MATH 71 Quiz # 4

(1) $x^2+y^2-2x-6y-10=0$

a) Sketch a graph.

b) Find the exact x- and y-intercepts (if any).

(2) $16x^2+9y^2=144$

a) Sketch a graph.

b) Find the following: foci, vertices, major axis, minor axis.

(3) $\begin{cases} x^2+y^2=9 \\ 4x^2+25y^2=100 \end{cases}$

a) First, solve the above system graphically. Highlight the solutions of the system on the graph. Label the axes and every point you are using.

b) Second, solve the system algebraically.

(4) $x^2-9y^2+9=0$

a) Sketch a graph.

b) Find the following: foci, vertices, and the equations of the asymptotes.

(5) Solve the following system algebraically:

$$\begin{cases} y^2-x=4 \\ x^2+y^2=4 \end{cases}$$

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SOLUTIONS

① $x^2+y^2-2x-6y-10=0$ (**)

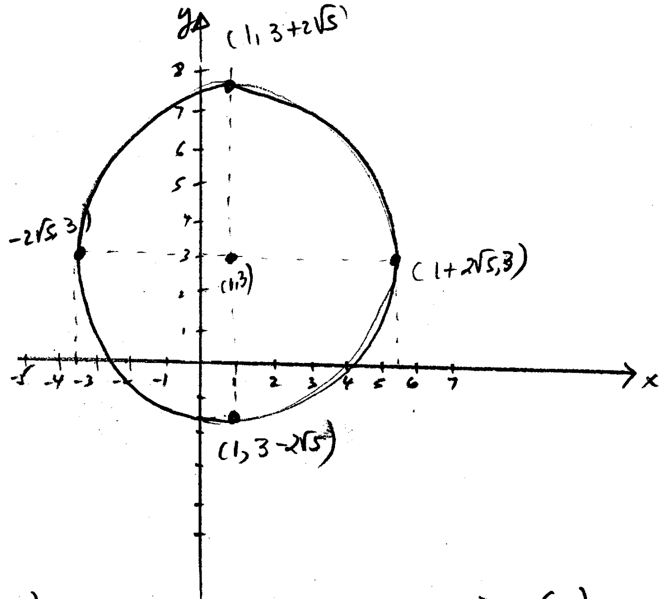
a) $x^2-2x+y^2-6y=10$

$x^2-2x+1+y^2-6y+9=10+1+9$

$(x-1)^2+(y-3)^2=20$ (*)

circle center $(1,3)$

radius $\sqrt{20}=2\sqrt{5}\approx 4.5$



b) x-n: let $y=0$ in (*)

$(x-1)^2+(-3)^2=20$

$(x-1)^2+9=20$

$(x-1)^2=11$

$x-1=\pm\sqrt{11}$, $x=1\pm\sqrt{11}$

$\boxed{\text{x-n: } (1-\sqrt{11},0) \text{ and } (1+\sqrt{11},0)}$

y-n: let $x=0$ in (**)

$y^2-6y=10$

$y^2-6y-10=0$

$y=\dfrac{6\pm\sqrt{36-4(-10)}}{2}=\dfrac{6\pm\sqrt{76}}{2}$

$=\dfrac{6\pm2\sqrt{19}}{2}=3\pm\sqrt{19}$

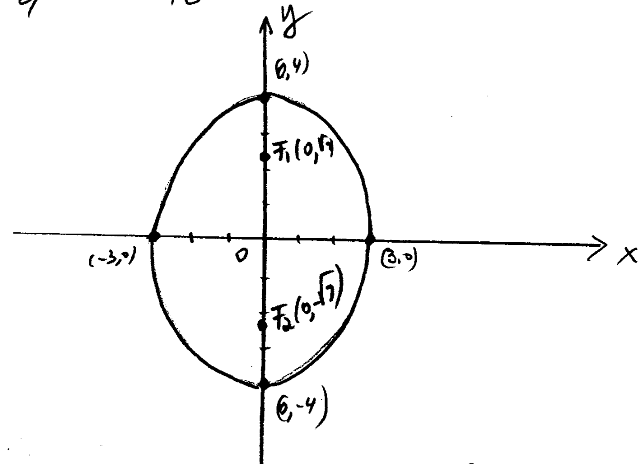
$\boxed{\text{y-n: } (0,3-\sqrt{19}) \text{ and } (0,3+\sqrt{19})}$

② $16x^2+9y^2=144$ $\big|\div 144$

ellipse

a) $\dfrac{16x^2}{144}+\dfrac{9y^2}{144}=1$

$\dfrac{x^2}{9}+\dfrac{y^2}{16}=1$



b) Major axis = vertical

minor axis = horizontal

Vertices $(0,\pm4)$

Foci $(0,\pm c)$ $\quad c^2=16-9=7$, $c=\sqrt{7}$

$F_1(0,\sqrt{7})$

$F_2(0,-\sqrt{7})$

Algebraically

③ ⓑ $\begin{cases} x^2+y^2=9 \\ 4x^2+25y^2=100 \end{cases}$ $\Big|\cdot(-4)$

$\begin{cases} -4x^2-4y^2=-36 \\ 4x^2+25y^2=100 \end{cases}$

$21y^2=64 \Rightarrow y^2=\dfrac{64}{21}$,

$y=\pm\dfrac{8}{\sqrt{21}}=\pm\dfrac{8\sqrt{21}}{21}$

$x^2+\dfrac{64}{21}=9$

$x^2=9-\dfrac{64}{21}=\dfrac{189-64}{21}=\dfrac{125}{21}=$

$x=\pm\sqrt{\dfrac{125}{21}}=\pm\dfrac{5\sqrt{5}}{\sqrt{21}}=\pm\dfrac{5\sqrt{105}}{21}$

Therefore, the solutions are

$A\left(\frac{5\sqrt{105}}{21}, \frac{8\sqrt{21}}{21}\right)$

$B\left(\frac{5\sqrt{105}}{21}, \frac{-8\sqrt{21}}{21}\right)$

$C\left(\frac{-5\sqrt{105}}{21}, \frac{8\sqrt{21}}{21}\right)$

$D\left(\frac{-5\sqrt{105}}{21}, \frac{-8\sqrt{21}}{21}\right)$

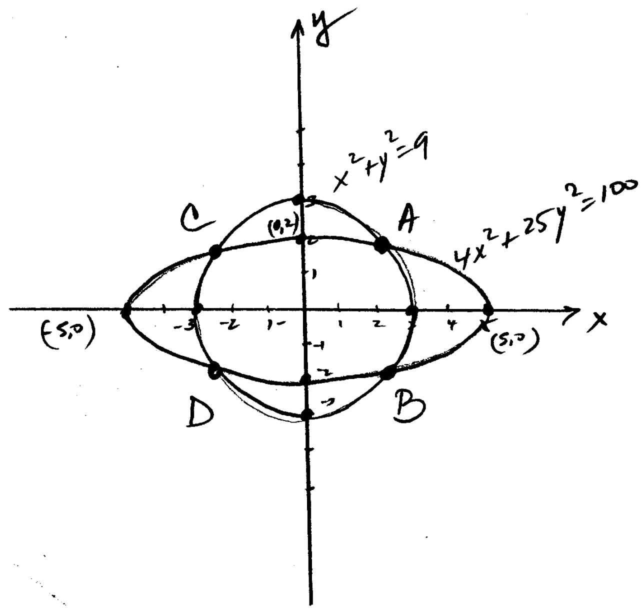
a) Graphically

$\boxed{x^2+y^2=9}$ circle center $(0,0)$ radius 3

$\boxed{4x^2+25y^2=100}$ ellipse

$$\frac{4x^2}{100}+\frac{25y^2}{100}=1$$

$$\frac{x^2}{25}+\frac{y^2}{4}=1$$



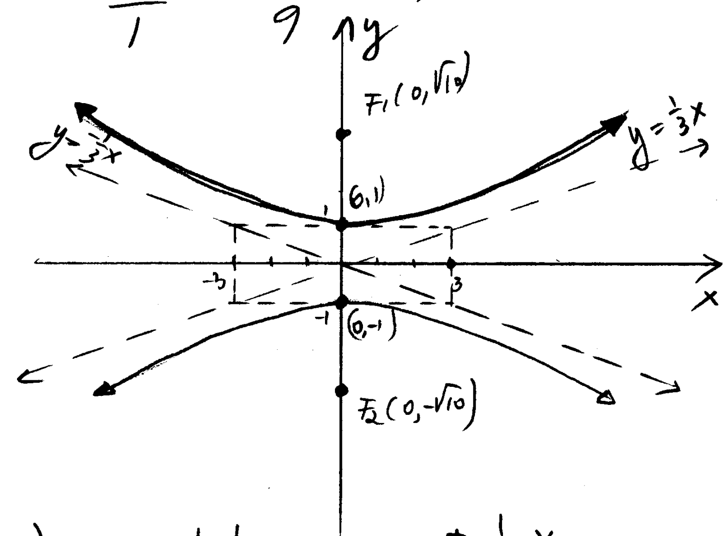
④ $x^2-9y^2+9=0$

hyperbola

a) $x^2-9y^2=-9 \quad | \div(-9)$

$$\frac{x^2}{-9}+\frac{9y^2}{9}=1$$

$$\frac{y^2}{1}-\frac{x^2}{9}=1$$



b) Asymptotes: $y=\pm\frac{1}{3}x$

Foci $F(0,\pm c)$

$c^2=1+9=10, \quad c=\sqrt{10}$

$F(0,\pm\sqrt{10})$

Vertices $(0,\pm 1)$

(5) $\begin{cases} y^2 - x = 4 \\ x^2 + y^2 = 4 \end{cases}$

$$\begin{cases} -x + y^2 = 4 \\ x^2 + y^2 = 4 \end{cases}$$

$\ominus$ $x^2 + x = 0$

$x(x+1) = 0$

$x = 0$ OR $x = -1$

if $x = 0$, then $y^2 - x = 4$

$y^2 = 4$

$y = \pm 2$

if $x = -1$, then $y^2 - x = 4$

$y^2 + 1 = 4$

$y^2 = 3$

$y = \pm\sqrt{3}$

The solutions are:

$(0, 2)$

$(0, -2)$

$(-1, \sqrt{3})$

$(-1, -\sqrt{3})$