Math 71 Fall 07 Name:

TEST 3 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

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1. Solve each equation in $\mathbb{C}$ (the set of complex numbers) by the indicated method.

   a) $2(x+3)^2+80=0$ by the square root property.

   b) $4y^2=3y-1$ by completing the square.

   c) $\frac{2t^2}{5}+\frac{t}{2}=\frac{1}{5}$ by the quadratic formula.

   d) $h=-16t^2+\frac{23}{3}t$ solve for t in terms of h.

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2. Solve the following equations. Give exact answers.

   a) $2x^4-3x^2+1=0$

   b) $\log_5(3x-1)-2=0$

   c) $4^x=13$ Give both exact and approximate answers.

   d) $\log_8(x+5)-\log_8 2=1$

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3. Solve the following inequalities.

   a) $x^2-6x+5\le 0$

   b) $\frac{1}{x-5}<\frac{3}{2-x}$

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4. Let $f(x)=3x-2$ and $g(x)=\frac{2-x}{x+1}$. Answer the following questions:

   a) Find $(g\circ f)(x)$.

   b) $(f\circ g)(2)$

   c) Find $f^{-1}(x)$.

   d) Find $g^{-1}(x)$.

5. Simplify the following expressions.

a) $4\ln x+7\ln y-3\ln z$

b) $\frac{1}{2}(\log_5 x+\log_5 y)-2\log_5(x+1)$

c) $\log_3 405-\log_3 5+\log 5+\log 2$

d) $\log_{10}(\log_3(\log_5 125))$

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6. For the equation given below, answer all the questions and graph the function (Be sure to label the axes and all points used). SHOW ALL WORK!

$y=-2x^2+x+3$

a) What type of curve is this?
b) What is the y-intercept?
c) What is the vertex
d) What are the x- intercept(s) (if any)?
e) What is the domain of the function?
f) What is the range of the function?
g) <u>Using the graph above</u>, solve the following inequality: $-2x^2+x+3<0$
h) What is the vertex form of the equation?

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7. Find the domain of each function:

a) $f(x)=-x^2+5x-7$

b) $g(x)=3^x-31$

c) $h(x)=\ln(6-5x)$

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8. Graph $f(x)=5^x$ and $f^{-1}(x)=\log_5 x$ on the same coordinate system showing the symmetry about the bisector line $y=x$. Label the axes and all the points.

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9. State whether each statement is TRUE or FALSE. Justify your answer.

a) $\log(a+b)=\log a+\log b$

b) $\log\left(\frac{a}{b}\right)\neq\frac{\log a}{\log b}$

c) $\log 5x^2=2\log 5x$

d) $\log(ab)\neq(\log a)(\log b)$

Choose any FOUR of the following problems. You may solve the fifth problem for extra credit.

1) The number of bacteria present in a culture after t hours is given by the formula $N = 500e^{0.59t}$.

a) How many bacteria will be there after ½ hour?

b) How long will it be before there are 500,000 bacteria?

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2) The owners of a small fruit orchard decide to produce gift baskets as a sideline. The cost per basket for producing *x* baskets is $C = 0.01x^2 - 2x + 120$. How many baskets should they produce in order to minimize the cost per basket? What will their total cost be at that production level?

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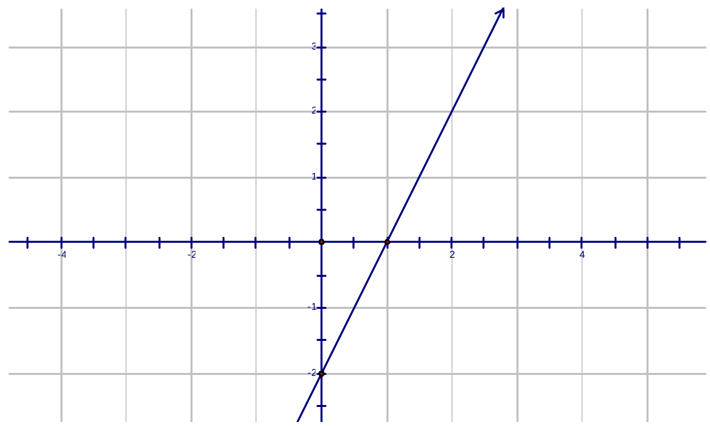
3) The graph of a function f is given.



a) Graph the inverse function f^{-1} on the same coordinate system, showing the symmetry about the bisector line y=x.

b) Using the graph, find an equation of the given line.

c) Find an equation of the inverse function.

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4) In a laboratory experiment, researchers establish a colony of 100 bacteria and monitor its growth. The experiments discover that the colony triples in population every day.

a) Fill in Table 1. showing the population, $P(t)$, of bacteria *t* days later.

b) Find a function that gives the population of the colony at any time *t* in days.

c) Graph the function. Label the axes and the points used.

d) How many bacteria will be present after 13 days?

e) How long will it take before there are 37,000 bacteria?

t	**P(t)**
0	
1	
2	
3	
4	

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5) India is currently one of the world's fastest-growing countries. By 2040, the population of India will be larger than the population of China; by 2050, nearly one-third of the world's population will live in these two countries alone. The exponential function

$$f(x) = 574(1.026)^x$$

models the population of India, $f(x)$, in millions, *x* years after 1974.

a) What was India's population in 1974?

b) Find $f(27)$ and its meaning.

c) Find India's population, to the nearest million, in the year 2028 as predicted by this function.

d) How long will it be until the population reaches 1000 million people?

MATH 71 TEST 3 - SOLUTIONS

① a) $2(x+3)^2+80=0$ square root property

$2(x+3)^2=-80$

$(x+3)^2=-40 \quad /\sqrt{\ }$

$\sqrt{(x+3)^2}=\sqrt{-40}$

$x+3=\pm\sqrt{40}\,i$

$\boxed{x=-3\pm 2\sqrt{10}\,i}$

b) $4y^2=3y-1$ by completing the square

1st isolate the constant

$4y^2-3y=-1 \quad / \div 4$

2nd leading coefficient 1

$y^2-\frac{3}{4}y=\frac{-1}{4} \quad / + \frac{9}{64}$

3rd missing term $\left(\frac{1}{2}\cdot\frac{3}{4}\right)^2=\left(\frac{3}{8}\right)^2=\frac{9}{64}$

$y^2-\frac{3}{4}y+\frac{9}{64}=\frac{-1}{4}+\frac{9}{64}$

$\left(y-\frac{3}{8}\right)^2=\frac{-16+9}{64}$

$\left(y-\frac{3}{8}\right)^2=\frac{-7}{64} \quad /\sqrt{\ }$

$\sqrt{\left(y-\frac{3}{8}\right)^2}=\sqrt{\frac{-7}{64}}$

$y-\frac{3}{8}=\pm\frac{\sqrt{7}\,i}{8}$

$\boxed{y=\frac{3}{8}\pm\frac{\sqrt{7}}{8}i}$

c) $\frac{2t^2}{5}+\frac{t}{2}=\frac{1}{5}$ by quadratic formula

Eliminate all fractions and write the equation in standard form:

$LCD=10$

$4t^2+5t=2$

$4t^2+5t-2=0$

$4t^2+5t-2=0$

$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\left\{\begin{array}{l} a=4 \\ b=5 \\ c=-2 \end{array}\right.$

$t=\frac{-5\pm\sqrt{25-4(4)(-2)}}{2(4)}$

$=\frac{-5\pm\sqrt{25+32}}{8}$

$\boxed{t=\frac{-5\pm\sqrt{57}}{8}}$

d) $h=-16t^2+\frac{23}{3}t$ solve for t

quadratic equation in t. Eliminate the fractions, write the equation in standard form and apply the quadratic formula

$3h=-48t^2+23t$

$48t^2-23t+3h=0$

$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\left\{\begin{array}{l} a=48 \\ b=-23 \\ c=3h \end{array}\right.$

$t=\frac{23\pm\sqrt{529-4(48)(3h)}}{2(48)}$

$\boxed{t=\frac{23\pm\sqrt{529-576h}}{96}}$

(2)

(a) $2x^4 - 3x^2 + 1 = 0$

let $x^2 = t$

then $x^4 = t^2$

The equation becomes:

$2t^2 - 3t + 1 = 0$

$t = \frac{3 \pm \sqrt{9 - 4(2)(1)}}{2(2)} = \frac{3 \pm 1}{4}$

$t = 1$ OR $t = \frac{1}{2}$

$x^2 = 1$ $\quad$ $x^2 = \frac{1}{2}$

$x = \pm 1$ $\quad$ $x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$\boxed{x \in \{\pm 1, \pm \frac{\sqrt{2}}{2}\}}$

(b) $\log_5(3x-1) - 2 = 0$

Condition: $3x - 1 > 0$

$3x > 1$

$\boxed{x > \frac{1}{3}}$

$\log_5(3x-1) = 2$

$\Longleftrightarrow$

$5^2 = 3x - 1$

$3x = 26 \Longrightarrow \boxed{x = \frac{26}{3}}$

(c) $4^x = 13 \quad / \ln$

$\ln 4^x = \ln 13$

$x \ln 4 = \ln 13$

$\boxed{x = \frac{\ln 13}{\ln 4}}$ $\quad$ $\boxed{x \approx 1.85}$

(d) $\log_8(x+5) - \log_8 2 = 1$

Condition: $x + 5 > 0$

$\boxed{x > -5}$

$\log_8 \frac{x+5}{2} = 1$

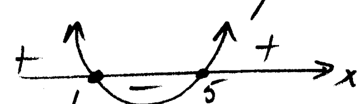
$\Longleftrightarrow$

$8^1 = \frac{x+5}{2}$

$x + 5 = 16 \Longrightarrow \boxed{x = 11}$

(3) (a) $x^2 - 6x + 5 \le 0$

let $y = x^2 - 6x + 5$ parabola opens up



x-$\cap$: $x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$ $\quad$ $\left\langle \begin{array}{l} x = 1 \\ x = 5 \end{array} \right.$

$x^2 - 6x + 5 \le 0$ iff $\boxed{x \in [1,5]}$

(b) $\frac{1}{x-5} < \frac{3}{2-x}$

$\frac{1}{x-5} - \frac{3}{2-x} < 0$ $\quad$ ($2-x$, $x-5$)

$LCD = (x-5)(2-x)$

$\frac{(2-x) - 3(x-5)}{(x-5)(2-x)} < 0$

$\frac{2 - x - 3x + 15}{(x-5)(2-x)} < 0$

$\frac{17 - 4x}{(x-5)(2-x)} < 0$

study the sign of each factor.

x	$-\infty$	2	$\frac{17}{4}$	5	∞
$17-4x$	+ + + +		0 − − − − −		
$x-5$	− − − − −			0 + + +	
$2-x$	+ + + 0 − − − − − −				
$\frac{17-4x}{(x-5)(2-x)}$	−	+ 0 −		+	

$$\frac{1}{x-5} < \frac{3}{2-x} \text{ iff}$$

$$\frac{17-4x}{(x-5)(2-x)} < 0 \text{ iff}$$

$$\boxed{x \in (-\infty, 2) \cup \left(\frac{17}{4}, 5\right)}$$

(4) $f(x) = 3x-2$, $g(x) = \frac{2-x}{x+1}$

(a) $(g \circ f)(x) = g(f(x))$

$$= g(3x-2)$$

$$= \frac{2-(3x-2)}{(3x-2)+1}$$

$$= \frac{2-3x+2}{3x-1}$$

$$\boxed{(g \circ f)(x) = \frac{4-3x}{3x-1}}$$

(b) $(f \circ g)(2) = f(g(2)) =$

$$= f(0)$$

$g(2) = \frac{2-2}{2+1} = 0$

$$= 3(0)-2$$

$$= -2$$

$$\boxed{(f \circ g)(2) = -2}$$

(c) $f(x) = 3x-2$

1st $y = 3x-2$

2nd solve for x

$$3x = 2+y$$

$$x = \frac{2+y}{3}$$

3rd $x \leftrightarrow y$

$$y = \frac{2+x}{3}$$

$$\boxed{f^{-1}(x) = \frac{2+x}{3}}$$

(d) $g(x) = \frac{2-x}{x+1}$

1st $y = \frac{2-x}{x+1}$

2nd solve for x

$$y(x+1) = 2-x$$

$$yx+y = 2-x$$

$$yx+x = 2-y$$

$$x(y+1) = 2-y$$

$$x = \frac{2-y}{y+1}$$

3rd $x \leftrightarrow y$

$$y = \frac{2-x}{x+1}$$

$$\boxed{g^{-1}(x) = \frac{2-x}{x+1}}$$

(5) (a) $4\ln x + 7\ln y - 3\ln z =$

$= \ln x^4 + \ln y^7 - \ln z^3$

$= \ln x^4 y^7 - \ln z^3$

$= \boxed{\ln \dfrac{x^4 y^7}{z^3}}$

(b) $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5 (x+1) =$

$= \frac{1}{2}\log_5 (xy) - \log_5 (x+1)^2$

$= \log_5 (xy)^{\frac{1}{2}} - \log_5 (x+1)^2$

$= \boxed{\log_5 \left(\dfrac{\sqrt{xy}}{(x+1)^2}\right)}$

(c) $\log_3 405 - \log_3 5 + \log 5 + \log 2 =$

$= \log_3 \frac{405}{5} + \log 10$

$= \log_3 81 + \log 10$

$= 4 + 1 = \boxed{5}$

(d) $\log_{10}\left(\log_3 \left(\log_5 125\right)\right) =$

$= \log_{10}(\log_3 3)$

$= \log_{10} 1$

$= \boxed{0}$

(6) $y = -2x^2 + x + 3$

(a) parabola that opens downwards $(a = -2 < 0)$

(b) y-int: $x = 0,\ y = 3$

$\boxed{\text{y-int: } (0,3)}$

(c) $V(x_v, y_v)$

$x_v = \frac{-b}{2a} = \frac{-1}{2(-2)} = \frac{1}{4}$

$y_v = -2\left(\frac{1}{4}\right)^2 + \frac{1}{4} + 3$

$= -2 \cdot \frac{1}{16} + \frac{1}{4} + 3$

$= \frac{-1}{8} + \frac{1}{4} + 3 = \frac{25}{8}$

$\boxed{V\left(\frac{1}{4}, \frac{25}{8}\right)}$

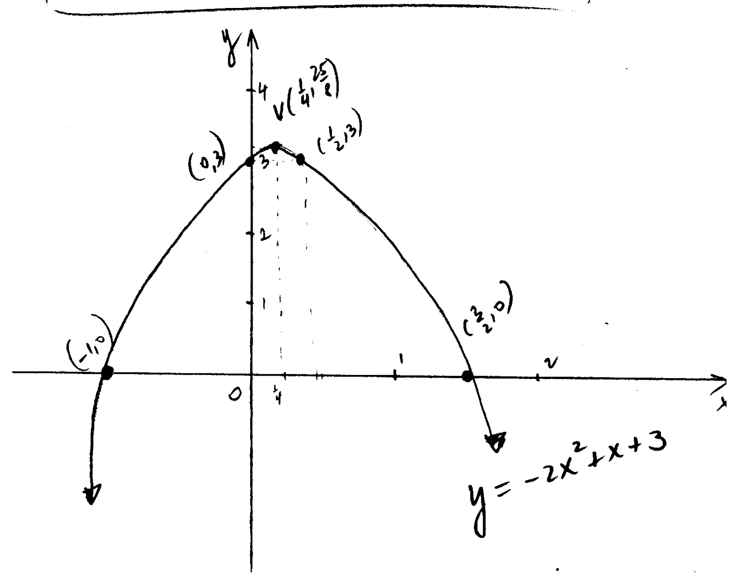
(d) x-int: $-2x^2 + x + 3 = 0 \quad / (-1)$

$2x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(2)(-3)}}{2(2)} = \frac{1 \pm 5}{4}$

$x = \frac{6}{4} = \frac{3}{2}$ OR $x = \frac{-4}{4} = -1$

$\boxed{\text{x-int: } \left(\frac{3}{2}, 0\right) \text{ and } (-1, 0)}$



e) Domain: $\boxed{x \in \mathbb{R}}$

f) Range: $\boxed{y \in (-\infty, \frac{25}{8}]}$

g) $-2x^2 + x + 3 < 0$

$x = ?$ $\quad y < 0$

$\boxed{x \in (-\infty, -1) \cup (\frac{3}{2}, \infty)}$

h) $y = a(x - x_v)^2 + y_v$

$V(\frac{1}{4}, \frac{25}{8})$, $\quad a = -2$

$\boxed{y = -2\left(x - \frac{1}{4}\right)^2 + \frac{25}{8}}$

(7) (a) $f(x) = -x^2 + 5x + 7$

Domain: $\boxed{x \in \mathbb{R}}$

(b) $g(x) = 3^x - 31$

Domain: $\boxed{x \in \mathbb{R}}$

(c) $h(x) = \ln(6 - 5x)$

Condition: $6 - 5x > 0$

$6 > 5x$

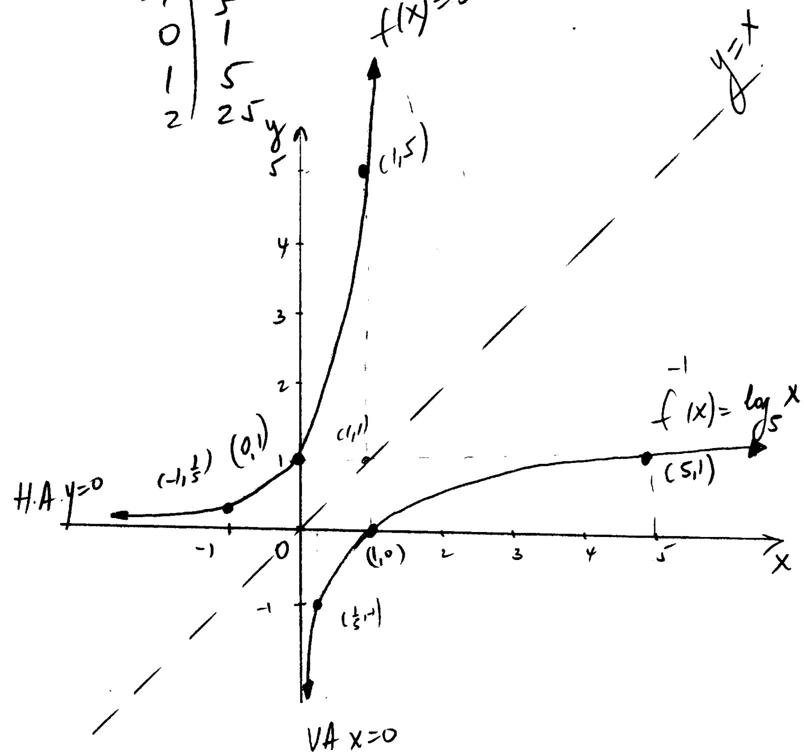
$x < \frac{6}{5}$

Domain: $\boxed{x \in (-\infty, \frac{6}{5})}$

(8) $f(x) = 5^x$

Domain: $x \in \mathbb{R}$

x	y
-1	$\frac{1}{5}$
0	1
1	5
2	25



(9) a) $\log(a+b) = \log a + \log b$

False $\quad \log a + \log b = \log(ab)$

b) $\log\left(\frac{a}{b}\right) \neq \frac{\log a}{\log b}$

TRUE $\quad \log\left(\frac{a}{b}\right) = \log a - \log b$

c) $\log 5x^2 = 2 \log 5x$

False

$\log 5x^2 = \log 5 + 2\log x$

d) $\log(ab) \neq (\log a)(\log b)$

TRUE $\quad \log(ab) = \log a + \log b$

PART II

(1) $N = 500\, e^{0.59t}$

t = time (# hours)

N = # bacteria

(a) $t = 0.5$ hrs, $N = ?$

$N = 500\, e^{0.59(0.5)} \approx 672$ bacteria

After $\frac{1}{2}$ hour, there will be 672 bacteria

(b) $t = ?$ if $N = 500{,}000$

$500{,}000 = 500\, e^{0.59t}$

$e^{0.59t} = 1000$

$\ln e^{0.59t} = \ln 1000$

$0.59t = \ln 1000$

$t = \frac{\ln 1000}{0.59} \approx 11.7$ hours

There will be 500,000 bacteria after approximately 11.7 hours.

(2) $C = 0.01x^2 - 2x + 120$

x = # baskets

C = cost per basket

The equation represents a parabola that opens up, therefore the minimum occurs at the vertex

$V(x_v, C_v)$

$x_v = \frac{-b}{2a} = \frac{-(-2)}{2(0.01)} = \frac{1}{0.01} = 100$ baskets

$C_v = 0.01(100)^2 - 2(100) + 120$

$= 20$ \$/basket

In order to minimize the cost per basket, they should produce 100 baskets. The cost per basket is then \$20.

The total cost at that production level is

100 baskets (20 \$/basket)

$= 2000$ \$

(3) (a) See graph on test.

(b) $y = mx + b$

$b = -2$

$m = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$

$y = 2x - 2$

$\boxed{f(x) = 2x - 2}$ the given line

(c) 1st $y = 2x - 2$

2nd $2x = y + 2$

$x = \frac{y+2}{2}$

3rd $y = \frac{x+2}{2}$

$\boxed{f^{-1}(x) = \frac{x+2}{2}}$

or it could be find using the ~~graph~~ $b = 2$

$m = \frac{1}{2}$

$y = \frac{1}{2}x + 2$

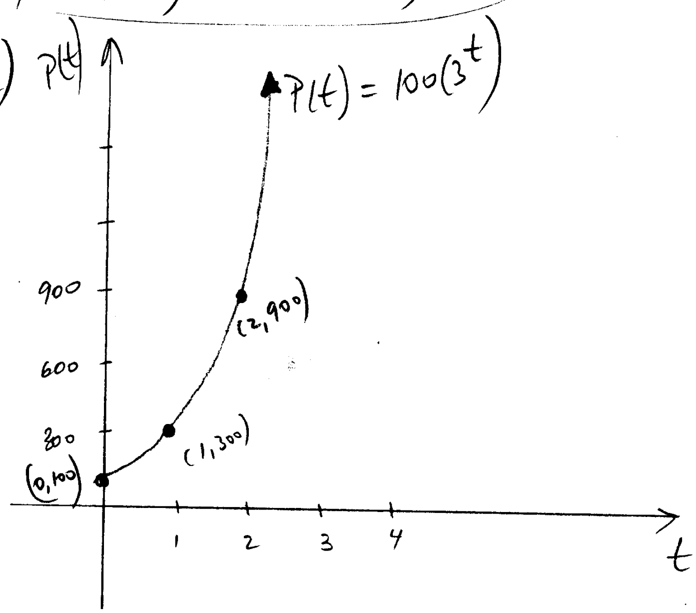
(4)

a)

t	$P(t)$	
0	100	$= 100$
1	$3(100)$	$= 300$
2	$3^2(100)$	$= 900$
3	$3^3(100)$	$= 2700$
4	$3^4(100)$	
t	$3^t(100)$	

b) $P(t) = 100(3^t)$

c)



d) $t = 13$ days, $P = ?$

$P(13) = 100(3^{13}) \approx 1.6 \times 10^8$

$= 160{,}000{,}000$ bacteria

e) $t = ?$ if $P = 37{,}000$

$37{,}0\cancel{00} = \cancel{100}(3^t)$

$3^t = 370$) ln

$\ln 3^t = \ln 370$

$t \ln 3 = \ln 370$

$t = \dfrac{\ln 370}{\ln 3} \approx 5.4$ days

(5) $f(x) = 574(1.026)^x$

x = # years after 1974

$f(x)$ = population (in millions)

a) $x = 0$, $f(0) = 574$ million

in 1974, the population was 574 million

b) $f(27) = 574(1.026)^{27}$

≈ 1147.86 million

In 2001, the population was 1147.86 million

c) 2028 $\quad x = 2028 - 1974$

$x = 54$

$f(54) = 574(1.026)^{54}$

≈ 2295 million

d) $x = ?$ if $f(x) = 1000$ million

$574(1.026)^x = 1000$

$(1.026)^x = \dfrac{1000}{574}$) ln

$\ln(1.026)^x = \ln\left(\dfrac{1000}{574}\right)$

$x \ln 1.026 = \ln\left(\dfrac{1000}{574}\right)$

$x = \dfrac{\ln\left(\frac{1000}{574}\right)}{\ln 1026} \approx 21.6$ years

The population reaches 1000 million in about 21.6 years