MATH 71 TEST 1 - SOLUTIONS

① a) $y = f(x)$

y is a function of x if and only if for every input x there is only one output y

b) Domain: $\{x \mid f(x) \in \mathbb{R}\}$

c) Range: $\{y \mid y = f(x), x \in \text{Domain}\}$

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② $f(x) = \dfrac{5x-2}{2x+1}$

a) $f(0) = \dfrac{5(0)-2}{2(0)+1} = \dfrac{-2}{1} = -2$

b) $f(-x) = \dfrac{5(-x)-2}{2(-x)+1} = \dfrac{-5x-2}{-2x+1}$

c) $f(a+h) = \dfrac{5(a+h)-2}{2(a+h)+1} = \dfrac{5a+5h-2}{2a+2h+1}$

d) Condition: $2x+1 \neq 0$

$x \neq -\frac{1}{2}$

Domain: $x \in \mathbb{R} \setminus \{-\frac{1}{2}\}$

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③ $f(x) = \begin{cases} x^2, & x \le 0 \\ 3x-1, & 0 < x < 5 \\ 2, & x \ge 5 \end{cases}$

$f(-1) = (-1)^2 = 1$ (b/c $x = -1 \le 0$)

$f(5) = 2$ (b/c $x = 5 \ge 5$)

$f(2) = 3(2)-1 = 5$ (b/c $x = 2 \in (0,5)$)

$f(0) = 0^2 = 0$ (b/c $x = 0 \le 0$)

$f(19) = 2$ (b/c $x = 19 \ge 5$)

④ given line: $2x+5y-3=0$

Find the slope of the given line:

$5y = -2x+3$ $/ \div 5$

$y = \frac{-2}{5}x + \frac{3}{5}$

$m = \frac{-2}{5}$

Then, the slope of a line perpendicular to the given line is

$m_{\perp} = \frac{-5}{3}$ / $y - y_1 = m(x - x_1)$

$(-2,3)$

$y - 3 = \frac{-5}{3}(x-(-2))$

$y - 3 = \frac{-5}{3}(x+2)$

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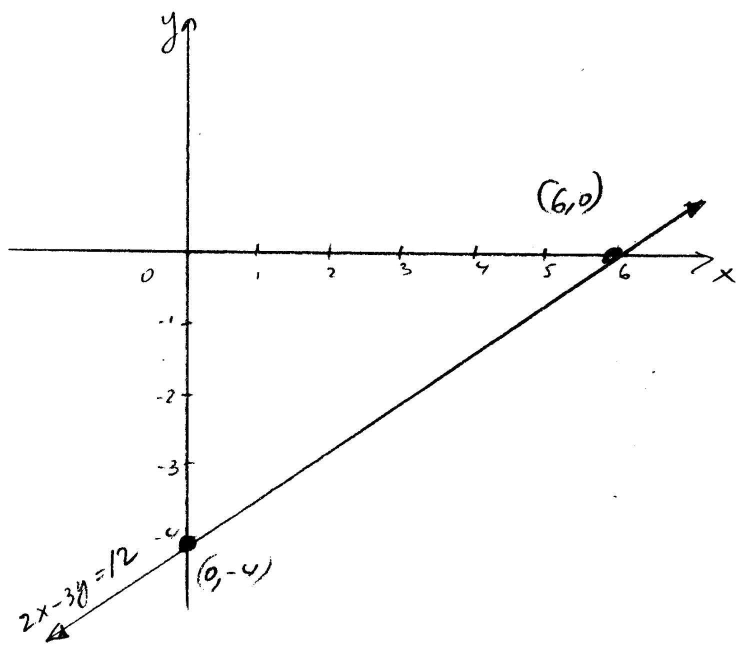
⑤ $2x - 3y = 12$

x	y
0	-4
6	0

$-3y = 12$, $y = -4$

$2x = 12$, $x = 6$



⑥ $\begin{cases} 6x+3y+5z=-1 & ① \\ 2x-6y+8z=-9 & ② \\ 4x-9y+3z=-4 & ③ \end{cases}$

Eliminate x:

$$\begin{matrix} ① \\ ② \end{matrix}\begin{cases} 6x+3y+5z=-1 \\ 2x-6y+8z=-9 \end{cases} \Big| \cdot -3$$

$$\begin{cases} 6x+3y+5z=-1 \\ -6x+18y-24z=27 \end{cases}$$

$(+)\quad 21y-19z=26 \quad ④$

$$\begin{matrix} ② \\ ③ \end{matrix}\begin{cases} 2x-6y+8z=-9 \\ 4x-9y+3z=-4 \end{cases} \Big| \cdot -2$$

$$\begin{cases} -4x+12y-16z=18 \\ 4x-9y+3z=-4 \end{cases}$$

$(+)\quad 3y-13z=14 \quad ⑤$

$$\begin{matrix} ④ \\ ⑤ \end{matrix}\begin{cases} 21y-19z=26 \\ 3y-13z=14 \end{cases} \Big| \cdot -7$$

Eliminate y:

$$\begin{cases} 21y-19z=26 \\ -21y+91z=-98 \end{cases}$$

$(+)\quad 72z=-72 \Rightarrow \boxed{z=-1}$

⑤ $3y-13z=14$

$3y+13=14 \Rightarrow \boxed{y=\frac{1}{3}}$

② $2x-6y+8z=-9$

$2x-2-8=-9$

$2x=1 \qquad \boxed{x=\frac{1}{2}}$

The solution is $\left(\frac{1}{2}, \frac{1}{3}, -1\right)$

⑦ a) Yes, they pass the vertical line test (no vertical line has more than one common point with the graph)

b) f: Domain: $x \in [-5,5]$

Range: $y \in [-2,2]$

c) g: Domain: $x \in [-4,5]$

Range: $y \in [-4,5]$

d) $(f+g)(-4) = f(-4)+g(-4)$

$= 0+3=3$

e) $(fg)(3) = f(3)\,g(3)$

$= 0 \cdot 3 = 0$

f) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1}{3}$

g) $f(x)=0$ when $x=-4$, $x=3$

$(-4,0)$ and $(3,0)$ are the x-intercepts of the graph

h) $g(0)=3$

$(0,3)$ is the y-intercept of the graph of g

8) a) $\left|\frac{1}{3} - 3x\right| = \frac{1}{5}$

$\frac{1}{3} - 3x = \frac{1}{5}$ OR $\frac{1}{3} - 3x = \frac{-1}{5}$

$3x = \frac{1}{3} - \frac{1}{5}$ $\qquad$ $3x = \frac{1}{3} + \frac{1}{5}$

$3x = \frac{2}{15} \;\Big|\cdot\frac{1}{3}$ $\qquad$ $3x = \frac{8}{15} \;\Big|\cdot\frac{1}{3}$

$x = \frac{2}{45}$ $\qquad$ $x = \frac{8}{45}$

$x \in \left\{\frac{2}{45}, \frac{8}{45}\right\}$

b) $|2x+3| = |x-1|$

$2x+3 = x-1$ OR $2x+3 = -(x-1)$

$x = -4$ $\qquad$ $2x+3 = -x+1$

$3x = -2$

$x = -\frac{2}{3}$

$x \in \left\{-4, \frac{-2}{3}\right\}$

c) $|15x+14| + 17 = 12$

$|15x+14| = -5$

not possible

$x \in \emptyset$

d) $\frac{1}{5}(t+2) = \frac{1}{2} + \frac{1}{6}(t-5)$

LCD = 30

$6(t+2) = 15 + 5(t-5)$

$6t + 12 = 15 + 5t - 25$

$6t - 5t = -22$

$t = -22$

e) $A = \frac{1}{2}h(a+b)$

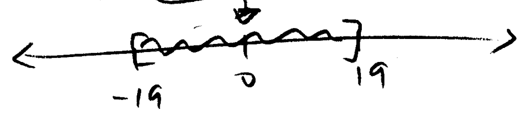
$2A = h(a+b)$

$a+b = \frac{2A}{h}$

$a = \frac{2A}{h} - b = \frac{2A - hb}{h}$

9) a) $|5x-1| + 2 \le 21$

$|5x-1| \le 19$

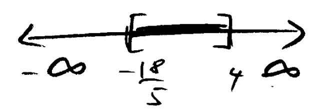


$-19 \le 5x-1 \le 19$
$\quad +1 \quad\; +1 \quad\; +1$

$-18 \le 5x \le 20 \quad \Big| \div 5$

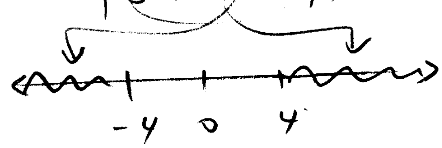
$-\frac{18}{5} \le x \le 4$

$x \in \left[-\frac{18}{5}, 4\right]$,



b) $4|5x+3| > 16$

$|5x+3| > 4$

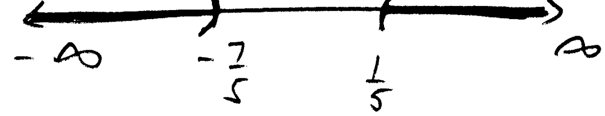


$5x+3 < -4$ OR $5x+3 > 4$

$5x < -7$ $\qquad$ $5x > 1$

$x < \frac{-7}{5}$ $\qquad$ $x > \frac{1}{5}$

$x \in \left(-\infty, \frac{-7}{5}\right) \cup \left(\frac{1}{5}, \infty\right)$



(10) $\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + 5y < 10 \\ 3x + 4y \leq 12 \end{cases}$ | Quadrant I

$\boxed{2x + 5y < 10}$

Boundary line: $2x + 5y = 10$

x	y
0	2
5	0

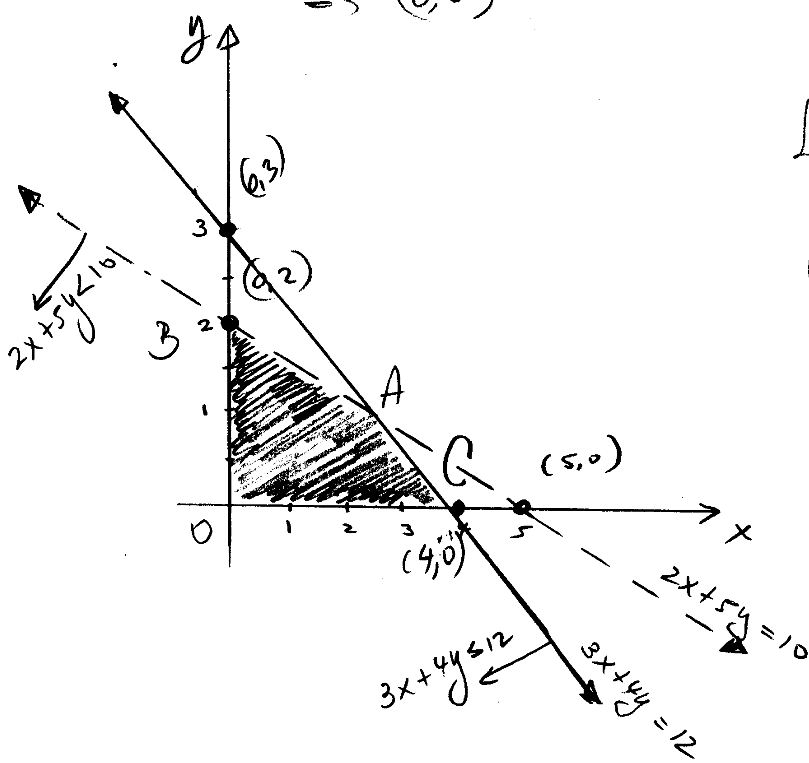
Test point: (0,0)
$2(0) + 5(0) < 10$ (true)
$\Rightarrow (0,0) =$ solution

$\boxed{3x + 4y \leq 12}$

Boundary line: $3x + 4y = 12$

x	y
0	3
4	0

Test point: (0,0)
$3(0) + 4(0) \leq 12$ (true)
$\Rightarrow (0,0) =$ solution.



Vertices: O (0,0)
B (0,2)
C (4,0)

A = intersection point

$\begin{cases} 2x + 5y = 10 \\ 3x + 4y = 12 \end{cases} \Big| \begin{matrix} 3 \\ -2 \end{matrix}$

$\begin{cases} 6x + 15y = 30 \\ -6x - 8y = -24 \end{cases}$

$7y = 6 \Rightarrow y = \frac{6}{7}$

$2x + 5 \cdot \frac{6}{7} = 10 \quad | \cdot 7$

$14x + 30 = 70$

$14x = 40 \Rightarrow x = \frac{40}{14} = \frac{20}{7}$

$A\left(\frac{20}{7}, \frac{6}{7}\right)$

(11A) let x = # nickels
y = # dimes
z = # quarters

$\begin{cases} x + y + z = 26 \\ 5x + 10y + 25z = 400 \\ z = (x + y) - 2 \end{cases} \Big| \div 5$

$\begin{cases} x + y + z = 26 \\ x + 2y + 5z = 80 \\ z = x + y - 2 \end{cases}$

$\begin{cases} x + y + (x + y - 2) = 26 \\ x + 2y + 5(x + y - 2) = 80 \end{cases}$

$\begin{cases} 2x + 2y = 28 \\ 6x + 7y = 90 \end{cases} \Big| -3$

$\begin{cases} -6x - 6y = -84 \\ 6x + 7y = 90 \end{cases}$

$y = 6$

$2x+2y=28$
$2x+12=28 \Rightarrow x=8$

$z=x+y-2$
$=8+6-2=12$

There will be 8 nickels
6 dimes
12 quarters.

$80 \le \frac{70+75+87+92+x}{5} < 90$
$400 \le 324+x < 450$
$-324 \quad -324 \quad -324$
$76 \le x < 126$

The student should get a score between [76,100]

(11B) W = weight (kg) - dependent
D = # days - independent variable

a)

D	W
2	3
31	9

$m = \frac{\Delta W}{\Delta D} = \frac{9-3}{31-2}$

$m = \frac{6}{29}$ kg/day

$\begin{cases} m = \frac{6}{29} \\ (2,3) \end{cases}$ $y-y_1 = m(x-x_1)$

$\boxed{W-3 = \frac{6}{29}(D-2)}$

b) $m = \frac{6}{29}$ kg/day
It shows the rate of growth of the pumpkin:
$\frac{6}{29}$ kg per day

(D) $W(x) = 0.07x + 4.1$
x = # years after 1984
W(x) = # women enrolled in colleges (in millions)
M(x) = # men enrolled (in millions)

a) $W(14) = 0.07(14)+4.1$
$W(14) = 5.08$ million women enrolled in 1998

b) $M(16) = 0.01(16)+3.9$
$M(16) = 4.06$ million men enrolled in 2000

c) $W(x)-M(x) =$
$= 0.07x+4.1-(0.01x+3.9)$
$= 0.06x+0.2$
So, $W(20)-M(20) = 0.06(20)+0.2$
$= 1.4$ million
in 2004, there were 1.4 million more women than men enrolled in colleges

(11C) I exam 70
II exam 75
III exam 87
IV exam 92
V exam X

We want $80 \le$ average < 90