Math 71 – Fall 2007 Name:

QUIZ #2 @ 50 points

Write in a neat and organized fashion. Write your complete solutions on SEPARATE PAPER. You should use a pencil. For an exercise to be complete there needs to be a detailed solution to the problem. Do not just write down an answer. No proof, no credit given! Clearly label each exercise.

1. If $f(x)=x^2-2x+3$, find (and simplify): $f(a+h)-f(a)$.

2. Factor each polynomial completely:

a) $15m^3-25m^2+10m$

b) $x^2-12x+20$

c) a^3+8b^3

d) $2x^{n+2}-5x^{n+1}+3x^n$

3. Solve each equation by factoring:

a) $2x^2-8x=90$

b) $(x-3)(x+8)=-30$

c) $3x^4-48x^2=0$

d) $2(3x-2)\left(2x+\frac{1}{5}\right)(x^2-7x)=0$

e) $\frac{1}{2}x^2+\frac{3}{5}x=0$

4. A baseball is thrown straight up from a rooftop 448 feet high. The function

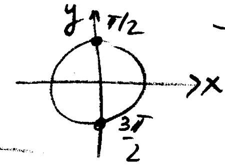
$$s(t)=-16t^2+48t+448$$

describes the ball's height above the ground, s(t), in feet, t seconds after it is thrown.

a) What is $s(0)$ and what is the meaning in the context of the problem?

b) How long will it take for the ball to hit the ground?

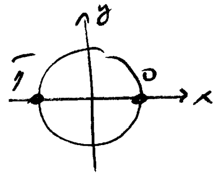
(4) (a) $\cos x = 0$



$$\left\{ \begin{array}{l} x = \frac{\pi}{2} + 2k\pi \\ \text{OR} \\ x = \frac{3\pi}{2} + 2k\pi \end{array} \right. , k \in \mathbb{Z}$$

(OR $x = \frac{\pi}{2} + k\pi$)

(b) $\sin x = 0$



$$x = k\pi , k \in \mathbb{Z}$$

(c) $\tan x = 0$ iff $\frac{\sin x}{\cos x} = 0$ iff $\sin x = 0$ iff $x = k\pi$, $k \in \mathbb{Z}$

(d) $\cot x = 0$ iff $\frac{\cos x}{\sin x} = 0$ iff $\cos x = 0$

iff $x = \frac{\pi}{2} + 2k\pi$ OR $x = \frac{3\pi}{2} + 2k\pi$, $k \in \mathbb{Z}$

(5) Recall that

$\sin^{-1} : [-1,1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

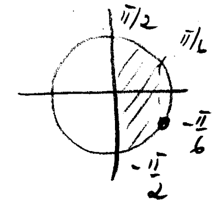
$\cos^{-1} : [-1,1] \rightarrow [0, \pi]$

$\tan^{-1} : \mathbb{R} \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

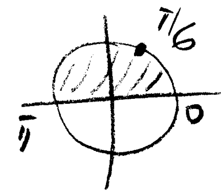
	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{2}/2$
cos	$\sqrt{3}/2$	$\frac{1}{2}$	$\sqrt{2}/2$
tan	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	1

(a) $\sin^{-1}(-\frac{1}{2}) = \boxed{-\frac{\pi}{6}}$

b/c $\sin(-\frac{\pi}{6}) = -\frac{1}{2}$ and $-\frac{\pi}{6} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

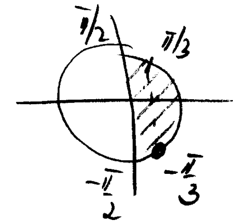


(b) $\cos^{-1}(\frac{\sqrt{3}}{2}) = \boxed{\frac{\pi}{6}}$



b/c $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{6} \in [0, \pi]$

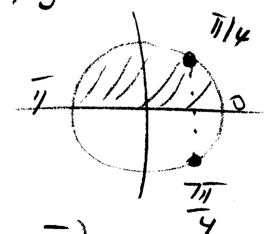
(c) $\tan^{-1}(-\sqrt{3}) = \boxed{-\frac{\pi}{3}}$



b/c $\tan(-\frac{\pi}{3}) = -\sqrt{3}$ and $-\frac{\pi}{3} \in (-\frac{\pi}{2}, \frac{\pi}{2})$

(d) $\cos^{-1}(\cos\frac{7\pi}{4}) \neq \frac{7\pi}{4}$ b/c $\frac{7\pi}{4} \notin [0, \pi]$

But, $\cos(\frac{7\pi}{4}) = \cos\frac{\pi}{4}$



Therefore,

$$\cos^{-1}(\cos\frac{7\pi}{4}) = \cos^{-1}(\cos\frac{\pi}{4}) = \boxed{\frac{\pi}{4}}$$

(e) $\sin(\sin^{-1}\frac{\sqrt{2}}{2}) = \boxed{\frac{\sqrt{2}}{2}}$

(f) $\sin(\cos^{-1}\frac{1}{2})$

Method I $\sin(\cos^{-1}\frac{1}{2}) =$

$= \sin(\frac{\pi}{3})$

$= \boxed{\frac{\sqrt{3}}{2}}$

TEST #2 @ 150 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given

1. Graph $f(x) = \sin x$ and $f^{-1}(x) = \sin^{-1}(x)$ on the same coordinate system, showing the relation between the two graphs (symmetry about the line $y = x$). Answer the following questions:

   a) What is the domain and range of $f(x) = \sin x$?

   b) What is the domain and range of $f^{-1}(x) = \sin^{-1}(x)$?

2. a) Graph $y = 1 + 3\sin(2x)$ between 0 and 2π. Identify the amplitude and period and label the axes accurately.

   b) Find the x-intercepts of the graph within the period graphed; that is, solve the equation $y = 0$ in $[0, 2\pi]$. Give exact answers as well as approximations.

3. Graph $y = \frac{3}{4}\cos\left(2x + \frac{2\pi}{3}\right)$ over one period. Identify the amplitude , period, and phase shift and label the axes accurately.

4. Find all real numbers x that satisfy each equation. Justify your answers.

   a) $\cos x = 0$

   b) $\sin x = 0$

   c) $\tan x = 0$

   d) $\cot x = 0$

5. Evaluate the following. Give exact answers whenever possible.

   a) $\sin^{-1}\left(-\frac{1}{2}\right)$

   b) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

   c) $\tan^{-1}\left(-\sqrt{3}\right)$

   d) $\cos^{-1}\left(\cos\frac{7\pi}{4}\right)$

   e) $\sin\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$

   f) $\sin\left(\cos^{-1}\frac{1}{2}\right)$