TEST #2 @ 130 points

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.
Write all solutions on separate paper. Label each problem clearly.

1. Answer the following questions. Do not prove.

   a) When is a quadrilateral a parallelogram?
   To receive full credit, give a case involving the sides, one involving the angles, and one involving the diagonals of the quadrilateral.

   b) How are the legs and the base angles of an isosceles trapezoid?
   Make a drawing and state the answer using math notation pertinent to your drawing .

   c) Draw a right triangle and write the Pythagorean theorem. Use math notation pertinent to your drawing.

   d) What do you know about the segment that joins the midpoints of two sides of a triangle?
   Make a drawing and state the answer using math notation pertinent to your drawing .

   e) What do you know about the segment that joins the midpoints of the legs of a trapezoid?
   Make a drawing and state the answer using math notation pertinent to your drawing .

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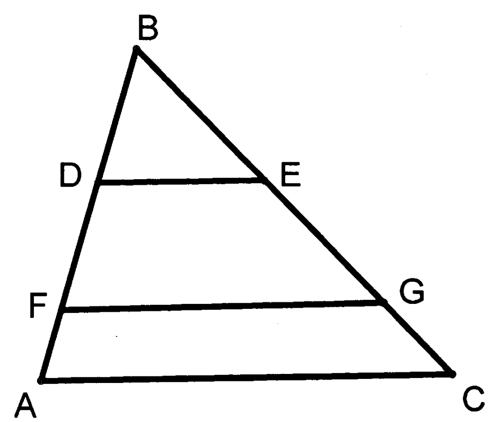
2. Given: $\triangle ABC$ with $\overline{DE} \parallel \overline{FG} \parallel \overline{AC}$ where $BE = 24, BD = 18, EG = 16, FA = 15$.
Find: DF and GC. Justify your answers.



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3. a) Draw a right triangle with right angle C. Then draw the altitude $\overline{CD}$ and the median $\overline{CE}$. Let D and E on $\overline{AB}$.

   b) If $BD = 8$ cm and $AC = 7$ cm, find AD. Justify your answers.

   c) If $CE = 7$ cm and $AD = 2$ cm, find AC. Justify your answers.

4. In a right triangle *FDG* with right angle *D*, the bisector of angle *D* intersects the hypotenuse at *E*. The acute angles of the triangle are congruent. Prove that *E* is the midpoint of the hypotenuse (formal proof).

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5. Given triangle ABC with $\overline{DE}$ parallel with $\overline{BC}$, *D* on side $\overline{AB}$ and *E* on side $\overline{AC}$. Prove (formal proof) that

$$\frac{DP}{BF} = \frac{PE}{FC}.$$

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6. Prove the following using an <u>indirect proof.</u> Make sure you make a drawing to illustrate the problem; write the hypothesis and conclusion using math notation pertinent to your drawing.

If two lines are parallel to a third line, then they are parallel to each other.

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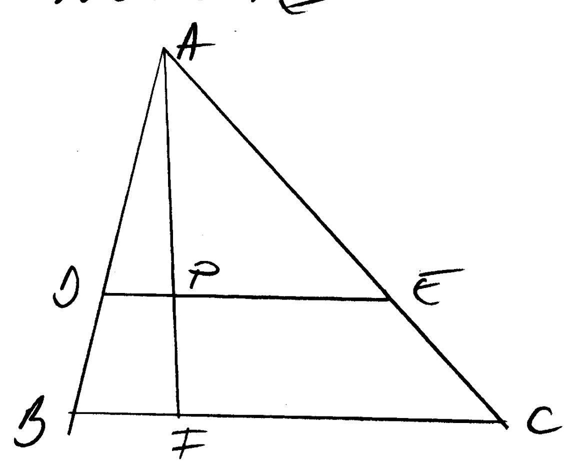
7. Prove the following theorem using a <u>formal proof.</u>
Make a drawing to illustrate the theorem; write the hypothesis and conclusion using math notation pertinent to your drawing.

The diagonals of a parallelogram bisect each other.

<u>Extra credit</u>

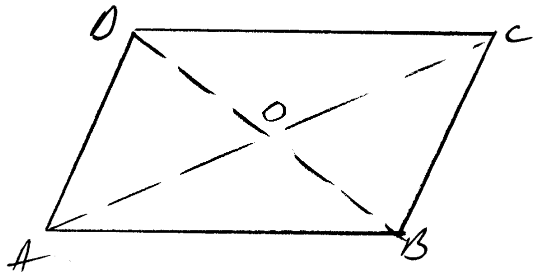
Prove that the area of an isosceles right triangle is one-fourth the square of the length of the hypotenuse.

FIG. PROBLEM 5



M61 TEST #2 - SOLUTIONS

(1)



(a) ABCD - quadrilateral

ABCD is a parallelogram if:

1°) the opposite sides are parallel

$$\begin{cases} \overline{AB} \parallel \overline{CD} \\ \overline{BC} \parallel \overline{AD} \end{cases}$$

OR

2°) a pair of opposite sides are parallel and congruent

$$\begin{cases} \overline{AB} \parallel \overline{CD} \\ \overline{AB} \cong \overline{CD} \end{cases}$$

OR

3°) the opposite angles are congr.

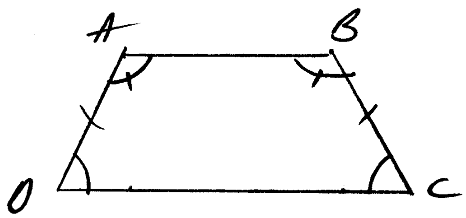
$$\begin{cases} \angle A \cong \angle C \\ \angle B \cong \angle D \end{cases}$$

OR

4°) diagonals bisect each other

$$\begin{cases} \overline{AC} \text{ bisects } \overline{BD} \\ \overline{BD} \text{ bisects } \overline{AC} \end{cases}$$

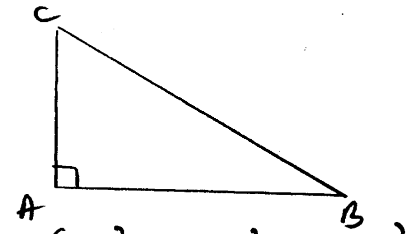
(b)



if ABCD isosc. trapezoid, then

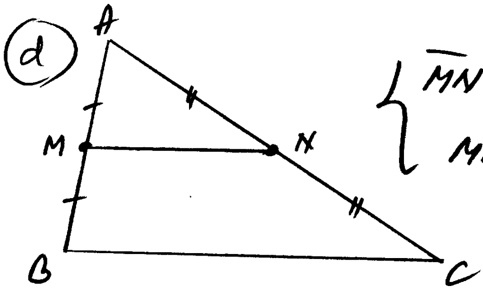
$$\begin{cases} \overline{AD} \cong \overline{BC} & \text{(legs)} \\ \angle A \cong \angle B & \text{(base angles)} \\ \angle D \cong \angle C & \end{cases}$$

(c)



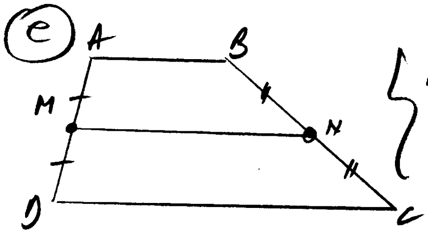
$$(AB)^2 + (AC)^2 = (BC)^2$$

(d)



$$\begin{cases} \overline{MN} \parallel \overline{BC} \\ MN = \frac{1}{2} BC \end{cases}$$

(e)



$$\begin{cases} \overline{MN} \parallel \overline{AB} \parallel \overline{DC} \\ MN = \frac{1}{2}(AB + DC) \end{cases}$$

(2) Solution

$\triangle BFG$: $\overline{DE} \parallel \overline{FG}$ $\Rightarrow$

$$\frac{BD}{DF} = \frac{BE}{EG}$$

$$\frac{18}{DF} = \frac{24}{16} \Rightarrow DF = \frac{16 \cdot 18}{24}$$

$$DF = 12$$

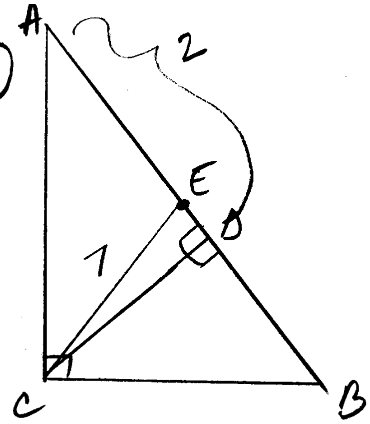
$\triangle BAC$: $\overline{FG} \parallel \overline{AC}$ $\Rightarrow$

$$\frac{BF}{FA} = \frac{BG}{GC}$$

$$\frac{18+12}{15} = \frac{24+16}{GC} \Rightarrow GC = \frac{15 \cdot 40}{30}$$

$$GC = 20$$

③ ⓐ



$\triangle ABC$ right at C
$\overline{CD}$ - altitude ($\overline{CD} \perp \overline{AB}$)
$\overline{CE}$ - median (E = midpt. $\overline{AB}$)

ⓒ $CE = 7$ cm
$AD = 2$ cm
$AC = ?$

Solution

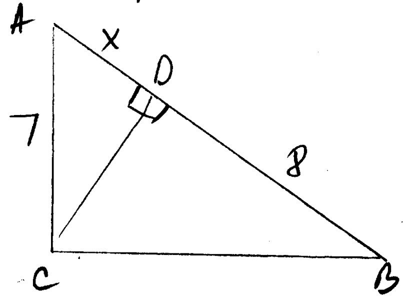
$\overline{CE}$ = median $\Rightarrow CE = \frac{1}{2} AB$

$7 = \frac{1}{2} AB$

$AB = 14$

$AC^2 = AD \cdot AB$
$AC^2 = 2(14)$
$AC^2 = 28$
$AC = \sqrt{28} = 2\sqrt{7}$
$AC = 2\sqrt{7}$ cm

ⓑ $BD = 8$ cm
$AC = 7$ cm
$AD = ?$



Solution

Let $AD = x$

$AC^2 = AD \cdot AB$
(leg = geom. mean of hypotenuse and adjacent segm. on hyp)

$7^2 = x(x+8)$
$49 = x^2 + 8x$
$x^2 + 8x - 49 = 0$

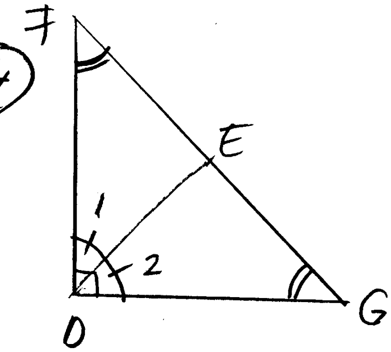
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-8 \pm \sqrt{64-4(49)}}{2}$$

$$x = \frac{-8 \pm \sqrt{260}}{2} = \frac{-8 \pm 2\sqrt{65}}{2} = -4 \pm \sqrt{65}$$

$x = -4 + \sqrt{65}$

$AD \approx 4.06$ cm

④



Given: $\triangle FDG$ - right at D
$\overline{DE}$ - bisector $\angle D$
$\angle F \cong \angle G$

Prove: E = midpoint $\overline{FG}$

We need to show $\overline{FE} \cong \overline{GE}$

We'll prove $\triangle FED \cong \triangle GED$

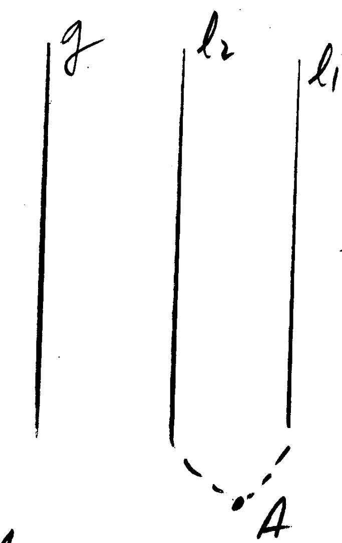
Proof

Statements	Reasons
1. $\triangle FDG$ - right at D	1. given
2. $\overline{DE}$ - bis. $\measuredangle D$	2. given
3. $\angle 1 \cong \angle 2$	3. def. $\measuredangle$ bisector
4. $\angle F \cong \angle G$	4. given
5. $\overline{DE} \cong \overline{DE}$	5. reflexive $\cong$
6. $\triangle FED \cong \triangle GED$	6. AAS
7. $\overline{FE} \cong \overline{GE}$	7. CPCTC
8. E = midpt $\overline{FG}$	8. def midpt.

(6).

Given: $l_1 \parallel g$
$l_2 \parallel g$

prove $l_1 \parallel l_2$



Proof

Assume $l_1 \nparallel l_2$

Then $l_1 \cap l_2 = A$

Given point A and a line g, there are two lines (l_1 and l_2) through A, parallel to g. Contradiction with the uniqueness of a line parallel to a given line through a point.

Therefore, $l_1 \parallel l_2$

(5) Given: $\triangle ABC$
$\overline{DE} \parallel \overline{BC}$
$D \in \overline{AB}$, $E \in \overline{AC}$

Prove: $\dfrac{DP}{BF} = \dfrac{PE}{FC}$

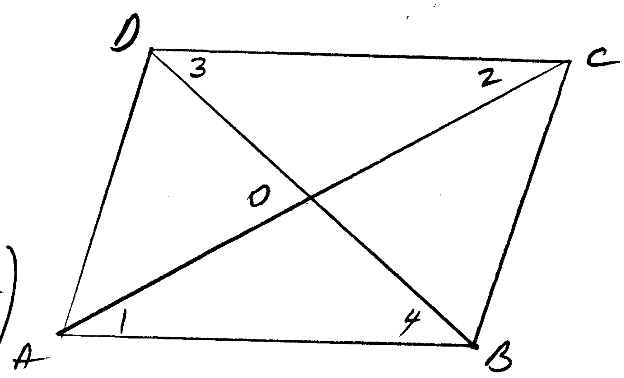
Proof

Statements	Reasons
1. $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$	1. given
2. $\dfrac{DP}{BF} = \dfrac{AP}{AF}$	2. $\triangle ABF$ with $\overline{DP} \parallel \overline{BF}$
3. $\dfrac{AP}{AF} = \dfrac{PE}{FC}$	3. $\triangle AFC$ with $\overline{PE} \parallel \overline{FC}$
4. (2,3) $\dfrac{DP}{BF} = \dfrac{PE}{FC}$	4. transitivity

(7) Given: ABCD - parallelogram
$\overline{AC}$, $\overline{BD}$ - diagonals

Prove: $\overline{AC}$ bisects $\overline{BD}$
$\overline{BD}$ bisects $\overline{AC}$

(Need to show: $\overline{DO} \cong \overline{BO}$; $\overline{AO} \cong \overline{CO}$
We'll prove $\triangle AOB \cong \triangle COD$)



Proof

1. ABCD - parallelogram — 1. given
2. $\overline{AB} \parallel \overline{DC}$ — 2. def. parallelogram
3. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$ — 3. $\parallel$ iff alt. int. $\angle$'s $\cong$ ($\overline{AB} \parallel \overline{CD}$ with transv. $\overline{AC}$, then with transv. $\overline{BD}$)
4. $\overline{AB} \cong \overline{CD}$ — 4. opp. sides ▱ $\cong$
5. $\triangle AOB \cong \triangle COD$ (3,4) — 5. ASA
6. $\overline{AO} \cong \overline{CO}$, $\overline{BO} \cong \overline{DO}$ — 6. CPCTC
7. O = midpt. $\overline{AC}$, O = midpt $\overline{BD}$ — 7. def. midpoint
8. $\overline{BD}$ bisects $\overline{AC}$, $\overline{AC}$ bisects $\overline{BD}$ — 8. def. segment bisector