Math 71 – Fall 2007 Name:

QUIZ #3 @ 50 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. Solve the following equations in the set of complex numbers ($\mathbb{C}$):

a) $x(3x+2)=(x+2)^2$

b) $(w+1)(2w-3)=3$

c) $4a=16a^3$

d) $3x^2\left(2x+\frac{3}{4}\right)(1-5x)(3x^2+5)=0$

2. Solve by extracting roots in the set of complex numbers ($\mathbb{C}$): $2(5x-12)^2+48=0$

3. Solve by completing the square in the set of complex numbers ($\mathbb{C}$): $8x^2+5x-1=0$

4. Solve by the quadratic formula in the set of complex numbers ($\mathbb{C}$): $-x^2+\frac{5}{2}x-\frac{1}{2}=0$

5. Solve the following equation by making an appropriate substitution: $x^4+2x^2-8=0$

6. $f(x)=x^2-2x-15$

a) Graph the function by finding the vertex, and y-and x-intercepts.

b) State the domain and range.

c) Using the graph, solve the following inequality: $x^2-2x-15>0$

7. Solve the following inequalities:

a) $x^2+6x+8\le 0$

b) $\frac{3x+5}{6-2x}\ge 0$

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Quiz # 3 - Solutions

1. (a) $x(3x+2) = (x+2)^2$

$3x^2 + 2x = x^2 + 4x + 4$

$3x^2 + 2x - x^2 - 4x - 4 = 0$

$2x^2 - 2x - 4 = 0 \quad / \div 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ OR $x = -1$

$\boxed{x \in \{2, -1\}}$

(b) $(w+1)(2w-3) = 3$

$2w^2 - w - 3 = 3$

$2w^2 - w - 6 = 0$

$w = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \begin{cases} a = 2 \\ b = -1 \\ c = -6 \end{cases}$

$w = \dfrac{1 \pm \sqrt{(-1)^2 - 4(2)(-6)}}{2(2)} = \dfrac{1 \pm \sqrt{49}}{4}$

$w = \dfrac{1 \pm 7}{4}$, $w = \dfrac{8}{4} = 2$, $w = \dfrac{-6}{4} = \dfrac{-3}{2}$

$\boxed{w \in \{2, -\tfrac{3}{2}\}}$

(c) $4a = 16a^3$

$16a^3 - 4a = 0$

$4a(4a^2 - 1) = 0$

$4a(2a-1)(2a+1) = 0$

$a = 0$ OR

$2a - 1 = 0 \Rightarrow a = \frac{1}{2}$ OR

$2a + 1 = 0 \Rightarrow a = -\frac{1}{2}$

$\boxed{a \in \{0, \tfrac{1}{2}, \tfrac{-1}{2}\}}$

(d) $3x^2(2x + \frac{3}{4})(1-5x)(3x^2+5) = 0$

$x = 0$ OR

$2x + \frac{3}{4} = 0 \Rightarrow 2x = -\frac{3}{4} \quad / \cdot \frac{1}{2}$

$x = \frac{-3}{8}$

OR

$1 - 5x = 0 \Rightarrow 5x = 1$

$x = \frac{1}{5}$

OR

$3x^2 + 5 = 0 \Rightarrow 3x^2 = -5$

$x^2 = \frac{-5}{3}$

$\sqrt{x^2} = \sqrt{\frac{-5}{3}}$

$x = \pm \frac{\sqrt{5}\, i}{\sqrt{3}}$

$x = \pm \frac{\sqrt{15}\, i}{3}$

$\boxed{x \in \{0, \tfrac{-3}{8}, \tfrac{1}{5}, \pm \tfrac{\sqrt{15}\, i}{3}\}}$

2. $2(5x-12)^2 + 48 = 0$

$2(5x-12)^2 = -48 \quad / \div 2$

$(5x-12)^2 = -24 \quad / \sqrt{\ }$

$\sqrt{(5x-12)^2} = \sqrt{-24}$

$5x - 12 = \pm 2\sqrt{6}\, i$

$5x = 12 \pm 2\sqrt{6}\, i$

$\boxed{x = \dfrac{12 \pm 2\sqrt{6}\, i}{5}}$

(3) $8x^2+5x-1=0$

1st isolate the constant

$$8x^2+5x=1 \qquad \Big| \div 8$$

2nd leading coefficient $=1$

$$x^2+\frac{5}{8}x=\frac{1}{8} \qquad \Big| + \frac{25}{256}$$

3rd find missing term

$$\left(\frac{1}{2}\,\text{coef.}\,x\right)^2 = \left(\frac{1}{2}\cdot\frac{5}{8}\right)^2 = \frac{25}{256}$$

$$x^2+\frac{5}{8}x+\frac{25}{256} = \frac{\overset{32}{1}}{8}+\frac{25}{256}$$

$$\left(x+\frac{5}{16}\right)^2 = \frac{57}{256} \qquad \Big| \sqrt{\ }$$

$$\sqrt{\left(x+\frac{5}{16}\right)^2} = \sqrt{\frac{57}{256}}$$

$$x+\frac{5}{16} = \pm\frac{\sqrt{57}}{16}$$

$$\boxed{x = -\frac{5}{16} \pm \frac{\sqrt{57}}{16}}$$

(4) $-x^2+\frac{5}{2}x-\frac{1}{2}=0 \qquad \Big| (-1)$

$$x^2-\frac{5}{2}x+\frac{1}{2}=0 \qquad \Big| \cdot 2$$

$$2x^2-5x+1=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad \begin{cases} a=2 \\ b=-5 \\ c=1 \end{cases}$$

$$x=\frac{5\pm\sqrt{25-8}}{4} = \frac{5\pm\sqrt{17}}{4}$$

$$\boxed{x=\frac{5\pm\sqrt{17}}{4}}$$

(5) $x^4+2x^2-8=0$

let $x^2=t$

then $x^4=t^2$

The equation becomes:

$$t^2+2t-8=0$$

$$(t+4)(t-2)=0$$

$t=-4$ OR $t=2$

$x^2=-4$ | $x^2=2$

$\sqrt{x^2}=\sqrt{-4}$ | $\sqrt{x^2}=\sqrt{2}$

$x=\pm 2i$ | $x=\pm\sqrt{2}$

$$\boxed{x\in\{\pm 2i, \pm\sqrt{2}\}}$$

(6) $f(x)=x^2-2x-15$

(a) vertex $V(x_v, y_v)$

$$x_v=\frac{-b}{2a}=\frac{-(-2)}{2}=1 \qquad \boxed{V(1,-16)}$$

$$y_v=1-2-15=-16$$

x-∩: $x^2-2x-15=0$

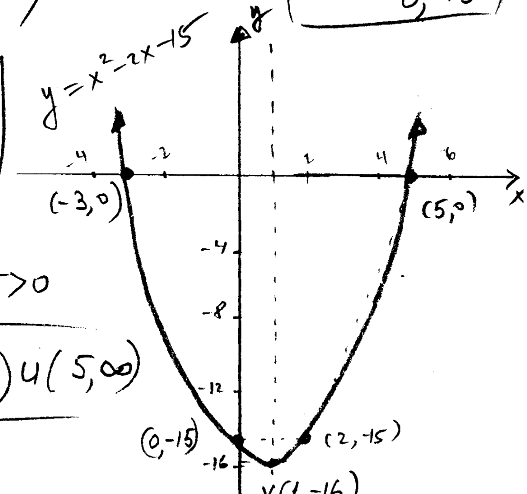
$$(x-5)(x+3)=0$$

$x=5$ OR $x=-3$

$\boxed{\text{x-∩: } (5,0), (-3,0)}$

y-∩: $x=0, y=-15$ $\boxed{\text{y-∩: } 0,-15}$

(b) $\boxed{\begin{array}{l} x\in\mathbb{R} \\ y\geq -16 \end{array}}$



(c) $x^2-2x-15>0$

iff

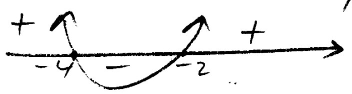
$$\boxed{x\in(-\infty,-3)\cup(5,\infty)}$$

(7) (a) $x^2+6x+8 \le 0$

$(x+4)(x+2) \le 0$

let $y=(x+4)(x+2)$

parabola opens up



x-int: $x=-4$, $x=-2$

$x^2+6x+8 \le 0$ iff $\boxed{x \in [-4,-2]}$

(b) $\dfrac{3x+5}{6-2x} \ge 0$

Study the sign of the numerator and denominator.

x	$-\infty$		$-\frac{5}{3}$		3		∞
$3x+5$		$-\ -\ -$	0	$+\ +\ +$	$+$	$+$	
$6-2x$		$+\ +\ +$	$+$	$+$	0	$-\ -$	
$\frac{3x+5}{6-2x}$		$-$	0	$+$	$\vert$	$-$	

$3x+5=0$ iff $x=-\frac{5}{3}$

$6-2x=0$ iff $x=3$

$\dfrac{3x+5}{6-2x} \ge 0$ iff $\boxed{x \in \left[-\frac{5}{3}, 3\right)}$