Math 150 Fall 07 Name:

Quiz #2 @ 85 points

Solve the problems on separate paper. Clearly label the problems. Show all steps in order to get credit. No proof, no credit given

1. Graph the function $y = \sin x$. Show the graph over two periods. Answer the following questions:
   a) What is the domain?
   b) What is the range?
   c) What is the period?
   d) What is the amplitude?
   e) What are the x-intercepts?
   f) What is the y-intercept?
   g) Is the function even or odd? How is that shown in the graph?

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2. Graph the following functions on graphing paper. In each case, identify the amplitude (when defined), the period, and phase shift (when defined) and label the axes accurately. Explain in words what and how you are graphing.

   a) $y = 1 + \cos x$ from -2π to 4π

   b) $y = -3\cos x$ over one period

   c) $y = 4\sin\frac{1}{3}x$ over one period

   d) $y = 2\sin\left(x - \frac{\pi}{3}\right)$ over one period

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3. Graph the following on graphing paper. Clearly label the axes.

   $y = x + \sin x$

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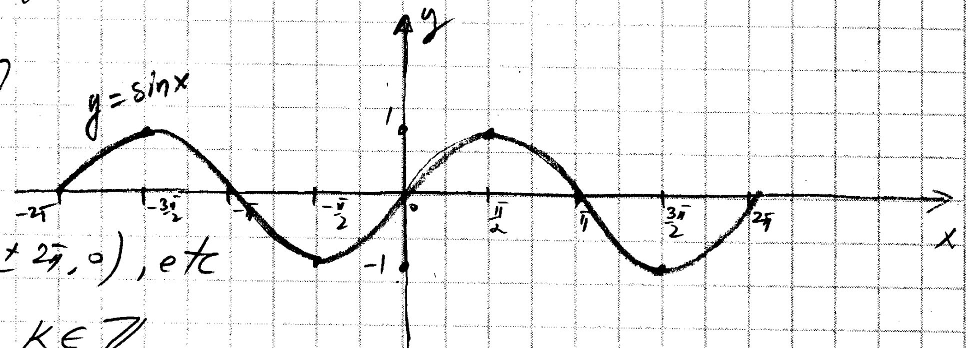
4. Evaluate the following. Give exact answers whenever possible.
   a) $\sin^{-1}\left(\frac{1}{2}\right)$
   b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
   c) $\tan^{-1}(-1)$
   d) $\cos\left(\sin^{-1}\frac{3}{5}\right)$
   e) $\sin^{-1}\left(\sin\frac{5\pi}{8}\right)$
   f) $\cos^{-1}\left(\cos\frac{2\pi}{7}\right)$
   g) $\tan\left(\tan^{-1}100.23\right)$
   h) $\tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right)$

SOLUTIONS

① $y = \sin x$ graph over two periods.

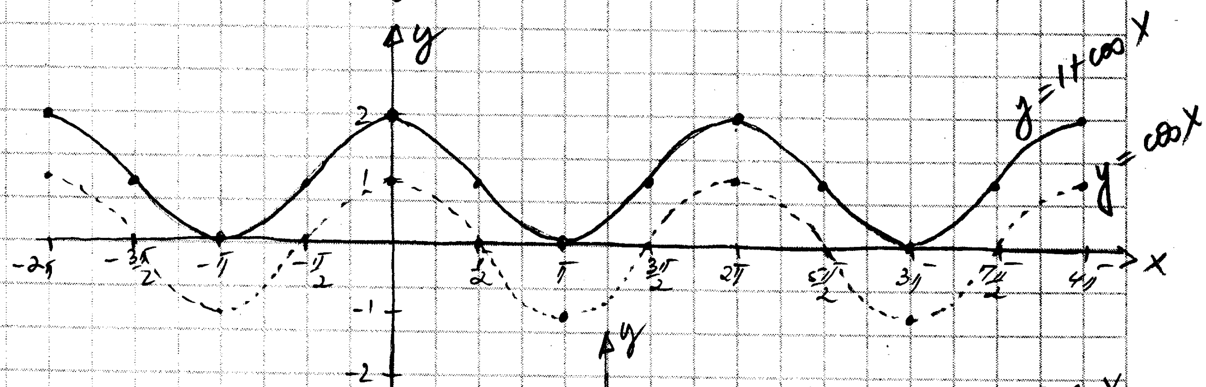
a) Domain: $x \in R$
b) Range: $y \in [-1, 1]$
c) $T = 2\pi$
d) $A = 1$
e) x-n:
$(0,0), (\pm\pi, 0), (\pm 2\pi, 0)$, etc
OR
$(k\pi, 0)$, $k \in \mathbb{Z}$



f) $(0,0)$
g) $y = \sin x$ is an odd function; the graph is symmetric about the origin.

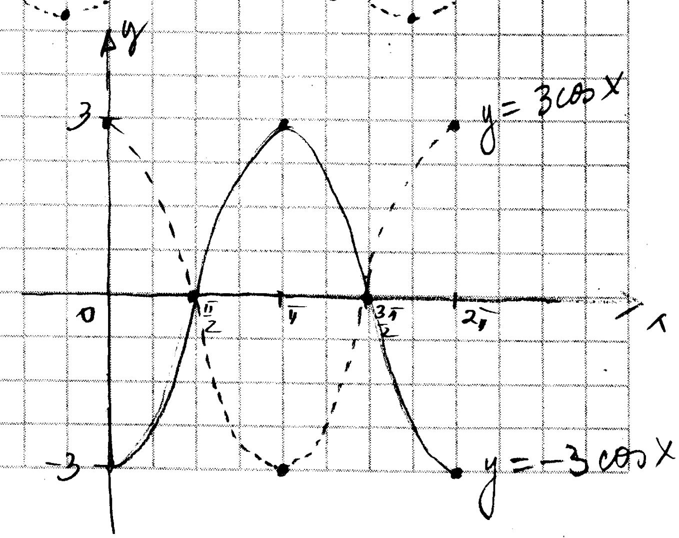
② ⓐ $y = 1 + \cos x$ from -2π to 4π

1st graph $y = \cos x$
and shift the graph of $y = \cos x$ one unit up
$T = 2\pi$
$A = 1$



ⓑ $y = -3\cos x$

$T = 2\pi$
$A = 3$
Take $[0, 2\pi]$, divide it into 4 equal parts (each of length $\frac{\pi}{2}$) sketch a cosine curve of ampl. 3, then reflect it about x-axis

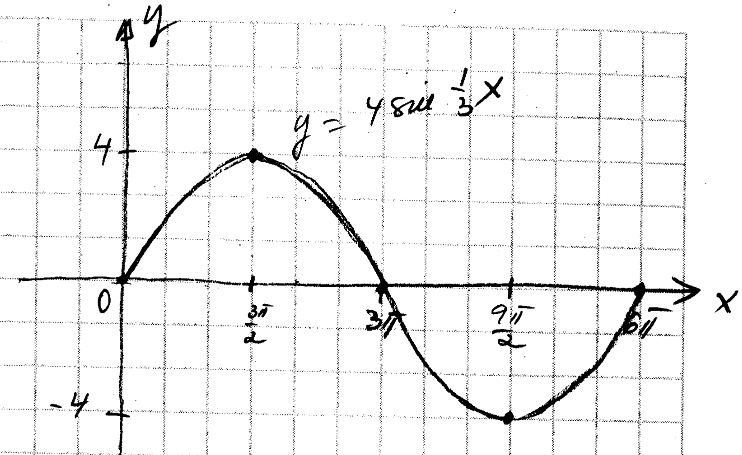


(c) $y = 4\sin\frac{1}{3}x$

$T = \frac{2\pi}{\frac{1}{3}} = 6\pi$

$A = 4$

Take $[0, 6\pi]$, divide it into 4 equal parts (each of length $\frac{6\pi}{4} = \frac{3\pi}{2}$) and sketch a sine curve of amplitude 4.



x-axis: $0, \frac{3\pi}{2}, 2\cdot\frac{3\pi}{2} = 3\pi, 3\cdot\frac{3\pi}{2} = \frac{9\pi}{2}, 4\cdot\frac{3\pi}{2} = 6\pi$

(d) $y = 2\sin\left(x - \frac{\pi}{3}\right)$

$A = 2$

$T = 2\pi$

phase shift $= \frac{\pi}{3}$

$[0, 2\pi] \longrightarrow \left[\frac{\pi}{3}, 2\pi + \frac{\pi}{3}\right]$

Take $\left[\frac{\pi}{3}, \frac{7\pi}{3}\right]$, divide it into 4 equal parts, each of length $\frac{\pi}{2}$, and sketch a sine curve of amplitude 2

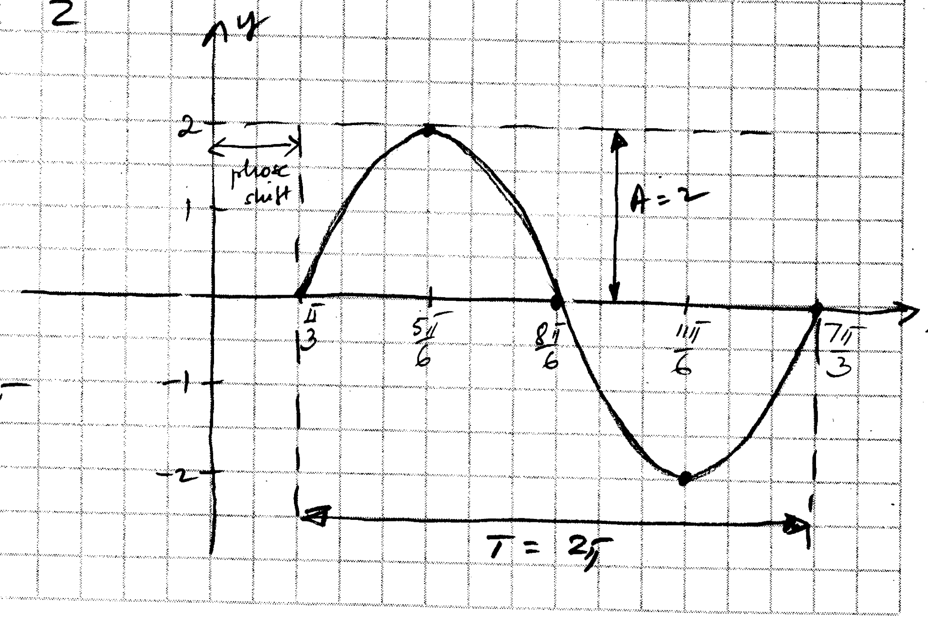
$\frac{\pi}{3}$

$\frac{\pi}{3} + \frac{\pi}{2} = \frac{5\pi}{6}$

$\frac{5\pi}{6} + \frac{\pi}{2} = \frac{8\pi}{6}$

$\frac{8\pi}{6} + \frac{\pi}{2} = \frac{11\pi}{6}$

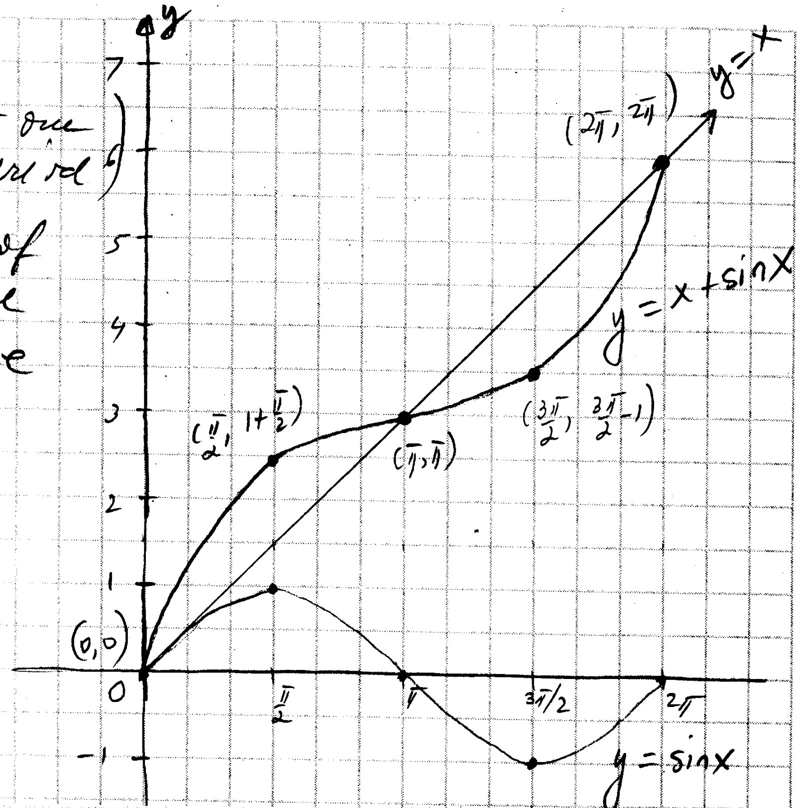
$\frac{11\pi}{6} + \frac{\pi}{2} = \frac{14\pi}{6} = \frac{7\pi}{3}$



(3) $y = x + \sin x$

graph $y_1 = x$, $y_2 = \sin x$ (over one period)

then add each value of y_2 in $y_2 = \sin x$ to the corresponding value of y_1 in $y_1 = x$.

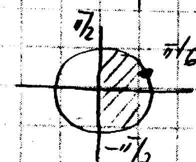


(4) NOTE THAT:

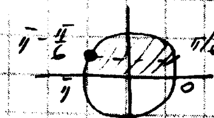
$$\begin{cases} \sin^{-1} : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}] \\ \cos^{-1} : [-1,1] \to [0, \pi] \\ \tan^{-1} : \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2}) \end{cases}$$

	$\pi/6$	$\pi/3$	$\pi/4$
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$

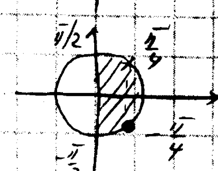
(a) $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$ b/c $\begin{cases} \frac{\pi}{6} \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ \text{and} \\ \sin\frac{\pi}{6} = \frac{1}{2} \end{cases}$



(b) $\cos^{-1}(-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}$ b/c $\begin{cases} \frac{5\pi}{6} \in [0, \pi] \\ \text{and} \\ \cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2} \end{cases}$



(c) $\tan^{-1}(-1) = \frac{-\pi}{4}$ b/c $\begin{cases} \frac{-\pi}{4} \in (-\frac{\pi}{2}, \frac{\pi}{2}) \\ \text{and} \\ \tan(\frac{-\pi}{4}) = -1 \end{cases}$



(d) $\cos(\sin^{-1}\frac{3}{5}) = ?$

let $\sin^{-1}\frac{3}{5} = u \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

then $\sin u = \frac{3}{5}$

$\sin^2 u + \cos^2 u = 1$

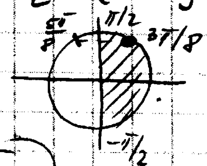
$\left(\frac{3}{5}\right)^2 + \cos^2 u = 1 \Rightarrow \cos^2 u = 1 - \frac{9}{25} = \frac{16}{25}$

$$\left.\begin{array}{l} \cos u = \pm \frac{4}{5} \\ \text{but } u \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \end{array}\right\} \Rightarrow \cos u = \frac{4}{5}$$

Therefore, $\cos u = \cos\left(\sin^{-1}\frac{3}{5}\right) = \frac{4}{5}$

(e) $\sin^{-1}\left(\sin\frac{5\pi}{8}\right) \neq \frac{5\pi}{8}$ b/c $\frac{5\pi}{8} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\frac{5\pi}{8} = \sin\frac{3\pi}{8}, \quad \frac{3\pi}{8} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Therefore,

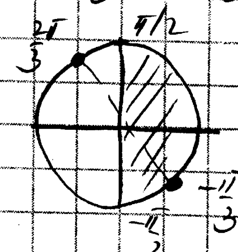
$\sin^{-1}\left(\sin\frac{5\pi}{8}\right) = \sin^{-1}\left(\sin\frac{3\pi}{8}\right) = \frac{3\pi}{8}$

(f) $\cos^{-1}\left(\cos\frac{2\pi}{7}\right) = \frac{2\pi}{7}$ b/c $\frac{2\pi}{7} \in [0, \pi]$

(g) $\tan\left(\tan^{-1} 100.23\right) = 100.23$ b/c $100.23 \in \mathbb{R}$

(h) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$ b/c $\frac{2\pi}{3} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan\frac{2\pi}{3} = \tan\frac{-\pi}{3}, \quad \frac{-\pi}{3} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$



Therefore,

$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) =$

$\tan^{-1}\left(\tan\frac{-\pi}{3}\right) = \frac{-\pi}{3}$