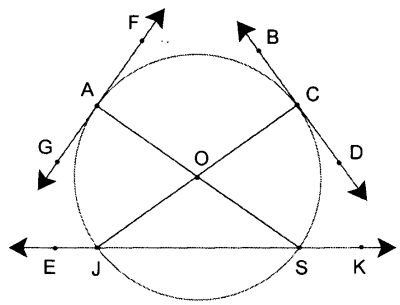
Math 61 Fall 2007 **QUIZ #4 @ 30 points** Name: SOLUTIONS

Write in a neat and organized fashion. Use a pencil. Show all work to get credit.

1. In the given figure, name:



a) four radii $\overline{OA}$, $\overline{OC}$, $\overline{OS}$, $\overline{OJ}$

b) two diameters $\overline{AS}$, $\overline{CJ}$

c) three chords $\overline{JS}$, $\overline{AS}$, $\overline{CJ}$

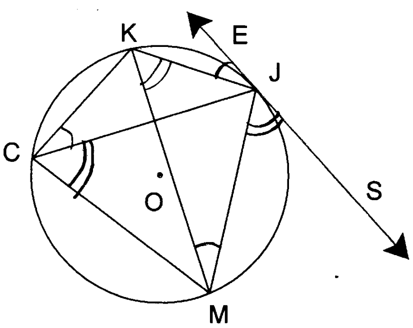
d) two tangents $\overleftrightarrow{AF}$, $\overleftrightarrow{BD}$

e) one secant $\overleftrightarrow{EK}$

2. Use the figure to answer the questions.

Given $\odot O$

tan $\overrightarrow{ES}$



a) Name two angles congruent to $\angle KJE$.

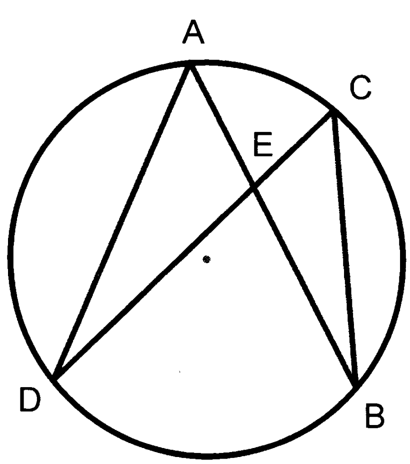
$m\angle KJE = \frac{1}{2} m\widehat{KJ}$, therefore $\angle KJE \cong \angle KCJ \cong \angle KMJ$

b) Name two angles congruent to $\angle JCM$.

$\angle JCM \cong \angle JKM \cong \angle SJM$

$(m\angle JCM = m\angle JKM = m\angle SJM = \frac{1}{2} m\widehat{MJ})$

3.



Given: DE = 12, EC = 5, AE = 8

Find: EB.

Solution

$\overline{AB}$, $\overline{CD}$ chords with $\overline{AB} \cap \overline{CD} = E$

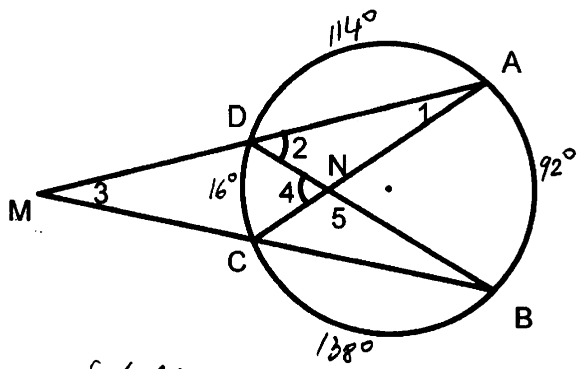
$AE \cdot EB = CE \cdot ED$

$8 \cdot EB = 5 \cdot 12$

$EB = \frac{5 \cdot \cancel{12}^{3}}{\cancel{8}_{2}} = \frac{15}{2}$

$EB = 7.5$

4.



Given: $m\overset{\frown}{AB} = 92^\circ$

$m\overset{\frown}{BC} = 138^\circ$

$m\overset{\frown}{DA} = 114^\circ$

Find:
$m\angle 1$ (<DAC)
$m\angle 2$ (< ADB)
$m\angle 3$ (<AMB)
$m\angle 4$ (< DNC)
$m\angle 5$ (< CNB)

Solution

$m\overset{\frown}{CD} = 360^\circ - (m\overset{\frown}{AB} + m\overset{\frown}{BC} + m\overset{\frown}{AD})$
$= 360^\circ - 344^\circ = 16^\circ$

$m<1 = \frac{1}{2} m\overset{\frown}{CD}$ (inscribed ∡)
$= \frac{1}{2} 16^\circ = 8^\circ$

$m<2 = \frac{1}{2} m\overset{\frown}{AB}$ (inscribed ∡)
$= \frac{1}{2} 92^\circ = 46^\circ$

$m<3 = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{CD})$
$= \frac{1}{2}(92^\circ - 16^\circ)$ 38°

$m<4 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$
$= \frac{1}{2}(92^\circ + 16^\circ)$
$= 54^\circ$

$m<5 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$
$= \frac{1}{2}(138^\circ + 114^\circ)$
$= 126^\circ$

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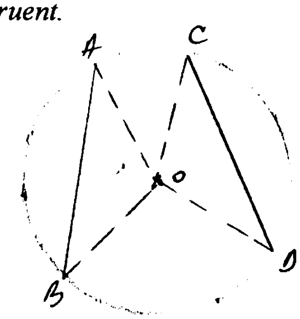
5. Prove the following theorem using a formal proof. Make a drawing and state the hypothesis (given) and conclusion(to prove) using math notation pertinent to your drawing – that is, do not state the hypothesis and conclusion in words!

If two chords in a circle are congruent, then their arcs are congruent.



Given: ⊙O, $\overline{AB}, \overline{CD}$ = chords
$\overline{AB} \cong \overline{CD}$

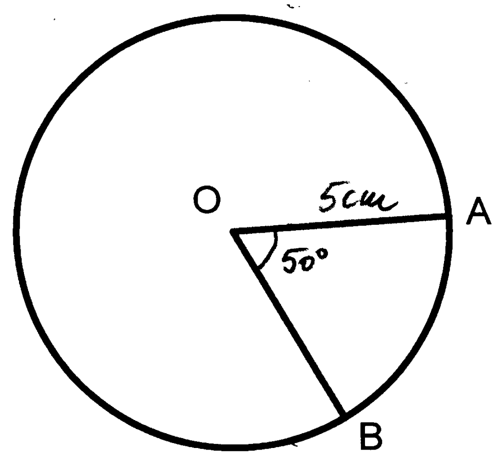
Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

Proof

Statements	Reasons
1. $\overline{AB}, \overline{CD}$ · chords, $\overline{AB} \cong \overline{CD}$	1. given
2. $\overline{AO}, \overline{BO}, \overline{CO}, \overline{DO}$ = radii	2. By construction
3 $\overline{AO} \cong \overline{BO} \cong \overline{CO} \cong \overline{DO}$	3. All radii are ≅
4 $\triangle AOB \cong \triangle COD$	4. SSS
5 $\angle AOB \cong \angle COD$	5 CPCTC
6. $m\angle AOB = m\angle COD$	6 def. ≅ ∡'s
7. $m\angle AOB = m\overset{\frown}{AB}$, $m\angle COD = m\overset{\frown}{CD}$	7. def measure of central ∡
8. $m\overset{\frown}{AB} = m\overset{\frown}{CD}$	8. transitivity / substitution
(6,7) 9 $\overset{\frown}{AB} = \overset{\frown}{CD}$	9. def ≅ arcs

6. Given $\odot O$ with $m\angle AOB = 50^\circ$ and $OA = 5cm$,

find the following (exact answers) and <u>use correct units:</u>



a) $m\widehat{AB}$ $\quad m\angle AOB = m\widehat{AB}$ (central ∠)

$m\widehat{AB} = 50^\circ$

b) $l\widehat{AB}$ $\quad \frac{l\widehat{AB}}{50^\circ} = \frac{2\pi r}{360^\circ} \Rightarrow$

$l\widehat{AB} = \frac{\pi(5)\cdot 50^\circ}{180^\circ} = \frac{25\pi}{18}$ $\quad \boxed{l\widehat{AB} = \frac{25\pi}{18}\,cm}$

c) Circumference of the circle

$C = 2\pi r = \boxed{10\pi\, cm}$

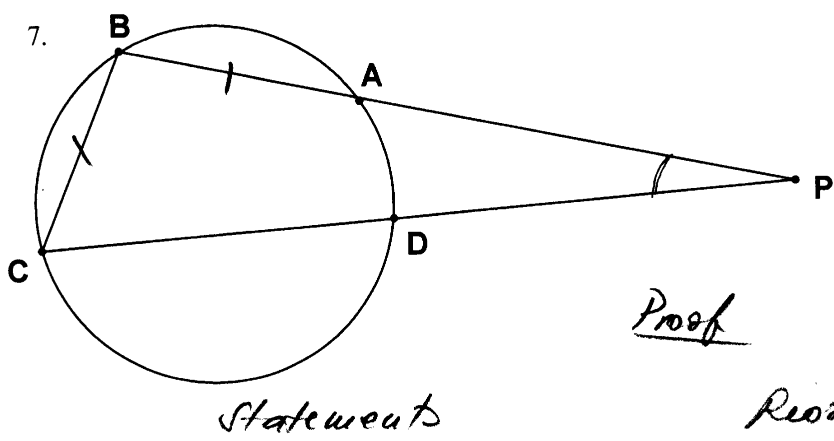
d) Area of the circle

$A = \pi r^2 = \boxed{25\pi\, cm^2}$

e) Area of the sector AOB

$\frac{A(AOB)}{50^\circ} = \frac{\pi r^2}{360^\circ} \Rightarrow A(AOB) = \frac{\pi\cdot 25\,cm^2\cdot 50^\circ}{360^\circ} = \boxed{\frac{125\pi}{36}\,cm^2}$

7.



Given: $\overline{AB} \cong \overline{BC}$

Prove: $m\angle BPC = \frac{1}{2}\left(m\widehat{AB} - m\widehat{AD}\right)$

(formal proof).

<u>Proof</u>

Statements	Reasons
1. $\overline{AB} \cong \overline{BC}$	1. given
2. $\widehat{AB} \cong \widehat{BC}$	2. ≅ chords iff ≅ arcs
3. $m\angle BPC = \frac{1}{2}(m\widehat{BC} - m\widehat{AD})$	3. ∠ formed by secants outside circle
4. (1,2,3) $m\angle BPC = \frac{1}{2}(m\widehat{AB} - m\widehat{AD})$	4. substitution