Math 71 Fall 07 Name:

TEST 2 @ 130 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

---

1. Factor each expression completely:

a) $x^3 - 3x^2 - 9x + 27$

b) $x^6 - x^2$

c) $3x^2 + 3x - 18$

d) $2d^{n+2} - 5d^{n+1} + 3d^n$

---

2. Do the following operations (simplify):

a) $5 + \dfrac{7}{x-2}$

b) $\dfrac{x+1}{x^2+x-2} - \dfrac{1}{x^2-3x+2} + \dfrac{2x}{x^2-4}$

c) $\dfrac{x^3-27}{4x^2-4x} \cdot \dfrac{4x}{x-3}$

d) $\left(\dfrac{x^{\frac{-5}{4}} y^{\frac{1}{3}}}{x^{-\frac{3}{4}}} \right)^{-6}$ and write the final answer using only positive exponents

e) $2\sqrt{75} + 4\sqrt{12} - \left(2\sqrt{2} - \sqrt{3}\right)\left(2\sqrt{2} + \sqrt{3}\right)$

f) $\dfrac{1+2i}{2-3i}$

g) $\dfrac{-14 + \sqrt{-128}}{16}$

h) Divide using long division: $\dfrac{4a^3 + 12a^2 + 7a - 1}{2a+3}$

i) $(1-3i)(2-i) - (4-2i)(3+i)$

---

3. If $f\left(x\right)=x^2-3x+1$, find $\frac{f\left(a+h\right)-f\left(a\right)}{h}$.

---

4. If $f\left(x\right)=\frac{4}{x-3}$ and $g\left(x\right)=\frac{10}{x^2+x-12}$, find all the values of a for which $f\left(a\right)=g\left(a\right)+1$.

---

5. $f\left(x\right)=2x^2-x+4$ Find the following:

a) $f\left(3i\right)$

b) $f\left(1-\sqrt{3}\right)$

---

6. Let $f\left(x\right)=\sqrt{x-2}$.

a) What is the domain of this function?

b) Sketch the graph of the function by plotting points. Label the axes and all the points used.

c) What is the range of this function.

---

7. Solve the following equations:

a) $\left(x-3\right)\left(x+8\right)=-30$

b) $\frac{3}{2}x^2+\frac{2}{3}x=0$

---

8. If $g\left(x\right)=\sqrt{x+9}-\sqrt{x-7}$, find x such that $g\left(x\right)=2$.

---

9. If $f\left(x\right)=\frac{x+6}{x+3}$ and $g\left(x\right)=\frac{3}{x+3}$, find x such that $f\left(x\right)=g\left(x\right)+2$.

---

10. Police use the function $f\left(x\right)=\sqrt{20x}$ to estimate the speed of a car, f(x), in miles per hour, based on the length, x, in feet, of its skid marks upon sudden braking on a dry asphalt road. A motorist is involved in an accident. A police officer measures the car's skid mark to be 45 feet long. Estimate the speed at which the motorist was traveling before braking. If the posted speed limit is 35 miles per hour and the motorist tells the officer she was not speeding, should the officer believe her? Explain.

---

11. A gymnast dismounts the uneven parallel bars at a height of 8 feet with an initial upward velocity of 8 feet per second. The function $s(t) = -16t^2 + 8t + 8$ describes the height of the gymnast's feet above ground, $s(t)$, in feet, t seconds after dismounting.

a) How long will it take the gymnast to reach the ground?

b) When will the gymnast be 8 feet above the ground?

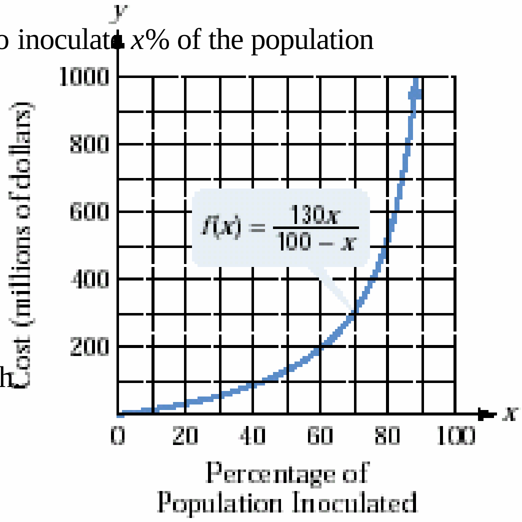
---

12. The rational function $f(x) = \dfrac{130x}{100 - x}$ models the cost, $f(x)$, in millions, to inoculat x% of the population against a particular strain of flu. Answer the following:

a) What is the domain of the function? What is the range?
What is the meaning of the domain of the function?

b) What happens to the cost as x approaches 100%?
How is this shown by the graph? Explain what it means.

c) Find and interpret $f(60)$. Identify your solution as a point on the graph



---

13. The Food Stamp Program is America's first line of defense against hunger for millions of families. Over half of all participants are children; one out of six is a low-income older adult. The next questions involve the number of participants in the program from 1990 through 2002.
The function

$$f(x) = -\frac{1}{4}x^2 + 3x + 17$$

models the number of people, $f(x)$, in millions, receiving food stamps x years after 1990.

a) In which year did 26 million people receive food stamps?

In which years did 25 million people receive food stamps?

b) In which years did 25 million people receive food stamps?

Math 71

TEST 2 - SOLUTIONS

① (a) $\underbrace{x^3-3x^2}\underbrace{-9x+27} =$

$= x^2(x-3) - 9(x-3)$

$= (x-3)(x^2-9)$

$= (x-3)(x-3)(x+3) = \boxed{(x-3)^2(x+3)}$

(b) $x^6 - x^2 = x^2(x^4-1)$

$= x^2((x^2)^2-1)$

$= x^2(x^2-1)(x^2+1)$

$= \boxed{x^2(x-1)(x+1)(x^2+1)}$

(c) $3x^2+3x-18 =$

$= 3(x^2+x-6)$

$= \boxed{3(x+3)(x-2)}$

(d) $2d^{n+2}-5d^{n+1}+3d^{n} =$

$= d^n(2d^2-5d+3)$

$= \boxed{d^n(2d-3)(d-1)}$

---

② (a) $5+\frac{7}{x-2} = \overset{x-2/}{\frac{5}{1}} + \frac{7}{x-2}$

$= \frac{5(x-2)+7}{x-2}$

$= \frac{5x-10+7}{x-2} = \boxed{\frac{5x-3}{x-2}}$

(b)

$\frac{x+1}{x^2+x-2} - \frac{1}{x^2-3x+2} + \frac{2x}{x^2-4} =$

$= \overset{x-2}{\frac{x+1}{(x+2)(x-1)}} - \overset{x+2}{\frac{1}{(x-1)(x-2)}} + \overset{x-1}{\frac{2x}{(x+2)(x-2)}}$

$LCD = (x+2)(x-1)(x-2)$

$= \frac{(x-2)(x+1) - (x+2) + 2x(x-1)}{(x+2)(x-1)(x-2)}$

$= \frac{x^2-x-2-x-2+2x^2-2x}{(x+2)(x-1)(x-2)}$

$= \frac{3x^2-4x-4}{(x+2)(x-1)(x-2)} = \frac{(3x+2)\cancel{(x-2)}}{(x+2)(x-1)\cancel{(x-2)}}$

$= \boxed{\frac{3x+2}{(x+2)(x-1)}}$

(c) $\frac{x^3-27}{4x^2-4x} \cdot \frac{4x}{x-3} =$

$= \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{4x}(x-1)} \cdot \frac{\cancel{4x}}{\cancel{x-3}}$

$= \boxed{\frac{x^2+3x+9}{x-1}}$

(d)
$$\left(\frac{x^{-\frac{5}{4}}y^{\frac{1}{3}}}{x^{-\frac{3}{4}}}\right)^{-6}=\left(x^{\overset{-2}{-\frac{5}{4}+\frac{3}{4}}}y^{\frac{1}{3}}\right)^{-6}$$
$$=\left(x^{-\frac{2}{4}}y^{\frac{1}{3}}\right)^{-6}=\left(x^{-\frac{1}{2}}y^{\frac{1}{3}}\right)^{-6}$$
$$=\left(x^{-\frac{1}{2}}\right)^{-6}\left(y^{\frac{1}{3}}\right)^{-6}$$
$$=x^{3}y^{-2}=\boxed{\frac{x^{3}}{y^{2}}}$$

(e)
$$2\sqrt{75}+4\sqrt{12}-(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})$$
$$=2\sqrt{25\cdot3}+4\sqrt{4\cdot3}-\left((2\sqrt{2})^{2}-(\sqrt{3})^{2}\right)$$
$$=2\cdot5\sqrt{3}+4\cdot2\sqrt{3}-(8-3)$$
$$=10\sqrt{3}+8\sqrt{3}-5=\boxed{18\sqrt{3}-5}$$

(f)
$$\frac{1+2i}{2-3i}\cdot\frac{2+3i}{}=\frac{(1+2i)(2+3i)}{(2-3i)(2+3i)}$$
$$=\frac{2+3i+4i+6i^{2}}{2^{2}-(3i)^{2}}$$
$$=\frac{2+7i+6(-1)}{4-9i^{2}}=\frac{-4+7i}{4-9(-1)}$$
$$=\boxed{\frac{-4+7i}{13}}$$

(g)
$$\frac{-14+\sqrt{-128}}{16}=\frac{-14+\sqrt{128}\sqrt{-1}}{16}$$
$$=\frac{-14+\sqrt{64\cdot2}\;i}{16}=\frac{-14+8\sqrt{2}i}{16}$$
$$=\frac{2(-7+4\sqrt{2}i)}{16}=\boxed{\frac{-7+4\sqrt{2}i}{8}}$$

(h)
$$\boxed{\frac{4a^{3}+12a^{2}+7a-1}{2a+3}=2a^{2}+3a-1+\frac{2}{2a+3}}$$
$$\begin{array}{rl} & 2a^{2}+3a-1 \\ 2a+3 \,\big) & 4a^{3}+12a^{2}+7a-1 \\ & -4a^{3}-6a^{2} \\ \hline & \quad 6a^{2}+7a-1 \\ & -6a^{2}-9a \\ \hline & \qquad -2a-1 \\ & \qquad +2a+3 \\ \hline & \qquad\quad 2 \end{array}$$

(i)
$$(1-3i)(2-i)-(4-2i)(3+i)=$$
$$=2-i-6i+3i^{2}-(12+4i-6i-2i^{2})$$
$$=2-7i+3(-1)-(12-2i-2(-1))$$
$$=2-7i-3-(14-2i)$$
$$=-1-7i-14+2i$$
$$=\boxed{-15-5i}$$

---

③ $f(x)=x^{2}-3x+1$
$$\frac{f(a+h)-f(a)}{h}=$$
$$=\frac{\left[(a+h)^{2}-3(a+h)+1\right]-(a^{2}-3a+1)}{h}$$
$$=\frac{\cancel{a^{2}}+2ah+h^{2}-\cancel{3a}-3h+\cancel{1}-\cancel{a^{2}}+\cancel{3a}-\cancel{1}}{h}$$
$$=\frac{2ah+h^{2}-3h}{h}=\frac{h(2a+h-3)}{h}$$
$$=\boxed{2a+h-3}$$

④ $f(x)=\frac{4}{x-3}$, $g(x)=\frac{10}{x^2+x-12}$

a = ? such that

$f(a) = g(a) + 1$

$$\frac{4}{a-3} = \frac{10}{a^2+a-12} + 1$$

$$\frac{\overset{a+4}{4}}{a-3} = \frac{10}{(a+4)(a-3)} \overset{(a+4)(a-3)}{+1}$$

$LCD = (a-3)(a+4)$

Conditions $\begin{cases} a \neq 3 \\ a \neq -4 \end{cases}$

$4(a+4) = 10 + (a+4)(a-3)$

$4a+16 = 10 + a^2+a-12$

$4a+16 = a^2+a-2$

$a^2+a-2-4a-16 = 0$

$a^2-3a-18=0$

$(a-6)(a+3)=0$

$a=6$ OR $a=-3$

$\boxed{a \in \{6,-3\}}$

---

⑤ $f(x) = 2x^2-x+4$

a) $f(3i) = 2(3i)^2-3i+4$

$= 2(9i^2)-3i+4$

$= 18(-1)-3i+4 = \boxed{-14-3i}$

b) $f(1-\sqrt{3}) = 2(1-\sqrt{3})^2-(1-\sqrt{3})+4$

$= 2(1-2\sqrt{3}+3)-1+\sqrt{3}+4$

$= 2(4-2\sqrt{3})+3+\sqrt{3}$

$= 8-4\sqrt{3}+3+\sqrt{3}$

$= \boxed{11-3\sqrt{3}}$

⑥ $f(x) = \sqrt{x-2}$

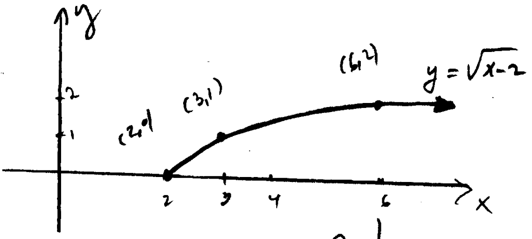
a) Condition: $x-2 \geq 0$

$x \geq 2$

Domain = $[2,\infty)$

b)

x	y
2	0
3	1
6	2



c) $y \geq 0$, Range = $[0,\infty)$

---

⑦ ⓐ $(x-3)(x+8) = -30$

$x^2+5x-24 = -30$

$x^2+5x+6 = 0$

$(x+2)(x+3) = 0$

$x=-2$ OR $x=-3$

$\boxed{x \in \{-2,-3\}}$

ⓑ $\frac{3}{2}x^2 + \frac{2}{3}x = 0$

$x\left(\frac{3}{2}x+\frac{2}{3}\right) = 0$

$x=0$ OR $\frac{3}{2}x + \frac{2}{3} = 0$

$\frac{3}{2}x = -\frac{2}{3} \quad \Big| \cdot \frac{2}{3}$

$x = \frac{-4}{9}$

$\boxed{x \in \{0, \frac{-4}{9}\}}$

⑧ $g(x)=\sqrt{x+9}-\sqrt{x-7}$

$g(x)=2$

$\sqrt{x+9}-\sqrt{x-7}=2$

$\sqrt{x+9}=2+\sqrt{x-7} \quad |^2$

$(\sqrt{x+9})^2=(2+\sqrt{x-7})^2$

$\cancel{x}+9=4+4\sqrt{x-7}+\cancel{x}-7$

$9=4\sqrt{x-7}-3$

$12=4\sqrt{x-7}$

$3=\sqrt{x-7} \quad |^2$

$9=x-7 \Rightarrow x=16$

check: $\sqrt{16+9}-\sqrt{16-7}\overset{?}{=}2$

$5-3=2$ TRUE

Therefore, $\boxed{x\in\{16\}}$

---

⑨ $f(x)=\frac{x+6}{x+3}$

$g(x)=\frac{3}{x+3}$

$f(x)=g(x)+2$

$\frac{x+6}{x+3}=\frac{3}{x+3}+2$

condition: $x\neq-3$

$\frac{x+6}{x+3}-\frac{3}{x+3}=2$

$\frac{x+6-3}{x+3}=2$

$\frac{x+3}{x+3}=2$

$\rightarrow 1=2$ Contradiction

$\boxed{x\in\emptyset}$

⑩ $f(x)=\sqrt{20x}$

x = length of skid mark (ft)

f(x) = speed of the car

x = 45 ft, find f(45)

$f(45)=\sqrt{20(45)}=\sqrt{900}=30\,mi/h$

She was traveling at

$\boxed{30\,mi/h} < 35\,mi/h$

so she was telling the truth

---

⑪ $s(t)=-16t^2+8t+8$

t = time (sec)

s(t) = height (ft)

a) $t=?\quad s(t)=0$

$-16t^2+8t+8=0 \quad |\div 8$

$16t^2-8t-8=0$

$2t^2-t-1=0$

$(2t+1)(t-1)=0$

$t=-\frac{1}{2}$ OR $\boxed{t=1\text{ second}}$

not possible

b) $t=?\quad s(t)=8$ ft

$-16t^2+8t+8=8$

$-16t^2+8t=0$

$16t^2-8t=0$

$8t(2t-1)=0$

$t=0$ OR $t=\frac{1}{2}$ seconds

The gymnast will be 8 ft above ground at the initial moment and again after 0.5 seconds.

⑫ $f(x) = \frac{130x}{100-x}$

x = % of the population

f(x) = cost (million \$)

a) Domain: $\boxed{x \in [0,100)}$

$100 - x \neq 0$

$x \neq 100$ and $x \geq 0$ (% of pop)

Range: $\boxed{y \geq 0}$

$x \in [0,100)$

b) when $x \to 100\%$, $y \to \infty$

c) $f(60) = \frac{130(60)}{100-60} = \frac{130 \cdot 6\cancel{0}}{4\cancel{0}}$

$f(60) = 195$

The cost of inoculating 60% of the population is 195 million dollars

⑬ $f(x) = -\frac{1}{4}x^2 + 3x + 17$

x = # years after 1990

f(x) = # people (in million)

a) $x = ?$ $f(x) = 26$

$-\frac{1}{4}x^2 + 3x + 17 = 26$

$\frac{1}{4}x^2 - 3x + 26 - 17 = 0$

$\frac{1}{4}x^2 - 3x + 9 = 0 \quad / 4$

$x^2 - 12x + 36 = 0$

$(x-6)^2 = 0$

$x = 6$

in 1996, 26 million people received food stamps.

b) $x = ?$ $f(x) = 25$

$-\frac{1}{4}x^2 + 3x + 17 = 25$

$\frac{1}{4}x^2 - 3x + 8 = 0 \quad |\cdot 4$

$x^2 - 12x + 32 = 0$

$(x-4)(x-8) = 0$

$x = 4$ or $x = 8$

in 1994 and 1998, 25 million people received food stamps