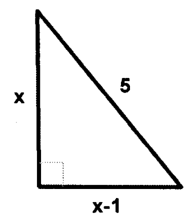
Math 150 Fall 07 Name:

TEST #1 @ 150 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given

---

1. Solve for x in the following right triangle:



---

2. Find the remaining sides of a $30^\circ - 60^\circ - 90^\circ$ triangle if the shortest side is 3 inches.

---

3. Let t a real number and $P\left(\frac{2\sqrt{2}}{3}, \frac{-1}{3}\right)$ a point on the unit circle that corresponds to t. Find the exact values of the six trigonometric functions of t.

---

4. Find the exact values of $\cos\theta$ and $\tan\theta$ if $\sin\theta = \frac{\sqrt{5}}{5}$ with θ in quadrant II.

---

5. Using your calculator and rounding your answers to the nearest hundredth, find $\cos\theta$ and if $\sin\theta = 0.23$ and θ is in quadrant I.

---

6. Do the following operations and simplify as much as possible:

a) $\frac{1}{\cos t} - \cos t$

b) $(2\cos a + 3)(4\cos a - 5)$

---

7. Find the exact values of the following:

a) $\sin 45^\circ + \cos(-30^\circ)$ b) $\sin\frac{4\pi}{3}$ c) $\tan\frac{\pi}{3} + \cos\frac{\pi}{3}$ d) $\cos\frac{11\pi}{6}$

e) $\sin\left(-\frac{3\pi}{4}\right)$ f) $\tan\left(-\frac{13\pi}{6}\right)$ g) $\cos\left(-\frac{5\pi}{6}\right)$ h) $\cot\frac{17\pi}{3}$

8. Use the unit circle to find all the values of θ between 0 and 2π for which

a) $\sin\theta = \dfrac{1}{2}$ b) $\cos\theta = -\dfrac{\sqrt{2}}{2}$

---

9. Prove the following trigonometric identities:

a) $\sin a \tan a + \cos a = \sec a$

b) $\sin(-\theta)\sec(-\theta)\cot(-\theta) = 1$

c) $\sin\theta(\sec\theta + \cot\theta) = \tan\theta + \cos\theta$

d) $\dfrac{\cos a}{1+\sin a} + \dfrac{1+\sin a}{\cos a} = 2\sec a$

---

10. Show that tangent is an odd function.

---

11. In a right triangle ABC with $C = 90^\circ$, angle $B = 55.33^\circ$ and side $b = 12.34\,yd$. Find the other two sides a and c and the remaining angle A.

---

12. A mixing blade on a food processor extends out 3 inches from its center. If the blade is turning at 600 revolutions per minute, what is the linear velocity of the tip of the blade in feet per minute?

---

13. From the top of a 200-ft lighthouse, the angle of depression to a ship in the ocean is 23°. How far is the ship form the base of the lighthouse?

---

14. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be 32°. One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is 35°. Estimate the height of the mountain.

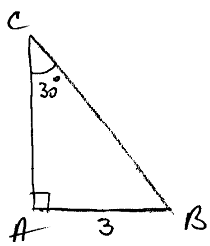
Math 150

TEST 1 - SOLUTIONS

(1) Pythagorean theorem:

$$x^2+(x-1)^2 = 5^2$$
$$x^2+x^2-2x+1 = 25$$
$$2x^2-2x-24=0 \quad / \div 2$$
$$x^2-x-12=0$$
$$(x-4)(x+3)=0$$
$$x=4 \quad \text{OR} \quad x=-3 \text{ not possible}$$
$$\boxed{x\in\{4\}}$$

(2)



shortest leg opposes smallest angle

so $AB=3$, $C=30°$

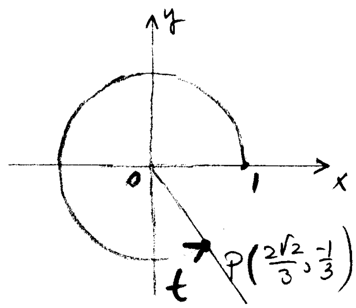
then $AB=\frac{1}{2}BC$

$BC=2AB=6$

$$AC^2=BC^2-AB^2$$
$$AC^2=6^2-3^2=27$$
$$AC=\sqrt{27}$$
$$AC=3\sqrt{3}$$

$$\boxed{\begin{array}{l} AB=3 \\ BC=6 \\ AC=3\sqrt{3}\end{array}}$$

(3) $P\left(\frac{2\sqrt{2}}{3}, \frac{-1}{3}\right)$



$$\boxed{\begin{array}{l} \cos t = \frac{2\sqrt{2}}{3} \\ \sin t = \frac{-1}{3}\end{array}}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{\frac{-1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{-1}{2\sqrt{2}}$$

$$\boxed{\tan t = \frac{-\sqrt{2}}{4}} \qquad \boxed{\cot t = -2\sqrt{2}}$$

$$\cot t = \frac{1}{\tan t} = -2\sqrt{2}$$

$$\sec t = \frac{1}{\cos t} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$
$$\boxed{\sec t = \frac{3\sqrt{2}}{4}}$$

$$\csc t = \frac{1}{\sin t} = -3$$
$$\boxed{\csc t = -3}$$

(4) $\underline{\sin\theta = \frac{\sqrt{5}}{5}}$, $\theta \in \text{II}$

$\underline{\cos\theta, \tan\theta = ?}$

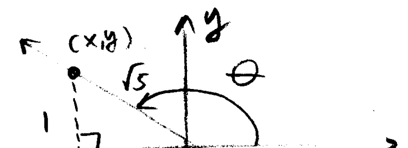
Method I

$$\sin^2\theta+\cos^2\theta=1$$
$$\left(\frac{\sqrt{5}}{5}\right)^2+\cos^2\theta=1$$
$$\frac{1}{5}+\cos^2\theta=1$$
$$\cos^2\theta=\frac{4}{5}$$
$$\left.\begin{array}{l}\cos\theta=\pm\frac{2\sqrt{5}}{5} \\ \theta\in \text{II}\end{array}\right\} \Rightarrow \boxed{\cos\theta=\frac{-2\sqrt{5}}{5}}$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{\sqrt{5}}{\cancel{5}}}{\frac{-2\sqrt{5}}{\cancel{5}}}=\frac{-1}{2}$$

$$\boxed{\tan\theta=\frac{-1}{2}}$$

Method II



let (x,y) point on the terminal side of θ

$$\left.\begin{array}{l}\sin\theta=\frac{\sqrt{5}}{5}=\frac{1}{\sqrt{5}} \\ \text{also } \sin\theta=\frac{y}{r}\end{array}\right\} \Rightarrow \begin{array}{l} y=1 \\ r=\sqrt{5}\end{array}$$

then $x^2=r^2-y^2=(\sqrt{5})^2-1=4$, $x=\pm 2$

but II $x=-2$

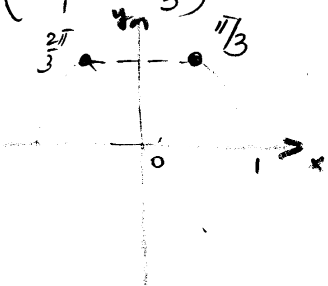
(h) $\cot \frac{17\pi}{3} = \cot\left(5\pi + \frac{2\pi}{3}\right)$

$= \cot \frac{2\pi}{3}$

$= -\cot \frac{\pi}{3}$

$= \frac{-1}{\tan \frac{\pi}{3}}$

$= \frac{-1}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$



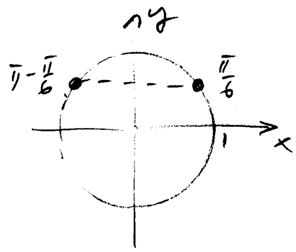
(8) (a) $\sin\theta = \frac{1}{2}$

we know $\sin\frac{\pi}{6} = \frac{1}{2}$

Therefore, $\theta = \frac{\pi}{6}$ OR

$\theta = \pi - \frac{\pi}{6}$

$\boxed{\theta \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}}$

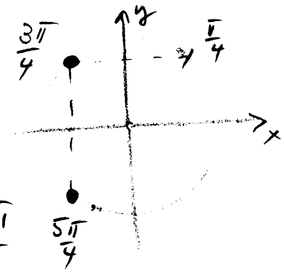


(b) $\cos\theta = \frac{-\sqrt{2}}{2}$

we know $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

need $\cos\theta < 0 \Rightarrow$ II and III

$\boxed{\theta \in \left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}}$



(9) (a) $\sin a \tan a + \cos a = \sec a$

LHS $= \sin a \tan a + \cos a$

$= \sin a \frac{\sin a}{\cos a} + \frac{\cos a}{\cos a} \cos a$

LCD $= \cos a$

$= \frac{\sin^2 a + \cos^2 a}{\cos a}$

$= \frac{1}{\cos a} = \sec a =$ RHS

So the given equation is an identity

(b) $\sin(-\theta)\sec(-\theta)\cot(-\theta) = 1$

LHS $= \sin(-\theta)\sec(-\theta)\cot(-\theta)$

$= \sin(-\theta)\frac{1}{\cos(-\theta)} \cdot \frac{\cos(-\theta)}{\sin(-\theta)}$

$= 1 =$ RHS

So, the given eq. is an identity

(c) $\sin\theta(\sec\theta + \cot\theta) = \tan\theta + \cos\theta$

LHS $= \sin\theta(\sec\theta + \cot\theta)$

$= \sin\theta\sec\theta + \sin\theta\cot\theta$

$= \sin\theta \cdot \frac{1}{\cos\theta} + \cancel{\sin\theta} \frac{\cos\theta}{\cancel{\sin\theta}}$

$= \tan\theta + \cos\theta =$ RHS

therefore the given equation is an identity

(d) $\frac{\cos a}{1+\sin a} + \frac{1+\sin a}{\cos a} = 2\sec a$

LHS $= \frac{\cos a}{1+\sin a}\cos a + \frac{1+\sin a}{\cos a}(1+\sin a)$

LCD $= \cos a(1+\sin a)$

$= \frac{\cos^2 a + (1+\sin a)^2}{\cos a(1+\sin a)}$

$= \frac{\cos^2 a + 1 + 2\sin a + \sin^2 a}{\cos a(1+\sin a)}$

$= \frac{1 + 1 + 2\sin a}{\cos a(1+\sin a)}$

$\cos\theta = \dfrac{x}{r}$

$\cos\theta = \dfrac{-2}{\sqrt{5}} = \dfrac{-2\sqrt{5}}{5}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-1}{2}$

(5) $\sin\theta = 0.23$

$\theta \in I$

$\cos\theta = ?$

$\sin^2\theta + \cos^2\theta = 1$

$(0.23)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - 0.529$

$\left.\begin{array}{l}\cos\theta = \pm\sqrt{0.9471} \\ \theta \in I\end{array}\right\} \Rightarrow \cos\theta = \sqrt{0.9471}$

$\boxed{\cos\theta \approx 0.97}$

(6) (a) $\dfrac{1}{\cos t} - \dfrac{\cos t}{\cos t}\cos t = \dfrac{1-\cos^2 t}{\cos t}$

$= \dfrac{\sin^2 t}{\cos t}$

(b) $(2\cos a + 3)(4\cos a - 5) =$

$= \boxed{8\cos^2 a + 2\cos a - 15}$

(7) (a) $\sin 45° + \cos(-30°) =$

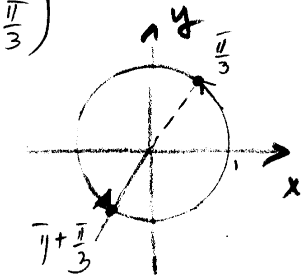
$= \sin 45° + \cos 30°$

$= \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{3}}{2} = \boxed{\dfrac{\sqrt{2}+\sqrt{3}}{2}}$

(b) $\sin\dfrac{4\pi}{3} = \sin\left(\pi + \dfrac{\pi}{3}\right)$

$= -\sin\dfrac{\pi}{3}$

$= \boxed{\dfrac{-\sqrt{3}}{2}}$

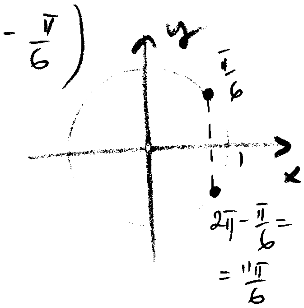


(c) $\tan\dfrac{\pi}{3} + \cos\dfrac{\pi}{3} = \boxed{\sqrt{3} + \dfrac{1}{2}}$

(d) $\cos\dfrac{11\pi}{6} = \cos\left(2\pi - \dfrac{\pi}{6}\right)$

$= \cos\dfrac{\pi}{6}$

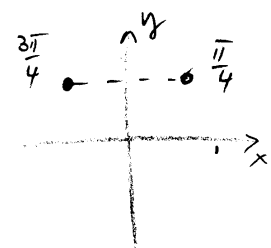
$= \boxed{\dfrac{\sqrt{3}}{2}}$



(e) $\sin\left(-\dfrac{3\pi}{4}\right) = -\sin\dfrac{3\pi}{4}$

$= -\sin\dfrac{\pi}{4}$

$= \boxed{-\dfrac{\sqrt{2}}{2}}$

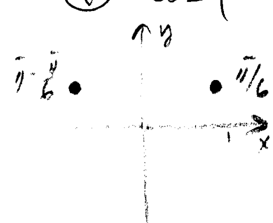


(f) $\tan\left(-\dfrac{13\pi}{6}\right) = -\tan\dfrac{13\pi}{6}$

$= -\tan\left(2\pi + \dfrac{\pi}{6}\right)$

$= -\tan\dfrac{\pi}{6} = \boxed{-\dfrac{\sqrt{3}}{3}}$

(g) $\cos\left(-\dfrac{5\pi}{6}\right) = \cos\dfrac{5\pi}{6}$



$= \cos\left(\pi - \dfrac{\pi}{6}\right)$

$= -\cos\dfrac{\pi}{6} = \boxed{-\dfrac{\sqrt{3}}{2}}$

$$= \frac{2+2\sin a}{\cos a\,(1+\sin a)}$$

$$= \frac{2\cancel{(1+\sin a)}}{\cos a\,\cancel{(1+\sin a)}} = \frac{2}{\cos a}$$

$$= 2\sec a = RHS$$

So, the given eq. is an identity

(10) tangent is odd iff

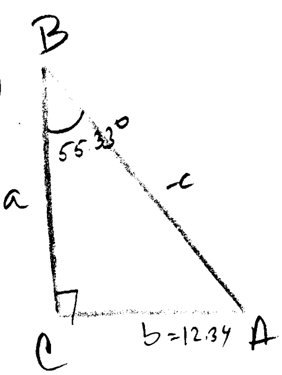
$$\tan(-\theta) = -\tan\theta$$

Proof

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$$

$$= \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

(11)



$C = 90°$

$B = 55.33°$

$b = 12.34\ yd$

$a, c = ?$

$A = ?$

$$A = 90° - 55.33°$$

$$\boxed{A = 34.67°}$$

$$\sin B = \frac{b}{c} \Rightarrow c = \frac{b}{\sin B}$$

$$c = \frac{12.34}{\sin(55.33°)} \qquad \boxed{c = 15\ yd}$$

$$a^2 = c^2 - b^2$$

$$a^2 = (15)^2 - (12.34)^2$$

$$\boxed{a \approx 8.53\ yd}$$

(12) $\omega = \frac{600\ rev}{min} = \frac{600 \cdot 2\pi\ rad}{min}$

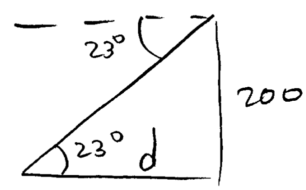
$$\omega = 1200\pi\ \frac{rad}{min}$$

$$V = \omega r = \left(1200\pi\ \frac{rad}{min}\right)(3\ in)$$

$$= 1200\pi\ \frac{rad}{min}\,(3\ \cancel{in}) \cdot \frac{1\ ft}{12\ \cancel{in}}$$

$$\boxed{V = 300\pi\ ft/min}$$

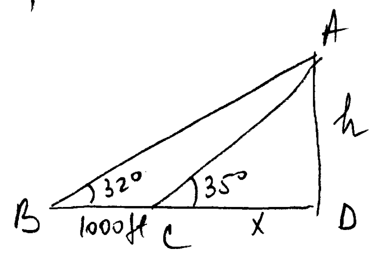
(13) $\tan 23° = \frac{200}{d}$



$$d = \frac{200}{\tan 23°}$$

$$\boxed{d \approx 471\ ft}$$

(14)



ΔACD: $\tan 35° = \frac{h}{x}$

$\Rightarrow h = x\tan 35°$

ΔABD: $\tan 32° = \frac{h}{1000+x}$

$\Rightarrow h = (1000+x)\tan 32°$

$\Rightarrow$

$$x\tan 35° = (1000+x)\tan 32°$$

$$x\tan 35° - x\tan 32° = 1000\tan 32°$$

$$x(\tan 35° - \tan 32°) = 1000\tan 32°$$

$$x = \frac{1000\tan 32°}{\tan 35° - \tan 32°}$$

$$x \approx 8294\ ft$$

$$h = x\tan 35°$$

$$h \approx 8294\tan 35°$$

$$\boxed{h \approx 5807\ ft}$$