Math 61 Fall 2007 Name: SOLUTIONS

TEST #1 @ 130 points

Write in a neat and organized fashion. Use a straightedge and compass for your drawings.

1) Write the inverse, converse, and contrapositive of the following statement and classify the statements as true or false. If true, state the definition, postulate, or theorem your conclusion is based on.
If false, say why or draw a counterexample.

Circle one — Justify your choice

"" *If M is the midpoint of* $\overline{AB}$, *then* $\overline{AM} \cong \overline{MB}$."

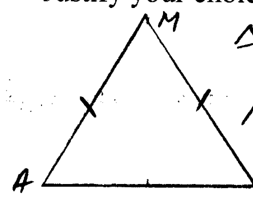
True — False

Definition of midpoint.

Circle one — Justify your choice

Inverse: If M is not the midpoint of $\overline{AB}$, then $\overline{AM} \ncong \overline{MB}$

True — False



$\triangle MAB$ isosceles
$M \neq$ midpt, but $\overline{MA} \cong \overline{MB}$

Circle one — Justify your choice

Converse: If $\overline{AM} \cong \overline{MB}$, then M is the midpoint of $\overline{AB}$

True — False

Not necessarily
In order for M to be the midpoint, we also need $M \in \overline{AB}$, A-M-B.
Otherwise, see the isosceles $\triangle$ above

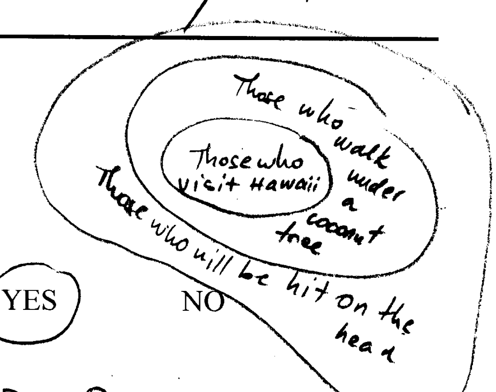
Circle one — Justify your choice

Contrapositive: If $\overline{AM} \ncong \overline{MB}$, then M is not the midpoint of $\overline{AB}$

True — False

Definition of midpoint

2) Study each argument carefully to decide whether or not it is valid.



a) If you walk under a coconut tree, you will probably be hit on the head.
If you visit Hawaii, then you will walk under coconut trees.
Therefore, if you visit Hawaii, you will probably be hit on the head.

VALID: YES NO

b) If you are using this book, then you must be able to read. $P \to Q$
If you are a geometry student, you must be able to read. $R \to Q$
Therefore, if you are using this book, you are a geometry students. $P \to R$

VALID: YES NO

3) a) Complete the following law:

$$\sim(P \vee Q) \equiv \underline{\sim P \wedge \sim Q}$$

b) Prove the law using a truth table. State clearly why we can conclude from the truth table that the law is valid.

P	Q	$\sim(P \vee Q)$		$\sim P$	$\wedge$	$\sim Q$
T	T	F	T	F	F	F
T	F	F	T	F	F	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T

The law is valid because $\sim(P \vee Q)$ and $\sim P \wedge \sim Q$ have exactly the same truth values

4) Answer true or false:

1) The hypotenuse is the side opposite one of the acute angles in a right triangle. False

2) An isosceles triangle can have an obtuse angle as one of its angles. True

3) If three angles of one triangle are congruent with three angles of a second triangle, then the two triangles are congruent. False

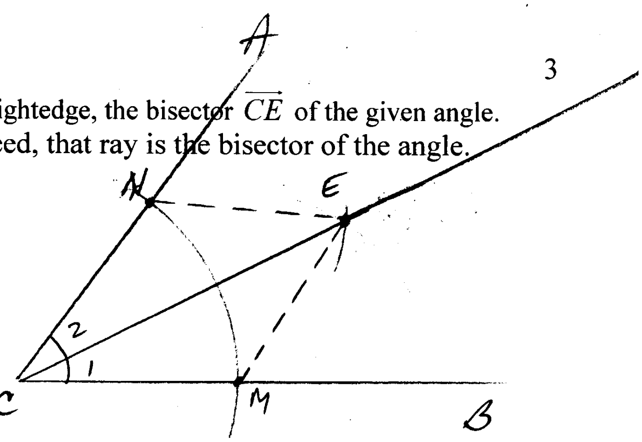
4) Triangles can be proved congruent using SSA. False

5) Corresponding parts of congruent triangles are congruent. True

6) An exterior angle of a triangle is the supplement of one of the interior angles of the triangle. True

7) The median to the base of an isosceles triangle bisects the vertex angle. True

5) Given an angle $\angle BCA$, construct using only a compass and a straightedge, the bisector $\overrightarrow{CE}$ of the given angle. Explain how you are constructing it and then prove that, indeed, that ray is the bisector of the angle.



Given: $< BCA$

Construct: $\overrightarrow{CE}$ bisector

(Condition: $E \in int < BCA$
$< BCE \cong < ECA$)

Solution

1. Construct a circle $\mathscr{C}$ with center C and a radius $r_1 > 0$
2. $\mathscr{C} \cap \overrightarrow{CB} = M$, $\mathscr{C} \cap \overrightarrow{CA} = N$
3. Construct two congruent circles < one with center M and radius r_2; one with center N and radius r_2
(note that r_2 can equal r_1 or not)
4. Let E = intersection of the two circles from (3)

We'll prove that $\overline{CE}$ = bisector of < ACB

1st - Note that the construction is unique (2 pts determine a line)

$\triangle NCE$, $\triangle MCE$ { $\overline{CE} \cong \overline{CE}$ (reflexive prop); $\overline{CN} \cong \overline{CM}$ ($CN = CM = r_1$); $\overline{NE} \cong \overline{ME}$ ($NE = ME = r_2$) } =>

=> $\triangle NCE \cong \triangle MCE$ (SSS)

=> $< 1 \cong < 2$ (CPCTC)

=> $\overline{CE}$ = bisector of angle ACB (definition of bisector)

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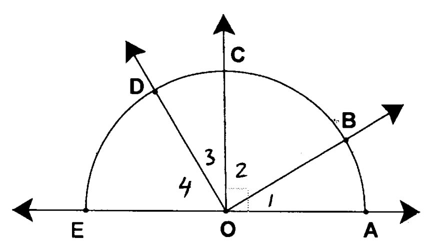
6) Given the figure, name:



a) three acute angles $<1, <2, <4$

b) Two right angles $<AOC$, $<COE$

c) One obtuse angle $<AOD$

d) One straight angle $<AOE$

e) Two complementary angles <1 and <2

f) Two supplementary angles $<AOB$ and $<BOE$

g) Two adjacent angles <2 and <3

h) Two nonadjacent angles <4 and <2

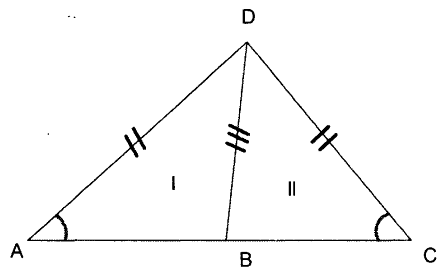
i) Two opposite rays $\overrightarrow{OA}$ and $\overrightarrow{OE}$

j) Three noncollinear points. D, O, A

7) i) Write the congruences given by the indicated measures or marks.

ii) State whether from the given congruences only you may conclude that triangles I and II are congruent.

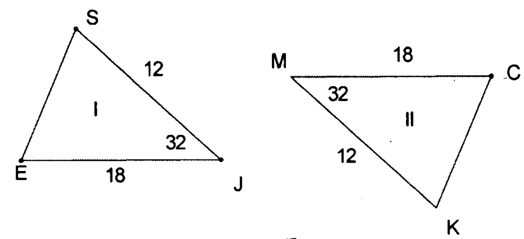
iii) If so, write what case of congruency applies.



i) $\overline{AD} \cong \overline{CD}$
$\overline{BD} \cong \overline{BD}$
$\angle A \cong \angle C$

ii) $\Delta I \ncong \Delta II$

iii) N/A

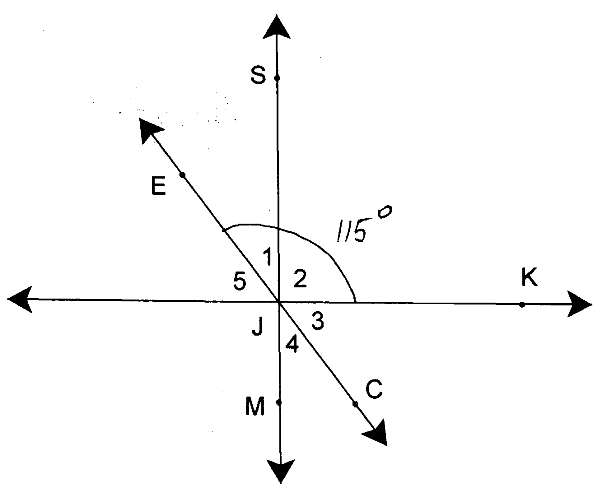


i) $\overline{SJ} \cong \overline{KM}$
$\overline{EJ} \cong \overline{CM}$
$\angle J \cong \angle M$

ii) $\Delta SJE \cong \Delta KMC$

iii) SAS

8)



Given $\overrightarrow{JK} \perp \overrightarrow{SM}$
$m\angle EJK = 115^\circ$

Find angles 1 through 5

INFORMAL PROOF – make sure , though, that you justify each step)

Solution

$m\angle 2 = 90^\circ$ ($\overline{JK} \perp \overline{SM}$)

$m\angle 1 = 115^\circ - 90^\circ = 25^\circ$

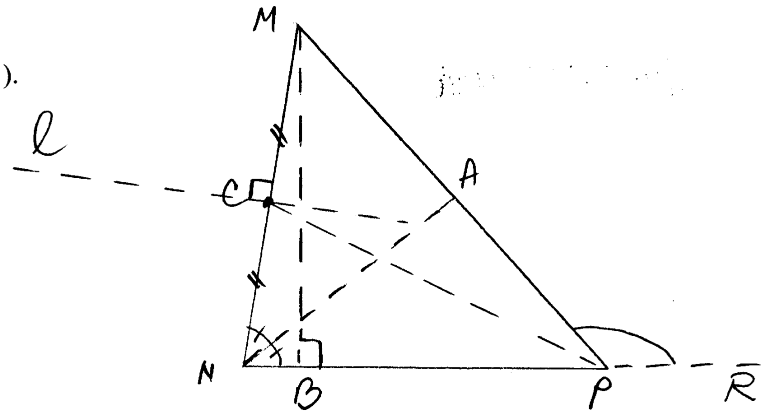
$m\angle 4 = m\angle 1 = 25^\circ$ (vertical <'s)

$m\angle 5 = 90^\circ - m\angle 1$
$= 90^\circ - 25^\circ$
$= 65^\circ$

$m\angle 3 = m\angle 5 = 65^\circ$ (vertical <'s)

9) A triangle MNP is given. All the questions below refer to the triangle MNP.

a) Draw a scalene triangle
(draw a relatively big triangle ☺).



b) Check all that applies:
A scalene triangle could be :

acute ☑ right ☑ obtuse ☑ none ☐

c) Name the following:

- the angle opposite side $\overline{MN}$ $\angle P$
- the side opposite angle MPN $\overline{MN}$
- the angle included by $\overline{NP}$ and $\overline{MP}$ $< P$
- an exterior angle of the triangle (make sure to mark it on the drawing) $< RPM$

d) Using your figure, draw the bisector of angle N, name it $\overline{NA}$, and state , using mathematical notation, that $\overline{NA}$ is the bisector of angle N (what does it mean?).

$\overline{NA}$ bisector $< N$ iff $< PNA \cong < ANM$

e) Using your above triangle, draw the altitude from vertex M to the opposite side, name it $\overline{MB}$, and state, using mathematical notation, that $\overline{MB}$ is an altitude (what does it mean?).

$\overline{MB}$ = altitude iff $\overline{MB} \perp \overline{NP}$, $B \in \overleftrightarrow{NP}$

f) Using your above triangle , draw the median from vertex P, name it $\overline{CP}$, and state, using mathematical notation, that $\overline{CP}$ is a median (what does it mean?).

$\overline{CP}$ = median iff C = midpoint of $\overline{MN}$
($\overline{MC} \cong \overline{CN}$)

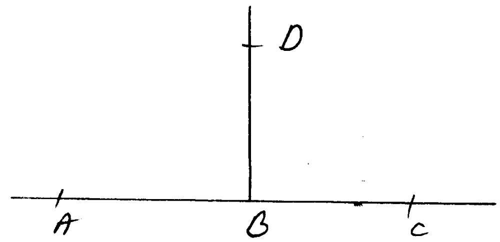
g) Using your above triangle, draw the perpendicular bisector of side $\overline{MN}$, name it l, and state, using mathematical notation, that l is the perpendicular bisector of $\overline{MN}$ (what does it mean?).

l = $\perp$ bisector $\overline{AB}$ iff $l \perp \overline{MN}$ at midpoint of $\overline{MN}$
(that is, $l \cap \overline{MN} = C$, $\overline{CM} \cong \overline{CN}$)

10) Prove the theorem (FORMAL proof).

Two equal supplementary angles are right angles.

Make sure you state the hypothesis and conclusion of the theorem (using notation pertinent to your drawing).



Given: $\angle ABD$, $\angle DBC$ = supplementary
$m\angle ABD = m\angle DBC$

Prove: $\angle ABD$, $\angle DBC$ = right $\angle$'s

(Need to draw $m\angle ABD = m\angle DBC = 90°$)

Proof

Statements	Reasons
1. $\angle ABD$, $\angle DBC$ = supplm.	1. given
2. $m\angle ABD + m\angle DBC = 180°$	2. definition suppl. $\angle$'s
3. $m\angle ABD = m\angle DBC$	3. given
4. (2,3) $m\angle DBC + m\angle DBC = 180°$	4. substitution
5. $2m\angle DBC = 180°$	5. simplifying (combining like terms using distributive property)
6. $m\angle DBC = 90°$	6. Division prop. of =
7. (3,6) $m\angle ABD = 90°$	7. Substitution
8. (6,7) $\angle DBC$, $\angle ABD$ = right $\angle$'s	8. Definition of right $\angle$'s

Q.E.D.

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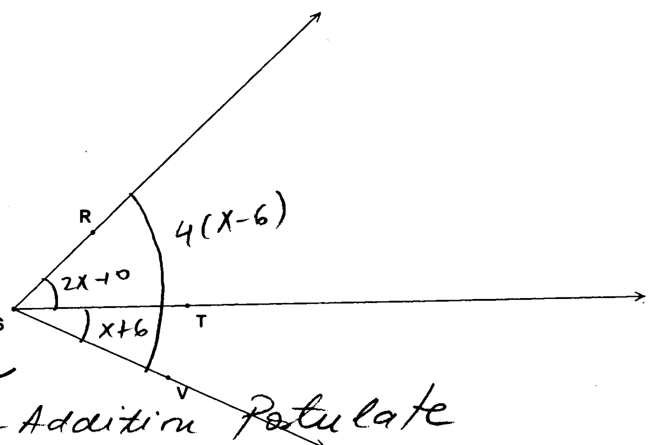
11) Given: $m\angle RST = 2x - 10$
$m\angle TSV = x + 6$
$m\angle RSV = 4(x-6)$

Find: x and $m\angle RSV$.

FORMAL PROOF.



Proof

Statements	Reasons
1. $m\angle RST = 2X-10$, $m\angle TSV = X+6$, $m\angle RSV = 4(X-6)$	1. given
2. $m\angle RST + m\angle TSV = m\angle RSV$	2. Angle-Addition Postulate
3. (2,1) $(2X-10)+(X+6) = 4(X-6)$	3. Substitution
4. $3X - 4 = 4X - 24$	4. Combining like terms & distributive prop.
5. $-4+24 = 4X - 3X$	5. Addition/Subtraction prop. of =
6. $20 = X$	6. Combining like terms (simplifying)
7. $X = 20$	7. symmetric prop. of =
8. (7,1) $m\angle RSV = 4(20-6) = 4(16) = 64°$	8. Substitution & Simplifying

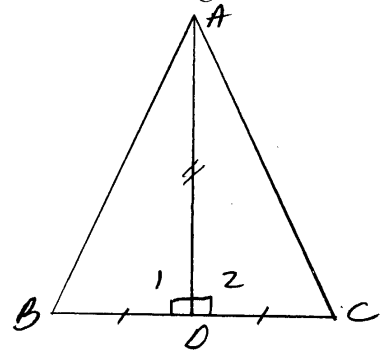
12) Draw a figure. Write the hypothesis and conclusion using notation pertinent to your drawing. Mark the figure and write a formal proof.

If the median of a triangle is perpendicular to one of its sides, then the triangle is isosceles.

Given: $\triangle ABC$
$\overline{AD}$ - median
$\overline{AD} \perp \overline{BC}$

Prove: $\triangle ABC$ - isosceles

We need to show that $\overline{AB} \cong \overline{AC}$



Proof

Statements	Reasons
1. $\triangle ABC$, $\overline{AD}$ - median	1. given
2. D - midpt $\overline{BC}$	2. definition of median
3. $\overline{BD} \cong \overline{DC}$	3. definition of midpoint
4. $\overline{AD} \perp \overline{BC}$	4. given
5. $\angle 1 \cong \angle 2$	5. $\perp$ iff $\cong$ adj. $\angle$'s (def $\perp$ lines)
6. $\triangle ADB$, $\triangle ADC$ $\begin{cases} \overline{AD} \cong \overline{AD} \\ \angle 1 \cong \angle 2 \\ \overline{BD} \cong \overline{CD} \end{cases}$	6. reflexive prop. $\cong$; (5) above; (3) above
7. $\triangle ADB \cong \triangle ADC$	7. SAS
8. $\overline{AB} \cong \overline{AC}$	8. CPCTC
9. $\triangle ABC$ - isosceles	9. def. of isosceles $\triangle$

Q.E.D.

EXTRA CREDIT

Given: $\overline{SR} \perp \overline{EJ}$
$\overline{SK}$ bisects $\angle ESR$
$\angle 6$ and $\angle 8$ are supplementary
$\angle 1 + \angle 5 + \angle 6 = 180^\circ$
$\angle SKR = 74^\circ$
$\angle 2 + \angle 7 = \angle 6$
$\triangle EKS \cong \triangle JTS$

Find angles 1 – 10.