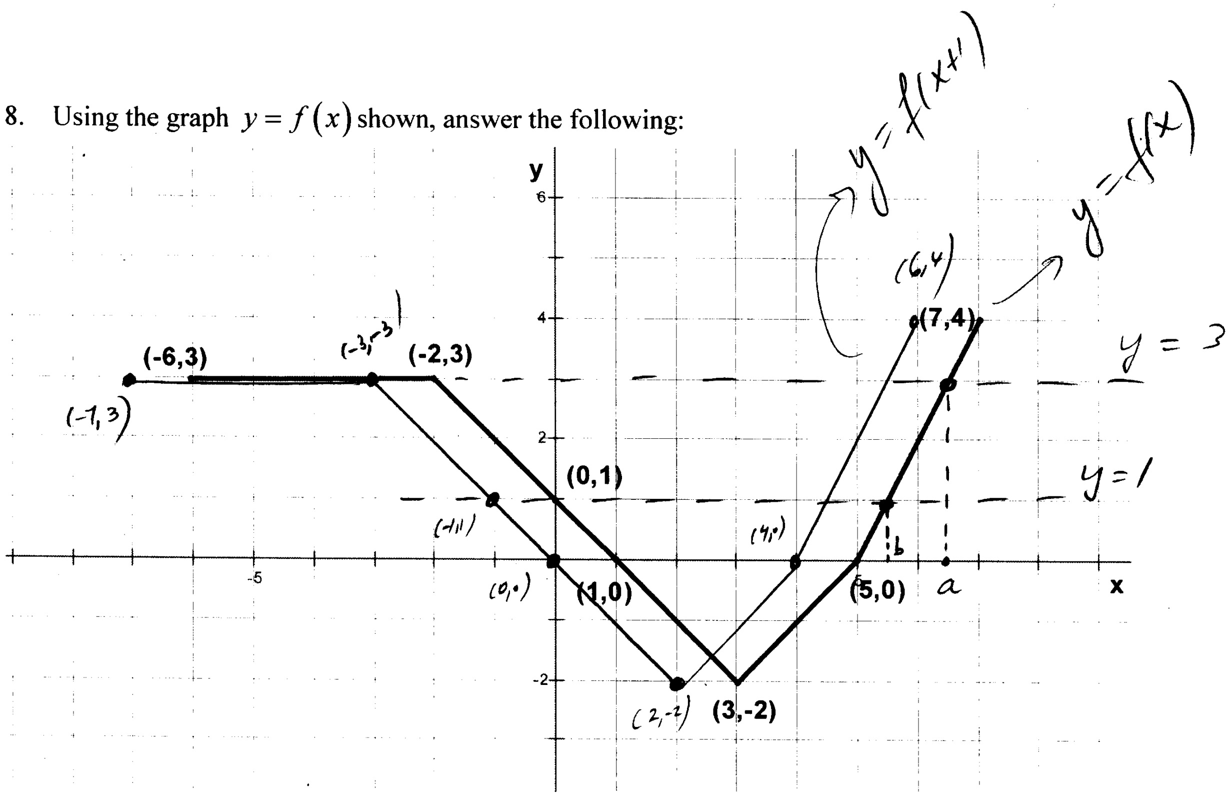
8. Using the graph $y = f(x)$ shown, answer the following:



a) Is y a function of x? Explain.

b) Find the domain and range of f.

c) List the x- and y- intercepts (as ordered pairs).

d) Find $f(3)$.

e) For what values of x does $f(x) = 3$

f) Estimate the values for which $f(x) > 1$.

g) Find $(f \circ f)(5)$.

h) Graph $y = f(x+1)$

Extra Credit @ 3 points

6. Let $s(t) = 11t^2 + t + 100$ be the position, in miles, of a car driving on a straight road at time t, in hours. The card's velocity at any time t is given by $v(t) = 22t + 1$.

a) Use function notation to express the car's position after 2 hours. Where is the car then?
b) Use function notation to express the question , "When is the car going 65 mph?"
c) Where is the car when it is going 67 mph?

SOLUTIONS - TEST 1

(1)

(a) $f(x) = \begin{cases} 2+x, & x<-3 \\ 1, & -3 \le x < 1 \\ \sqrt{x} - 1, & x \geq 1 \end{cases}$

I $y = 2+x$ when $x<-3$

x	y
-3	-1
-4	-2

open

II $y=1$ when $-3 \le x < 1$

horizontal line

III $y = \sqrt{x} - 1$ when $x \geq 1$

x	y
1	0
4	1

closed

(b) Domain: $x \in \mathbb{R}$

Range $y \in (-\infty, -1) \cup [0, \infty)$

(c) f is increasing on $[1, \infty)$ and on $(-\infty, -3)$

f is constant on $[-3, 1)$

(d) $f(-3) = 1$ because $-3 \in [-3, 1)$

$f(\sqrt{10}) = \sqrt{\sqrt{10}} - 1 = 10^{\frac{1}{4}} - 1$ because $\sqrt{10} \in [1, \infty)$

$f(3) = \sqrt{3} - 1$ because $3 \in [1, \infty)$

(2) $4x^2 + 4y^2 + 4x - 4y - 7 = 0$ (*) $\Big| \div 4$

(a) $x^2 + x + y^2 - y - \frac{7}{4} = 0$

$x^2 + x + y^2 - y = \frac{7}{4}$

$\left(\frac{1}{2} \text{coef } x\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\left(\frac{1}{2} \text{coef } y\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$x^2 + x + \frac{1}{4} + y^2 - y + \frac{1}{4} = \frac{7}{4} + \frac{1}{4} + \frac{1}{4}$

$$\boxed{\left(x+\frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}} \quad (**)$$

So the equation represents a circle with center $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and radius $\sqrt{\frac{9}{4}} = \frac{3}{2}$

(b) No, because a circle doesn't pass the vertical line test

(c) x-I: let $y=0$ in (*) or (**)

$4x^2 + 4x - 7 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(4)(-7)}}{2(4)}$

$= \frac{-4 \pm \sqrt{16(1+7)}}{8} = \frac{-4 \pm 4\sqrt{8}}{8}$

$= \frac{-4 \pm 8\sqrt{2}}{8} = \frac{4(-1 \pm 2\sqrt{2})}{8} = \frac{-1 \pm 2\sqrt{2}}{2}$

so $\boxed{\text{x-I: } \left(\frac{-1 \pm 2\sqrt{2}}{2}, 0\right)}$

y-I: let $x = 0$ in (*) or (**)

$\left(\frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

$\left(y - \frac{1}{2}\right)^2 = 2$

$y - \frac{1}{2} = \pm\sqrt{2}$

$y = \frac{1}{2} \pm \sqrt{2}$

$$\boxed{\text{y-I: } \left(0, \frac{1}{2} \pm \sqrt{2}\right)}$$

(3) (a) $(3a+1)^2 + \frac{1}{9} = 0$

$(3a+1)^2 = \frac{-1}{9} \quad / \sqrt{}$

$3a+1 = \pm\sqrt{\frac{-1}{9}}$

$3a+1 = \pm \frac{i}{3}$

$3a = -1 \pm \frac{i}{3} \quad / \cdot \frac{1}{3}$

$\boxed{a = \frac{-1}{3} \pm \frac{i}{9}}$ or $\boxed{a = \frac{-3 \pm i}{9}}$

(b) $\overset{4}{\frac{2}{3}}t^2 - \overset{12}{4} = \overset{3}{\frac{-1}{4}}t$

LCD = 12

$8t^2 - 48 = -3t$

$8t^2 + 3t - 48 = 0$

$t = \frac{-3 \pm \sqrt{9 - 4(8)(-48)}}{2(8)}$

$\boxed{t = \frac{-3 \pm \sqrt{1545}}{16}}$

$5 \,|\, 1545$
$3 \,|\, 309$
$103 \,|\, 103$

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(4) (a) $f(x) = x^2 - 3x + 2$

Domain: $\boxed{x \in \mathbb{R}}$

$g(x) = 3x - 7 \quad \boxed{x \in \mathbb{R}}$

$f(x) = \sqrt{10-x}$

condition: $10 - x \geq 0$

$x \leq 10$

so $\boxed{x \in (-\infty, 10]}$

$\ell(x) = \frac{x+2}{5x-2}$

Condition: $5x - 2 \neq 0$

$x \neq \frac{2}{5}$

so $\boxed{x \in \mathbb{R} \setminus \{\frac{2}{5}\}}$

(b) $g(x) = 3x - 7$

$g(3x) = 3(3x) - 7$

$\boxed{g(3x) = 9x - 7}$

(c) $\frac{g(x+h) - g(x)}{h} =$

$= \frac{(3(x+h) - 7) - (3x - 7)}{h}$

$= \frac{\cancel{3x} + 3h \cancel{- 7} \cancel{- 3x} \cancel{+ 7}}{h} = \frac{3h}{h} = \boxed{3}$

(d) $f(x) = x^2 - 3x + 2$

$f(\sqrt{x}+1) = (\sqrt{x}+1)^2 - 3(\sqrt{x}+1) + 2$

$= x + 2\sqrt{x} + 1 - 3\sqrt{x} - 3 + 2$

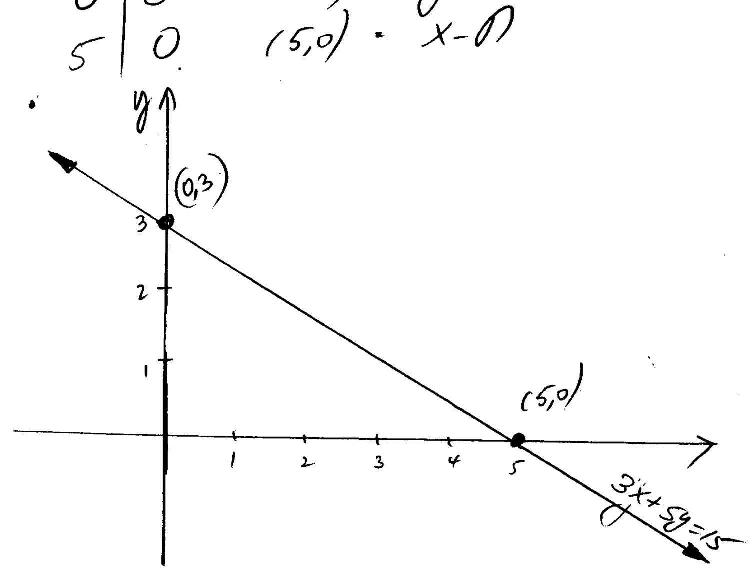
$\boxed{f(\sqrt{x}+1) = x - \sqrt{x}}$

(5) $3x + 5y = 15$

(a)

x	y
0	3
5	0

$(0,3)$ - y-I
$(5,0)$ - x-I



(b) Method I

$3x + 5y = 15$

$5y = -3x + 15$

$y = \frac{-3}{5}x + 3$

$\boxed{m = \frac{-3}{5}}$

Method II

Using (a) =>

$m = \frac{\Delta y}{\Delta x} = \frac{3-0}{0-5}$

$m = -\frac{3}{5}$

(c) $m_{\perp} = \frac{5}{3}$

$(2,-3)$

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{5}{3}(x-2) \quad / \cdot 3$

$3y + 9 = 5(x-2)$

$3y + 9 = 5x - 10$

$\boxed{5x - 3y = 19}$

(6) $C(x) = x^2 - 40x + 610$

x = # units of tacos

$C(x)$ = cost

The equation is quadratic, therefore its graph is a parabola opening up ($a=1>0$) therefore the minimum occurs at the vertex

V=min

$V(x_v, C_v)$ x_v = # tacos need to minimize cost

C_v = minimum cost

$x_v = \frac{-b}{2a} = \frac{-(-40)}{2(1)} = 20$ tacos

$C_{min} = (20)^2 - 40(20) + 610$

$C_{min} = 210$ dollars

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(7) (a) $f(x) = 2(x+3)^2 - 3$

This is a quadratic function whose graph is a parabola that opens up, as $a = 2 > 0$

(b) $V(-3,-3)$ because the equation is given in vertex form $y = a(x-x_v)^2 + y_v$

(c) 1st $y = x^2$ basic parabola

2nd $y = (x+3)^2$ shift graph (1) 3 units left

3rd $y = 2(x+3)^2$ stretch graph (2) vertically by 2

4th $y = 2(x+3)^2 - 3$ shift graph (3) down 3 units

(d) $x \in \mathbb{R}$
$y \in [-3, \infty)$

(e) 1st - find x-int:

$y=0, \quad 2(x+3)^2 - 3 = 0$

$(x+3)^2 = \frac{3}{2}$

$x+3 = \pm\sqrt{\frac{3}{2}}$

$x = -3 \pm \frac{\sqrt{6}}{2} = \frac{-6 \pm \sqrt{6}}{2}$

$f(x) > 0$ iff
$x \in (-\infty, \frac{-6-\sqrt{6}}{2}) \cup (\frac{-6+\sqrt{6}}{2}, \infty)$

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(8) (a) yes, the graph passes the Vertical Line Test.

(b) $x \in [-6, 7]$, $y \in [-2, 4]$

(c) x-int: $(1,0)$ and $(5,0)$
y-int: $(0,1)$

(d) $f(3) = -2$

(e) $f(x) = 3$ when $x \in [-6, -2] \cup \{6.5\}$

(f) $f(x) > 1$ when $x \in [-6, 0) \cup (5.5, 7]$

(g) $f(f(5)) = f(0) = 1$

(h) $y = f(x+1)$
shift the graph of $y = f(x)$ one unit left.

Extra credit

$s(t) = 11t^2 + t + 100$
$v(t) = 22t + 1$

t = time (in hours)
$v(t)$ = velocity (mi/h)
$s(t)$ = position (mi)

(a) The car's position after 2 hours is given by $s(2)$

$s(2) = 11(2^2) + 2 + 100$
$s(2) = 146$ miles

(b) $v(t) = 65$ mph

(c) find $s(t)$ when $v(t) = 67$ mph

$v(t) = 67$
$22t + 1 = 67$
$22t = 66$
$t = 3$ h

so, after 3 hours the car is at $s(3) = 11(3^2) + 3 + 100$
$s(3) = 202$ miles