Math 130 Fall 2008 Name:

QUIZ #3 @ 85 points

Write neatly. Show all work. **Write all responses on separate paper.**

1. Solve the following systems using matrices.

a) $\begin{cases} x+y-z=6 \\ 2x-y+z=-9 \\ x-2y+3z=1 \end{cases}$

b) $\begin{cases} x-y+2z+w=4 \\ y+z=3 \\ z-w=2 \\ x-y=0 \end{cases}$

2. Let

$$A=\begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} \quad B=\begin{pmatrix} 1 & 6 & -7 \\ -2 & 1 & 3 \end{pmatrix} \quad C=\begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix} \quad D=\begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix}$$

Do the following operations. If not defined, say so and explain why.

a) $A-C$

b) $3A$

d) AC

e) AD

3. Graph the solution set of the following system of inequalities. Then find the coordinates of the vertices (if any).

$$\begin{cases} x+y<4 \\ x-y\le 5 \\ 4x+y\le -4 \end{cases}$$

SOLUTIONS

(1a) $\begin{cases} x+y-z=6 \\ 2x-y+z=-9 \\ x-2y+3z=1 \end{cases}$

$$\begin{pmatrix} 1 & 1 & -1 & | & 6 \\ 2 & -1 & 1 & | & -9 \\ 1 & -2 & 3 & | & 1 \end{pmatrix} \xrightarrow[R_3 \to -R_1+R_3]{R_2 \to -2R_1+R_2} \begin{pmatrix} 1 & 1 & -1 & | & 6 \\ 0 & -3 & 3 & | & -21 \\ 0 & -3 & 4 & | & -5 \end{pmatrix} \xrightarrow[R_3 \to -R_2+R_3]{R_2 \to \frac{1}{3}R_2} \begin{pmatrix} 1 & 1 & -1 & | & 6 \\ 0 & -1 & 1 & | & -7 \\ 0 & 0 & 1 & | & 16 \end{pmatrix}$$

3rd row: $z=16$

2nd row: $-y+z=-7 \Rightarrow -y+16=-7$, $y=23$

1st row: $x+y-z=6$

$\Rightarrow x+23-16=6 \Rightarrow x=-1$

The solution is $(-1, 23, 16)$.

(1b)

$$\begin{pmatrix} 1 & -1 & 2 & 1 & | & 4 \\ 0 & 1 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & -1 & | & 2 \\ 1 & -1 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{R_4 \to -R_1+R_4} \begin{pmatrix} 1 & -1 & 2 & 1 & | & 4 \\ 0 & 1 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & -1 & | & 2 \\ 0 & 0 & -2 & -1 & | & -4 \end{pmatrix} \xrightarrow{R_4 \to 2R_3+R_4} \begin{pmatrix} 1 & -1 & 2 & 1 & | & 4 \\ 0 & 1 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & -1 & | & 2 \\ 0 & 0 & 0 & -3 & | & 0 \end{pmatrix}$$

4th row: $-3w=0 \Rightarrow w=0$

3rd row: $z-w=2 \Rightarrow z=2$

2nd row: $y+z=3 \Rightarrow y=1$

1st row: $x-y+2z+w=4$

$x-1+4=4 \Rightarrow x=1$

The solution is $(1, 1, 2, 0)$.

(a) $A-C =$

$$= \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-1 & 1-1 & 0-3 \\ -1+2 & 2-3 & -3+5 \\ 0-1 & 5-0 & -1+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & -3 \\ 1 & -1 & 2 \\ -1 & 5 & 0 \end{pmatrix}$$

b) $3A = 3\begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix}$

$= \begin{pmatrix} 6 & 3 & 0 \\ -3 & 6 & -9 \\ 0 & 15 & -3 \end{pmatrix}$

c) $AC = ?$

$\dim A = 3\times 3$

$\dim C = 3\times 3$

so $\dim AC = 3\times 3$

$AC = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix} =$

$= \begin{pmatrix} 0 & 5 & 1 \\ -8 & 5 & -10 \\ -11 & 15 & -24 \end{pmatrix}$

$a_{11} = 2\cdot 1 + 1(-2) + 0(1) = 0$

$a_{12} = 2\cdot 1 + 1\cdot 3 + 0\cdot 0 = 5$

$a_{13} = 2\cdot 3 + 1(-5) + 0(-1) = 1$

$a_{21} = -1(1) + 2(-2) + (-3)1 = -8$

$a_{22} = (-1)1 + 2\cdot 3 + (-3)0 = 5$

$a_{23} = (-1)3 + 2(-5) + (-3)(-1) = -10$

$a_{31} = 0(1) + 5(-2) + (-1)1 = -11$

$a_{32} = 0(1) + 5(3) + (-1)0 = 15$

$a_{33} = 0(3) + 5(-5) + (-1)(-1) = -24$

e) $AD = ?$

$\dim A = 3\times 3$

$\dim D = 3\times 2$

so $\dim AD = 3\times 2$

$AD = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 3 & 2 \\ -13 & -7 \\ -8 & -2 \end{pmatrix}$

(3) $\boxed{x+y<4}$

Boundary line: $x+y=4$

x	y
0	4
4	0

Test point: (0,0) $0+0<4$ true

so (0,0) = solution

$\boxed{x-y\leq 5}$

Boundary line: $x-y=5$

x	y
0	-5
5	0

Test point (0,0): $0-0\leq 5$ true

so (0,0): solution

$\boxed{4x+y\leq -4}$

Boundary line: $4x+y=-4$

x	y
0	-4
-1	0

Test point (0,0)

$4(0)+0\leq -4$

false

so (0,0) $\neq$ solution

The solution set has two vertices: A and B

A $\begin{cases} 4x+y=-4 \\ x+y=4 \end{cases}$

$-3x=-8 \Rightarrow x=\frac{-8}{3}$

$x+y=4$

$-\frac{8}{3}+y=4 \Rightarrow y=4+\frac{8}{3}=\frac{21}{3}$

$\boxed{A\left(\frac{-8}{3},\frac{21}{3}\right)}$

B $\begin{cases} x-y=5 \\ 4x+y=-4 \end{cases}$

$+\ 5x=1 \Rightarrow x=\frac{1}{5}$

$x-y=5$

$\frac{1}{5}-y=5 \Rightarrow y=\frac{1}{5}-5=\frac{-24}{5}$

$\boxed{B\left(\frac{1}{5},\frac{-24}{5}\right)}$