TEST #2 @ 150 points

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1. Consider the polynomial function

$$f(x) = 2x^4 - 19x^3 + 57x^2 - 64x + 20.$$

Questions $a - g$ below relate to this polynomial function.
You may use the given grid to graph. Write all the answers and show ALL your work on separate paper.

a) Describe the long-term behavior of this function; that is, what happens as $|x| \to \infty$.

b) Compute and compare the values of $f(0)$ and $f(1)$. What can you conclude using the Intermediate value theorem?

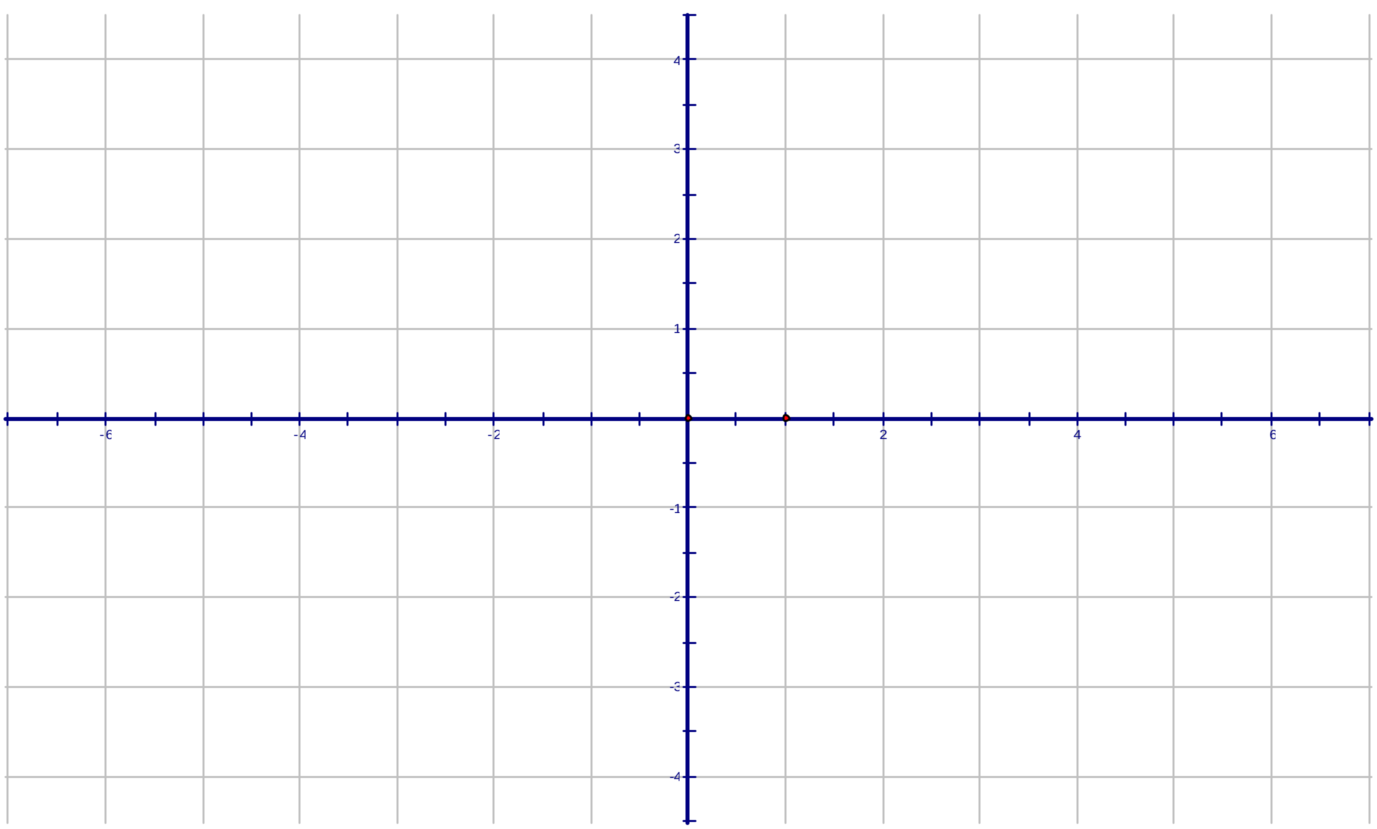
c) Using Descartes' rule of signs, determine the number of positive real zeros and the number of negative real zeros for $f(x)$.

d) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list possible rational zeros.

e) Find all the real zeros of $f(x)$ and factor $f(x)$.

f) What are the intercepts of the graph of $f(x)$? Write each intercept as an ordered pair.

g) Sketch a graph of $f(x)$ showing how it passes through its intercepts. Plot additional points, as necessary, to get the shape of the graph. Clearly label all the points.



2. Consider $f\left(x\right)=\dfrac{3x}{x^2-x-2}$.

Questions a – e below relate to this polynomial function.

You may use the given grid to graph. Write all the answers and show ALL your work on separate paper.

a) Factor the denominator.

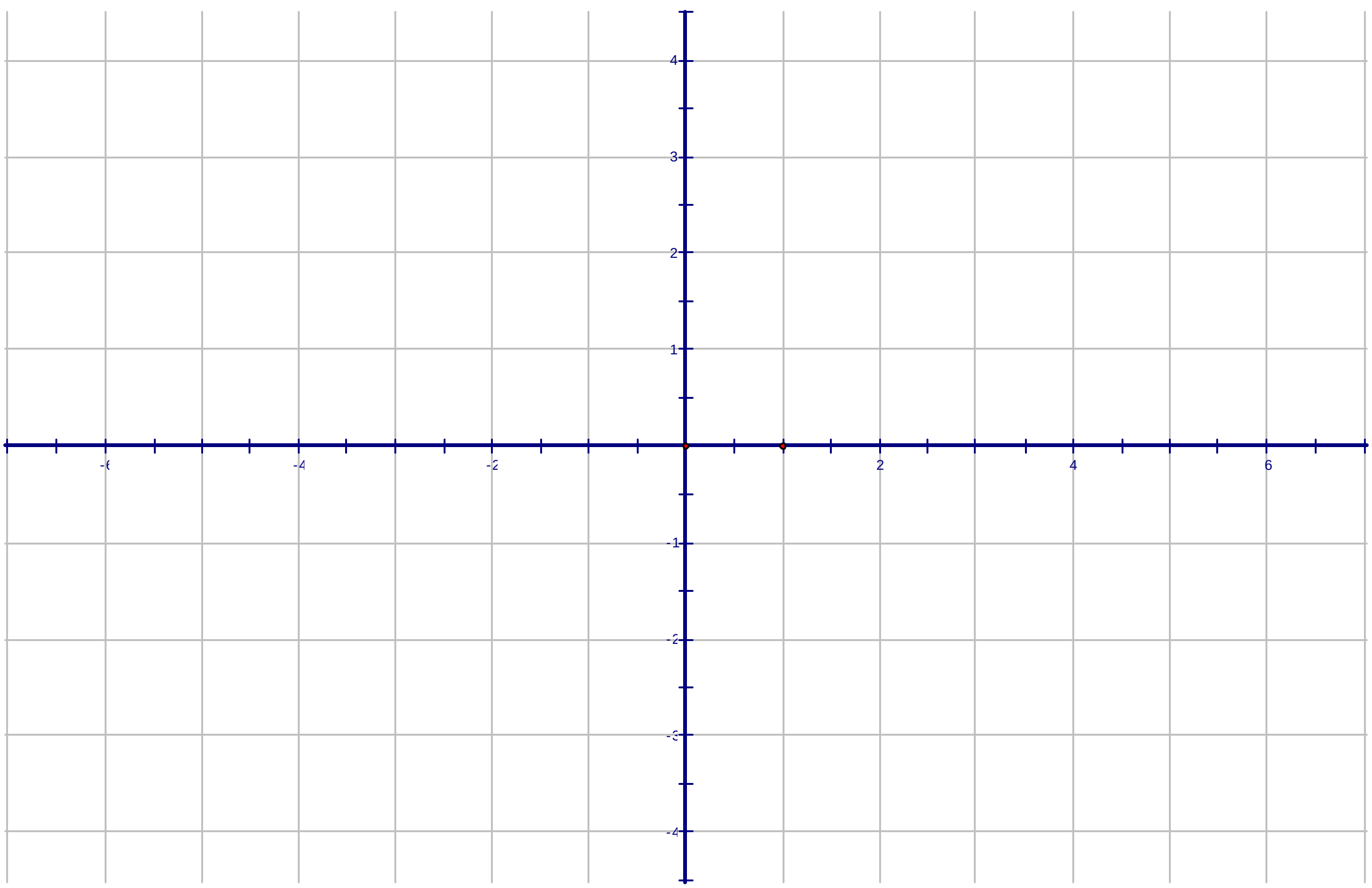
b) What is the domain of the function?

c) What are the vertical asymptotes?

d) What is the horizontal asymptote?

e) What are the intercepts for this function? Write them as ordered pairs.

e) Plot additional points, as necessary, to get the shape of this function and sketch a graph.



3. Let $f(x) = 1 + \log_2(x-1)$.

a) Graph the function (using table of values or transformations). Clearly show how you're obtaining the graph. If you choose transformations, show all equations and their meaning.

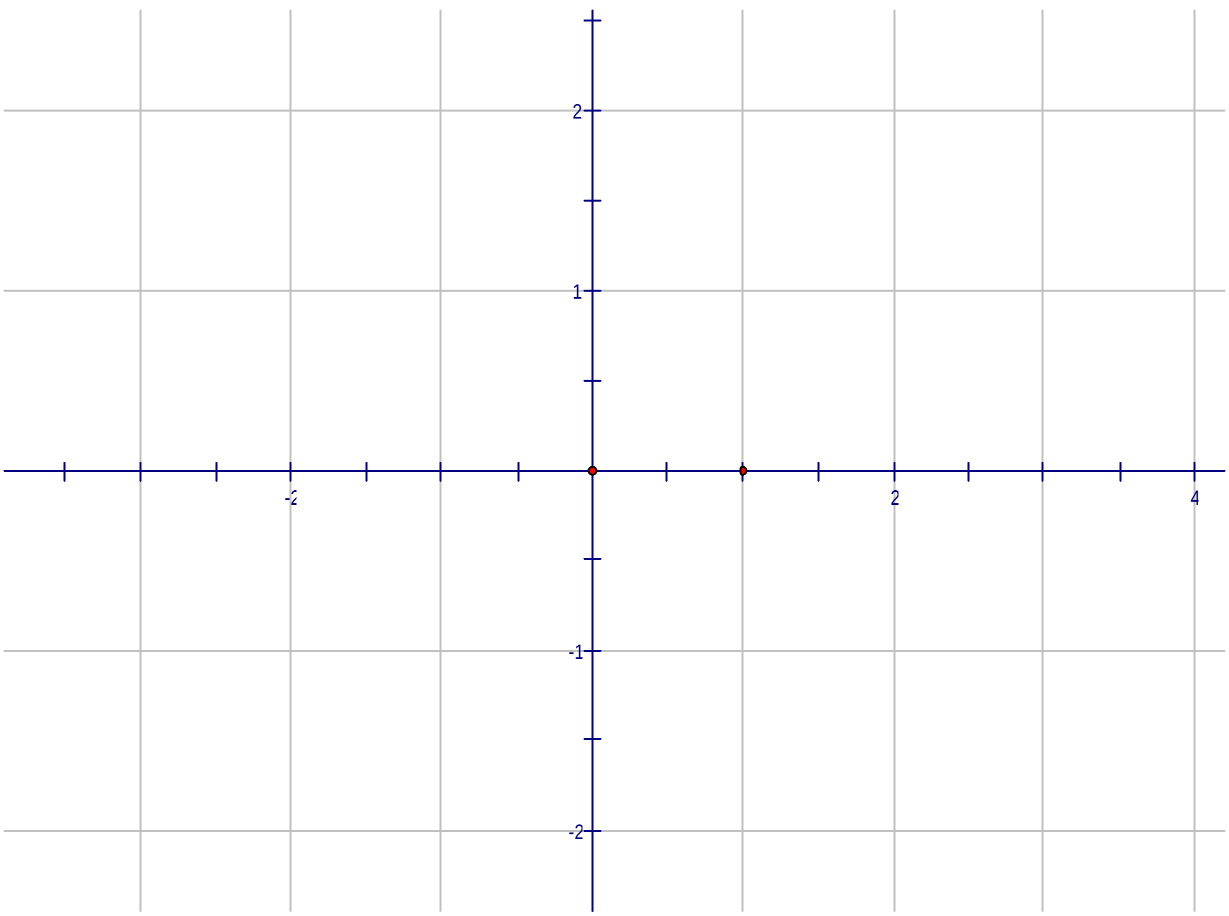
b) State the domain, range, and vertical asymptote.

c) Find the exact x- and y-intercepts (if any).

d) Does the function have an inverse? Explain.

e) Graph the inverse $f^{-1}(x)$ showing the symmetry through $y = x$.

f) State the domain, range, and horizontal asymptote for the inverse function $f^{-1}(x)$.



4. Solve the following equations. Give exact answer(s).

a) $2^{x^2-2x}=8$

b) $2^x=3^{3x-2}$

c) $\log_3(x+5)-\log_3 2=1$

d) Solve for *t:* $r=p-k\ln t$

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5. State whether each statement is TRUE or FALSE. DO NOT prove.

a) $\log(ab)=\log a+\log b$

b) $\log\left(\frac{a}{b}\right)\neq\frac{\log a}{\log b}$

c) $\log 3x^2\neq 2\log 3x$

d) $\log(xy)=(\log x)(\log y)$

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6. Revenues in the United States from all forms of legal gambling increased between 1991 and 1995. The function represented by

$$f(x)=26.6e^{0.131x}$$

models these revenues in billions of dollars x years after 1991.

a) What was the revenue in 1991?
b) Estimate gambling revenues in 1995.
c) Determine the year when these revenues reached \$30 billion (round to the nearest whole number).

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7. A group of agricultural scientists has been studying how the growth of a particular type of bacteria is affected by the acidity level of the soil. One colony of the bacteria is placed in a soil that is slightly acidic. A second colony of the same size is placed in a neutral soil. Suppose that after analyzing the data, the scientists determine that the size of each population over time can be modeled by the following functions.

Colony of neutral soil: $y=\frac{2t+1}{t+1}, t\geq 0$ Colony of acidic soil: $y=\frac{4t+3}{t^2+3}, t\geq 0$

In both cases, *y* represents the population, in thousands, after t hours.

a) What is the initial population for each colony?

b) Determine the long-term behavior of each colony.

M130 TEST 2 - SOLUTIONS

① $f(x) = 2x^4 - 19x^3 + 57x^2 - 64x + 20$

(a) The long-term behavior is given by the leading term $2x^4$

when $x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to \infty$

(b) $f(0) = 20 > 0$

$f(1) = 2 - 19 + 57 - 64 + 20$

$f(1) = -4 < 0$

Therefore, according to the intermediate value theorem, there is $c \in (0,1)$ such that $f(c) = 0$

(c) There are 4 variations of sign in $f(x)$, so there are 4 or 2 or 0 positive real zeros.

$f(-x) = 2x^4 + 19x^3 + 57x^2 + 64x + 20$

There is no variation in sign in $f(-x)$, so there is no negative real zero.

(d) All coefficients are integers, so we can apply the Rational Zeros theorem

possible rational zeros:

$$\frac{p}{q} = \frac{\text{factors of } 20}{\text{factors of } 2} = \frac{\pm1, \pm2, \pm4, \pm5, \pm10, \pm20}{\pm1, \pm2}$$

Possible rational zeros:

$$\frac{p}{q} \in \left\{\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm\frac{1}{2}, \pm\frac{5}{2}\right\}$$

(e)

$$\begin{array}{r|rrrrr} & 2 & -19 & 57 & -64 & 20 \\ \hline 2 & 2 & -15 & 27 & -10 & 0 \end{array}$$

$f(x) = (x-2)(2x^3 - 15x^2 + 27x - 10)$

Factor $2x^3 - 15x^2 + 27x - 10$

$$\frac{p}{q} = \frac{\pm1, \pm2, \pm5, \pm10}{\pm1, \pm2}$$

$$\begin{array}{r|rrrr} & 2 & -15 & 27 & -10 \\ \hline 5 & 2 & -5 & 2 & 0 \end{array}$$

$f(x) = (x-2)(x-5)(2x^2 - 5x + 2)$

$2x^2 - 5x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25-16}}{2(2)}$$

$$= \frac{5 \pm 3}{4} \quad \begin{cases} x = 2 \\ \text{or} \\ x = \frac{1}{2} \end{cases}$$

$f(x) = (x-2)(x-5)\,2\left(x-2\right)\left(x-\frac{1}{2}\right)$

$\boxed{f(x) = (x-2)^2(x-5)(2x-1)}$ factored form

The zeros of $f(x)$ are

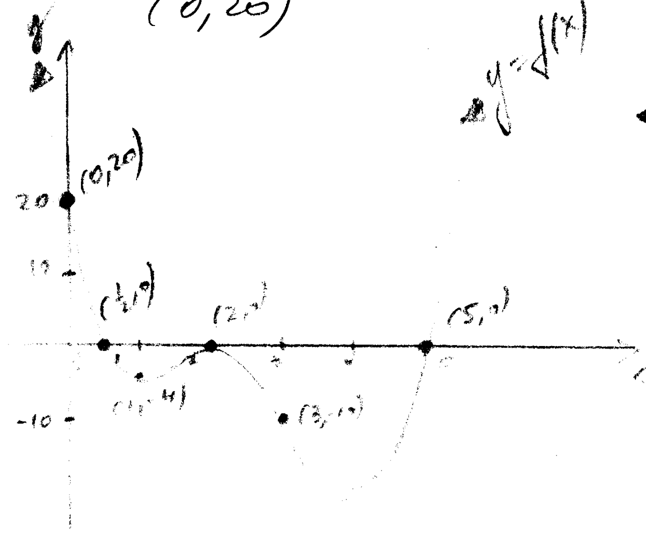
$x = 2$, $x = 5$, $x = \frac{1}{2}$

(7) x-n: $(2,0)$, $(5,0)$, $(\frac{1}{2},0)$

y-n: $x=0$, $y=20$

$(0,20)$

(8)



Test points

(2) $f(x) = \dfrac{3x}{x^2-x-2}$

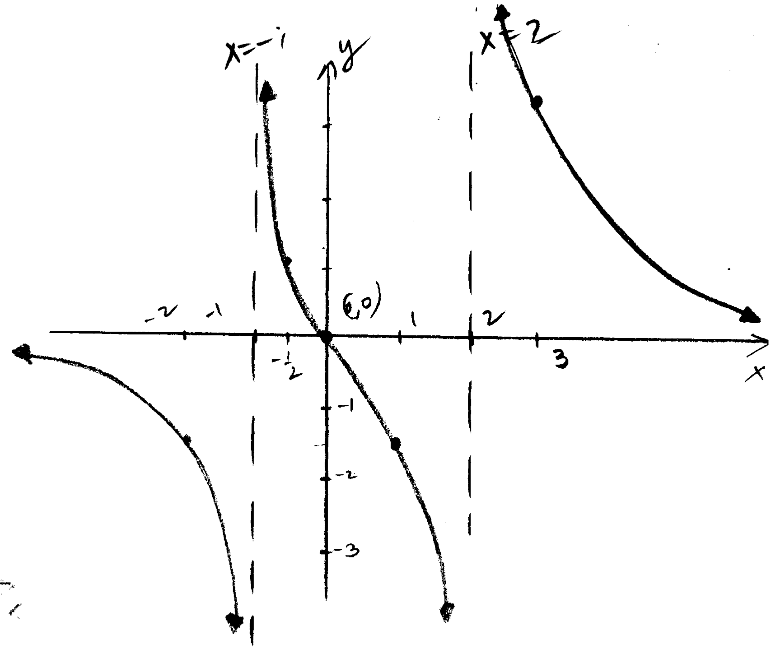
(a) $f(x) = \dfrac{3x}{(x-2)(x+1)}$

(b) $x \in \mathbb{R} \setminus \{2,-1\}$

(c) V.A. $x=2$, $x=-1$

(d) H.A $y=0$

(e) x-n: y-n: $x=0$, $y=0$ $(0,0)$



Test points $x=-2$, $f(-2) = \dfrac{-6}{-4(-1)} = \dfrac{-3}{2}$

$x=\frac{-1}{2}$, $f(\frac{-1}{2}) = \dfrac{-\frac{3}{2}}{-\frac{5}{2}\cdot\frac{1}{2}} = \dfrac{6}{5}$

$x=1$, $f(1) = \dfrac{3}{-1(2)} = \dfrac{-3}{2}$

$x=3$, $f(3) = \dfrac{9}{1(4)} = \dfrac{9}{4}$

(3) $f(x) = 1 + \log_2(x-1)$

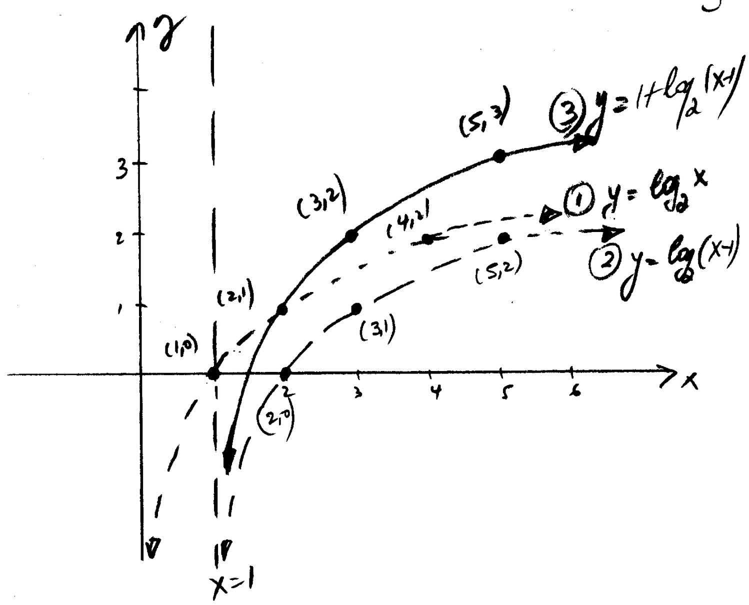
1st: $y = \log_2 x$ $x>0$, VA $x=0$

2nd $y = \log_2(x-1)$ shift right 1 unit VA $x=1$

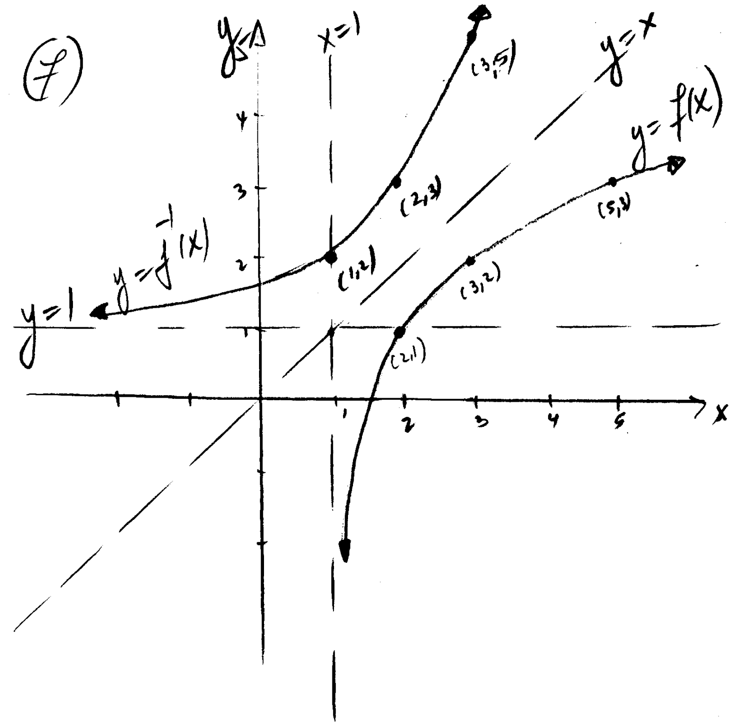
3rd $y = \log_2(x-1)+1$ shift up 1 unit

Graph $y = \log_2 x$

x	0	1	2	4	∞
y	$-\infty$	0	1	2	∞



(f)



(b) Domain: $x \in (1, \infty)$
Range: $y \in \mathbb{R}$
V.A. $x = 1$

(c) x-int: let $y = 0$

$1 + \log_2(x-1) = 0$
$\log_2(x-1) = -1$
$x - 1 = 2^{-1}$
$x = 1 + \frac{1}{2} = \frac{3}{2}$ $\boxed{\text{x-int}: \left(\frac{3}{2}; 0\right)}$

y-int: $x > 1$ so no y-int

(d) yes, $y = f(x)$ has an inverse because it is a one-to-one function (its graph passes the HLT)

(f) $y = f^{-1}(x)$
Domain: $x \in \mathbb{R}$
Range: $y \in (1, \infty)$
H.A. $y = 1$

(4) (a) $2^{x^2-2x} = 8$

$2^{x^2-2x} = 2^3$ iff

$x^2 - 2x = 3$
$x^2 - 2x - 3 = 0$
$(x-3)(x+1) = 0$

$x - 3 = 0$, $x = 3$
or
$x + 1 = 0$, $x = -1$

$\boxed{x \in \{3, -1\}}$

(b) $2^x = 3^{3x-2}$ / ln

$\ln 2^x = \ln 3^{3x-2}$
$x \ln 2 = (3x-2) \ln 3$
$x \ln 2 = 3x \ln 3 - 2 \ln 3$
$2 \ln 3 = 3x \ln 3 - x \ln 2$
$2 \ln 3 = x(3 \ln 3 - \ln 2)$
$\ln 3^2 = x(\ln 3^3 - \ln 2)$

$x = \dfrac{\ln 9}{\ln 27 - \ln 2}$ $\boxed{x = \dfrac{\ln 9}{\ln \frac{27}{2}}}$

(c) $\log_3(x+5) - \log_3 2 = 1$

Condition: $x+5>0$

$x>-5$

$\log_3 \frac{x+5}{2} = 1$ iff

$\frac{x+5}{2} = 3$

$x+5=6$

$x=1$

$\boxed{x \in \{1\}}$

(d) $r = p - k \ln t$

$k \ln t = p - r$

$\ln t = \frac{p-r}{k}$

$\boxed{t = e^{\frac{p-r}{k}}}$

(5) (a) $\log(ab) = \log a + \log b$ TRUE

(b) $\log\left(\frac{a}{b}\right) \neq \frac{\log a}{\log b}$ TRUE

$\left(\log\left(\frac{a}{b}\right) = \log a - \log b\right)$

(c) $\log 3x^2 \neq 2\log 3x$ TRUE

$\left(\log 3x^2 = \log 3 + 2\log x\right)$

(d) $\log(xy) = (\log x)(\log y)$ FALSE

$(\log xy = \log x + \log y)$

(6) $f(x) = 26.6\, e^{0.131x}$

x = # years after 1991

$f(x)$ = revenues (in billion \$)

(a) $f(0) = 26.6e^0 = 26.6$ billion \$

The revenues in 1991 was 26.6 billion \$.

(b) $f(4) = 26.6e^{0.131(4)}$

≈ 44.9 billion \$

The revenues in 1995 was about 44.9 billion \$.

(c) $x=?$ when $f(x) = 30$

$26.6\, e^{0.131x} = 30$

$e^{0.131x} = \frac{30}{26.6}$ $\Big/ \ln$

$\ln e^{0.131x} = \ln \frac{30}{26.6}$

$0.131x = \ln \frac{30}{26.6}$

$x = \frac{\ln \frac{30}{26.6}}{0.131} \approx 0.91$

$x \approx 1$

The revenues were 30 billion \$ approximately in 1992.

(7) neutral soil: $y = \dfrac{2t+1}{t+1}, \ t \geq 0$

acidic soil: $y = \dfrac{4t+3}{t^2+3}, \ t \geq 0$

t = # hours

y = population (in thousands)

(a) $t=0, \ y_0 = 1$ (neutral soil)

$t=0, \ y_0 = 1$ (acidic soil)

For both, the initial population was 1000 bacteria

(b) Long-term behavior - when $t \to \infty, \ y = ?$

The horizontal asymptote of the neutral soil colony function is $y = \frac{2}{1} = 2 \quad y = 2$

when $t \to \infty, \ y \to 2$

Therefore, over time, the neutral soil colony approaches 2000 bacteria

The horizontal asymptote of the acidic soil colony function is $y = 0$

Therefore, when $t \to \infty, \ y \to 0$

So that population becomes extinct.