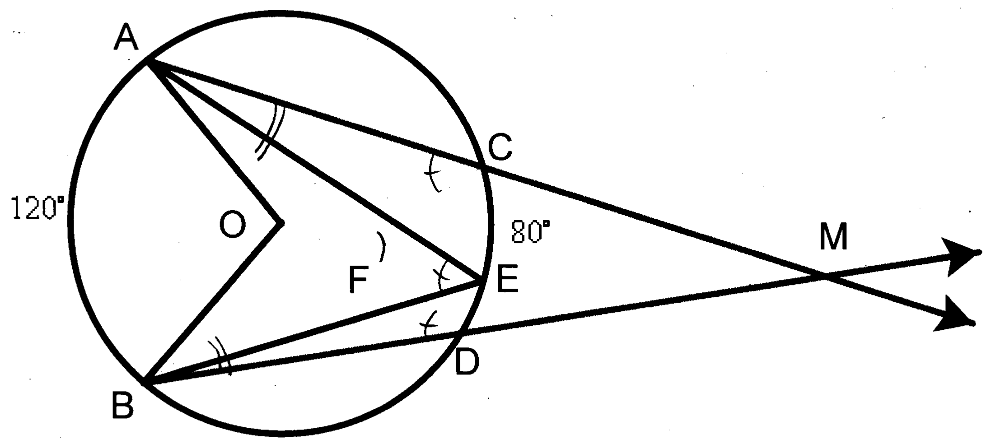
Math 61 Fall 2007 Name: Solutions

TEST #3 @ 130 points

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.

1)



Given arcs: $m\overset{\frown}{AB} = 120^\circ$ and $m\overset{\frown}{CD} = 80^\circ$

Find the following (use correct units):

a) $m\angle AOB = m\overset{\frown}{AB} = 120^\circ$ (central ∡)

b) $m\angle CFD = \frac{1}{2}\left(m\overset{\frown}{CD} + m\overset{\frown}{AB}\right) = \frac{1}{2}(200^\circ) = 100^\circ$

Name another angle that is congruent with $\angle CFD$: $\angle AFB$

c) $m\angle CBD = \frac{1}{2}\, m\overset{\frown}{CD} = 40^\circ$ (inscribed ∡)

Name another angle that is congruent with $\angle CBD$: $\angle CAD$

d) $m\angle AEB = \frac{1}{2}\, m\overset{\frown}{AB} = 60^\circ$

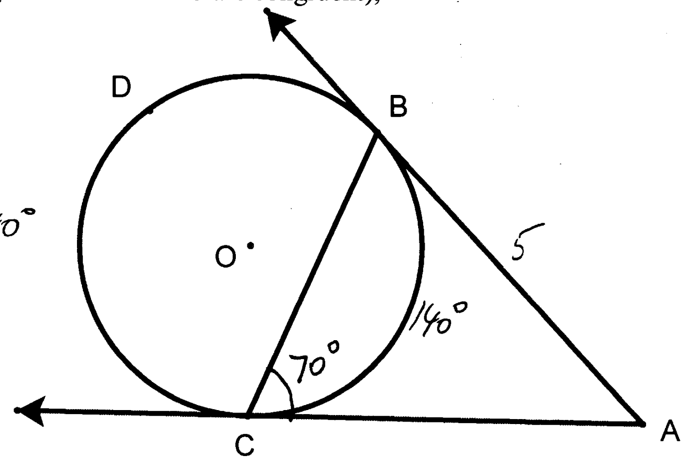
Name two other angles that are congruent with $\angle AEB$: $\angle ACB$ and $\angle ADB$

e) $m\angle AMB = \frac{1}{2}\left(m\overset{\frown}{AB} - m\overset{\frown}{CD}\right)$

$= \frac{1}{2}(40^\circ)$

$= 20^\circ$

2) Given: $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents to $\odot O$ (that is, segments AB and AC are congruent), $m\angle ACB = 70^\circ$, $AB = 5cm$.



Find:

a) $m\overset{\frown}{BC}$

$m\angle ACB = \frac{1}{2} m\overset{\frown}{BC} \Rightarrow m\overset{\frown}{BC} = 2m\angle ACB$
$= 2(70^\circ) = 140^\circ$

b) $m\overset{\frown}{BDC}$ $= 360^\circ - m\overset{\frown}{BC}$
$= 360^\circ - 140^\circ$
$= 220^\circ$

c) $m\angle ABC$

$\triangle ABC$ isosceles $\Rightarrow m\angle ABC = 70^\circ$
OR $m\angle ABC = \frac{1}{2} m\overset{\frown}{BC} = 70^\circ$

d) $m\angle A$

$= 180^\circ - 70^\circ - 70^\circ = 40^\circ$ (in $\triangle ABC$)

e) AC

$\triangle ABC$ isosceles $\Rightarrow AC = AB = 5$ cm

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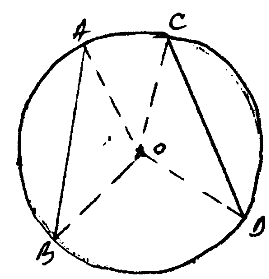
3) Prove the following theorem using a formal proof. Make a drawing and state the hypothesis (given) and conclusion(to prove) using math notation pertinent to your drawing – that is, do not state the hypothesis and conclusion in words!



Given: $\odot O$, $\overline{AB}, \overline{CD}$ = chords
$\overline{AB} \cong \overline{CD}$

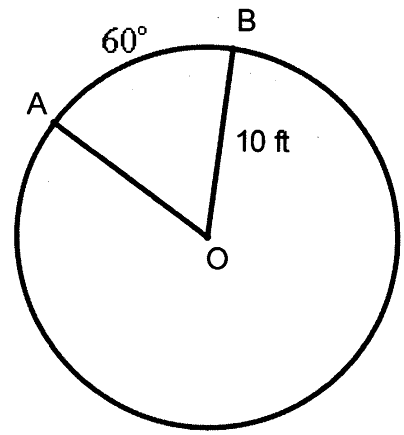
Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

Proof

Statements	Reasons
1. $\overline{AB}, \overline{CD}$ chords, $\overline{AB} \cong \overline{CD}$	1. given
2. $\overline{AO}, \overline{BO}, \overline{CO}, \overline{DO}$ = radii	2. By construction
3 $\overline{AO} \cong \overline{BO} \cong \overline{CO} \cong \overline{DO}$	3. All radii are $\cong$
4 $\triangle AOB \cong \triangle COD$	4. SSS
5 $\angle AOB \cong \angle COD$	5 CPCTC
6. $m\angle AOB = m\angle COD$	6 def. $\cong \angle$'s
7. $m\angle AOB = m\overset{\frown}{AB}$, $m\angle COD = m\overset{\frown}{CD}$	7. def measure of central $\angle$
8. $m\overset{\frown}{AB} = m\overset{\frown}{CD}$	8. transitivity/substitution
(6,7) 9 $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	9. def $\cong$ arcs

4)

a) Find the circumference of the given circle.
Give exact answer using correct units.



Circumference $= 2\pi r = 2\pi \cdot 10\ ft = 20\pi\ ft$

b) Find the area of the given circle.
Give exact answer using correct units.

Area $= \pi r^2 = \pi (10 ft)^2 = 100\pi\ ft^2$

c) Find the length of the arc AB. Give exact answer using correct units.

$$\frac{\ell(\overset{\frown}{AB})}{60^\circ} = \frac{\cancel{2}\pi r}{\cancel{360^\circ}_{180^\circ}} \Rightarrow \ell(\overset{\frown}{AB}) = \frac{\pi \cdot 10 ft \cdot \cancel{60^\circ}}{\cancel{180^\circ}\ 3}$$

$$\ell(\overset{\frown}{AB}) = \frac{10\pi}{3}\ ft$$

d) Find the area of the sector AOB. Give exact answer using correct units.

$$\frac{A(AOB)}{60^\circ} = \frac{\pi r^2}{360^\circ} \Rightarrow A(AOB) = \frac{100\pi\ ft^2 \cdot \cancel{60^\circ}}{\cancel{360^\circ}\ 6} = \frac{100\pi}{6}\ ft^2 = \frac{50}{3}\pi\ ft^2$$

$$A(AOB) = \frac{50}{3}\pi\ ft^2$$

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5) Simplify the following trigonometric expressions:

a) $\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x(1+\sin x) + \cos^2 x}{\cos x(1+\sin x)}$

LCD $= \cos x(1+\sin x)$

$$= \frac{\sin x + \underbrace{\sin^2 x + \cos^2 x}_{1}}{\cos x(1+\sin x)}$$

$$= \frac{\cancel{\sin x + 1}}{\cos x\cancel{(1+\sin x)}}$$

$$= \frac{1}{\cos x}$$

b) $\cos^3\theta + \sin^2\theta\cos\theta =$

$$= \cos\theta(\underbrace{\cos^2\theta + \sin^2\theta}_{1})$$

$$= \cos\theta$$

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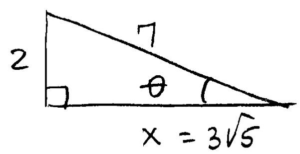
6) Sketch a right triangle that has one acute angle θ, and find the other five trigonometric ratios of θ knowing that

$$\sin\theta = \frac{2}{7}$$



$x^2 = 7^2 - 2^2$

$x^2 = 45$

$x = \sqrt{45} = 3\sqrt{5}$

$\sin\theta = \frac{2}{7}$ (given)

$\cos\theta = \frac{3\sqrt{5}}{7}$

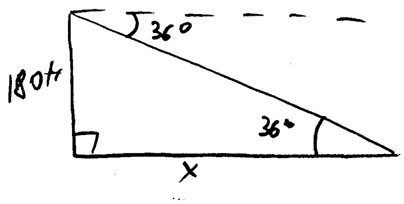
$\tan\theta = \frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{15}$

$\cot\theta = \frac{3\sqrt{5}}{2}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{7}{2}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{7}{3\sqrt{5}} = \frac{7\sqrt{5}}{15}$

7) From the top of a 180-ft lighthouse, the angle of depression to a ship in the ocean is 36°. How far is the ship from the base of the lighthouse?



$$\tan 36^\circ = \frac{180\text{ ft}}{x}$$

$$x = \frac{180\text{ ft}}{\tan 36^\circ} \approx 248\text{ ft}$$

8) Prove the following identities:

a) $\sin a \cot a = \cos a$

$$\sin a \cot a = \cancel{\sin a} \cdot \frac{\cos a}{\cancel{\sin a}}$$

$$= \cos a$$

b) $\dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\csc x} = 1$

$$\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = \frac{\cos x}{\frac{1}{\cos x}} + \frac{\sin x}{\frac{1}{\sin x}}$$

$$= \cos^2 x + \sin^2 x$$

$$= 1$$

c) $\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

$$(\sec\theta - \tan\theta)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos x}\right)^2$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^{\cancel{2}}}{\cancel{(1-\sin\theta)}(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta}$$

EXTRA CREDIT

Choose ONE or TWO of the following problems:

(1) @ 10 points

A water tower 30 m tall is at the top of a hill. From a distance of 120 m down the hill it is observed that the angle formed between the top and the base of the tower is 8°. Find the angle of inclination of the hill.

(2) @ 5 points

A circle is inscribed in the triangle ABC as shown.
$AB = 14, BC = 16, AC = 12$.
Find AM, PC, BN.