Math 130 Fall 2008 Name:

QUIZ #2 @ 85 points

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1. Let $f(x)=6x^4+7x^3-12x^2-3x+2$. Answer the following questions:

   a) What is the maximum number of real zeros?
   b) Determine the possible number of positive real zeros.
   c) Determine the possible number of negative real zeros.
   d) Explain why the Rational Zeros Theorem can be applied; use the teomre to list all possible rational zeros.
   e) Find all rational zeros.
   f) Factor the polynomial into linear factors.

2. Let $f(x)=(3x-1)(x+2)^2$. Graph the function showing the following:
   a) Domain.
   b) Intercepts and their multiplicities.
   c) End-behavior (explain or prove; do not just write an answer).
   d) Test points (only of necessary).

   Use a table of values to record all the information found in a) – d) .

3. Let $f(x)=\dfrac{x^2-2x-15}{x^2-3x-4}$. Graph the function showing the following:

   a) Domain.
   b) Asymptotes.
   c) Intercepts.
   d) Intersection of the function with the horizontal or oblique asymptote.
   e) Test points (when necessary).

M130-70108

Quiz #2 - SOLUTIONS

① $f(x) = 6x^4 + 7x^3 - 12x^2 - 3x + 2$

(a) maximum number of real zeros is 4

(b) There are 2 variations in the sign of $f(x)$, so there could be 2 or 0 positive zeros

(c) $f(-x) = 6x^4 - 7x^3 - 12x^2 + 3x + 2$

There are 2 variations in the sign of $f(-x)$, so there could be 2 or 0 negative zeros.

(d) We can apply the Rational zeros theorem because all coefficients are integers and the constant term $\neq 0$.

Possible rational zeros $= \frac{p}{q}$

$$\frac{p}{q} = \frac{\text{factor of } 2}{\text{factor of } 6} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$$

$$\frac{p}{q} \in \left\{ \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 2, \pm \frac{2}{3} \right\}$$

(e) Note that $f(1) = 0$

x	6	7	-12	-3	2
1	6	13	1	-2	0

$$f(x) = (x-1)\underbrace{(6x^3 + 13x^2 + 1 \cdot x - 2)}_{\text{possible rational zeros } \frac{p}{q} = \frac{\text{factor } 2}{\text{factor } 6} \text{ - same list}}$$

	6	13	1	-2
-2	6	1	-1	0

$f(x) = (x-1)(x+2)(6x^2 + x - 1)$

$f(x) = (x-1)(x+2)(3x-1)(2x+1)$

The zeros are:

$$\begin{cases} x = 1 \\ x = -2 \\ x = \frac{1}{3} \\ x = -\frac{1}{2} \end{cases}$$

each with multiplicity 1

(f) $f(x) = (x-1)(x+2)(3x-1)(2x+1)$

(2) $f(x) = (3x-1)(x+2)^2$

x	$-\infty$	-2	0	$\frac{1}{3}$	∞
$f(x)$	$-\infty$	0	-4	0	∞
		$m=2$		$m=1$	

(a) Domain: $x \in \mathbb{R}$

(b) x-n: $f(x)=0 \Rightarrow$

$x=\frac{1}{3}, \quad x=-2$

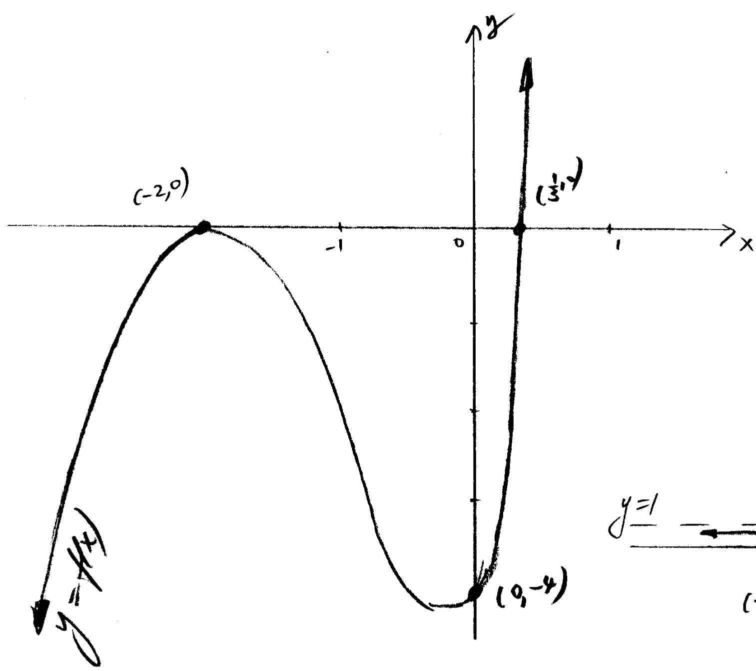
$m=1 \quad\quad m=2$

y-n: $x=0 \Rightarrow y=(-1)(2^2)=-4$

(c) End-behavior is given by the leading term $3x(x^2)=3x^3$

so when $x \to \infty, \; y \to \infty$

$x \to -\infty, \; y \to -\infty$



(3) $f(x) = \dfrac{x^2-2x-15}{x^2-3x-4}$

$f(x) = \dfrac{(x-5)(x+3)}{(x-4)(x+1)}$

(a) Domain: $x \in \mathbb{R} \setminus \{4, -1\}$

(b) V.A. $x=4, \; x=-1$

H.A. $y=1$

(c) x-n: $y=0 \Rightarrow x=5, \; x=-3$

$(5,0)$ and $(-3,0)$

y-n: $x=0, \Rightarrow y=\frac{15}{4}$

$(0, \frac{15}{4})$

(d) 'n' $f(x)$ with the H.A. $y=1$

$\dfrac{x^2-2x-15}{x^2-3x-4}=1$

$\cancel{x^2}-2x-15 = \cancel{x^2}-3x-4$

$x=11 \quad\quad (11,1)$ common point