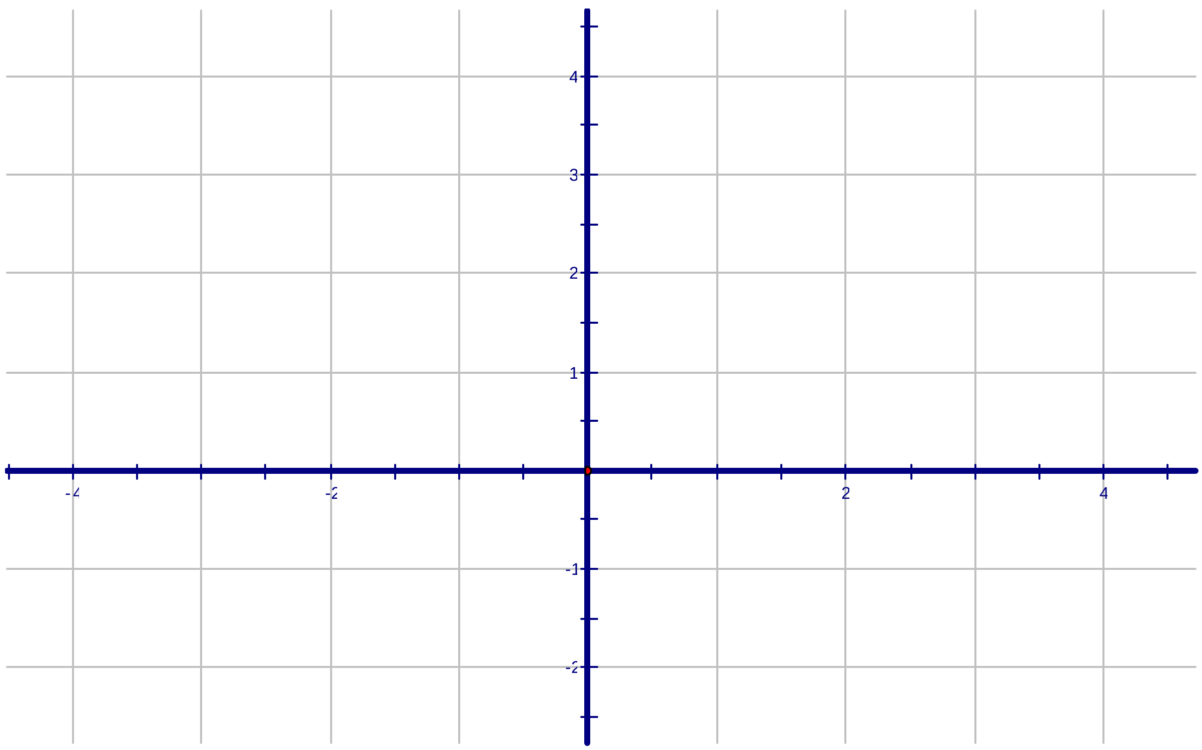
Math 130 Fall 2007 Name:

TEST #1 @ 150 points

Write neatly. Show all work. **Write all responses on separate paper.** **Clearly label the exercises.**

1. A piecewise-defined function is given.

$$f(x)=\begin{cases} x-2 & if \quad x<-1 \\ 2x & if \quad -1\le x<1 \\ x^2 & if \quad x\ge 1 \end{cases}$$



You may use the above grid to graph. Write all the answers and show ALL your work on separate paper.

a) Sketch a graph for the function. Clearly show how you obtain the points you are using for the graph. Label the axes and all points used.

b) State its domain and range in interval notation.

c) On what interval(s) is the function increasing, decreasing, constant?

d) Find $f(-3)$, $f(0)$, and $f(3)$.

e) Locate the x – and y – intercepts (if any). Write each intercept as an ordered pair.

f) Find the values of $f(f(1))$ and $(f\circ f)(-2)$.

2. Let $A(2,-1)$ and $B(-3,1)$ be two points in a plane.

a) Find an equation of the circle with diameter AB (note that the diameter is twice the radius). Show how you obtain the equation.

b) Does the equation from (a) represent y as a function of x? Explain.

c) Find the exact x- and y-intercepts (if any).

d) Find the equation of the line that passes through the two given points.

e) Does the equation from (d) represent y as a function of x? Explain.

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3. Solve the following equations in the set of complex numbers:

a) $\left(t-\frac{1}{2}\right)^2=\frac{3}{4}$

b) $\frac{1}{2}a^2-3=-\frac{1}{4}a$

c) $x^3+27=0$

d) $2x^4-7x^2+5=0$

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4. Let $f(x)=4x^2+2x+8$ and $g(x)=x^2+5$ two functions. Do the following.

a) Find $f(x+2)$.

b) Find $g(-2x)$

c) Find $(f\circ g)(x)$

d) Find $(f-g)(x)$.

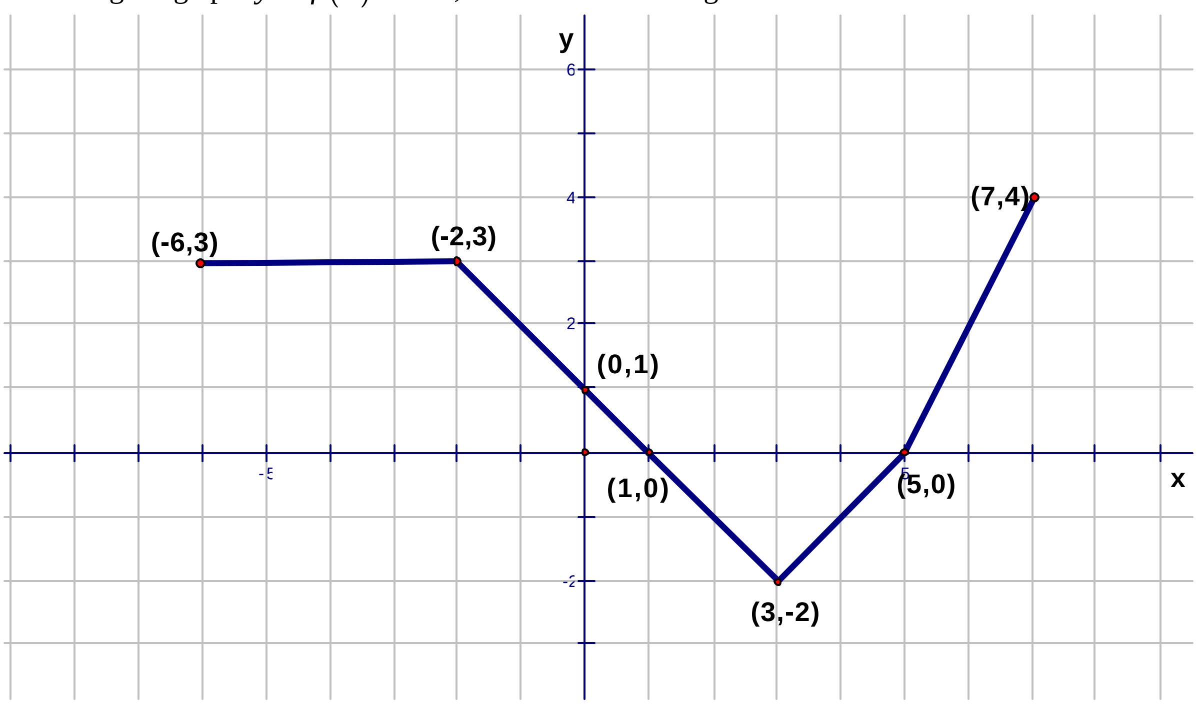
e) Determine (algebraically) whether g has a graph that is symmetric with respect to the x-axis, the y-axis, the origin, or none of these. Show all work.

f) Determine (algebraically) whether f is even, odd, or neither.

5. Graph the following function using transformations. You may use the grid to graph Clearly show all the steps: the equations and their meaning on separate paper. Graph all steps.

$$f(x) = 2\sqrt{x-1} + 3$$

6. Using the graph $y = f(x)$ shown, answer the following:



a) Is y a function of x? Explain.

b) Find the domain and range of f.

c) List the x- and y- intercepts (as ordered pairs).

d) Find $f(-2)$.

e) For what values of x does $f(x) = -2$

f) Solve $f(x) > 0$.

g) Find $(f \circ f)(3)$.

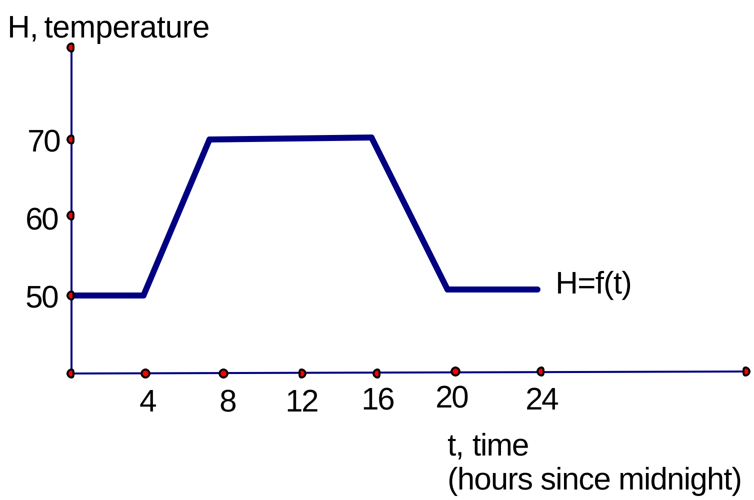
h) Graph $y = f(x) + 1$

Extra Credit

The function $H(t)$ shown gives the heating schedule of an office building during the winter months. $H(t)$ is the building's temperature in degrees Fahrenheit t hours after midnight.

a) If the company decides to schedule its heating according to the function $H(t)-2$, what has it decided to do?

b) If the company decides to schedule its heating according to the function $H(t-2)$, what has it decided to do?



Math 130

TEST #1 - SOLUTIONS

① $f(x)=\begin{cases} x-2, & x<-1 \\ 2x, & -1\le x<1 \\ x^2, & x\ge 1 \end{cases}$

ⓐ (1) $\boxed{y=x-2}$ when $x<-1$

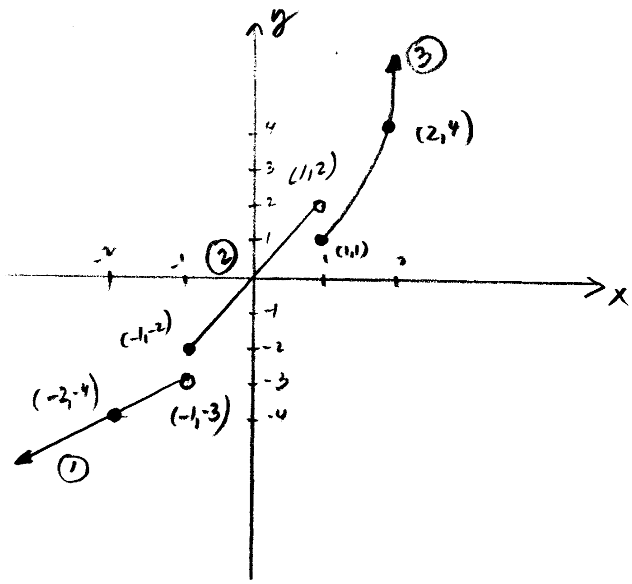
x	y
-1	-3
-2	-4

(2) $\boxed{y=2x}$ when $-1\le x<1$

x	y
-1	-2
1	2

(3) $\boxed{y=x^2}$ when $x\ge 1$

x	y
1	1
2	4



ⓑ Domain: $x\in(-\infty,\infty)$

Range: $y\in(-\infty,-3)\cup[-2,\infty)$

ⓒ The function is increasing on every interval:

$(-\infty,-1)$

$[-1,1)$

$[1,\infty)$

ⓓ $f(-3)=-3-2=-5$

$f(0)=2(0)=0$

$f(3)=3^2=9$

ⓔ x-int and y-int: $(0,0)$

ⓕ $f(f(1))=f(1)=1$

$(f(1)=1^2=1)$

$(f\circ f)(-2)=f(f(-2))$

$=f(-4)$

$=-6$

$\left(\begin{array}{l} f(-2)=-2-2=-4 \\ f(-4)=-4-2=-6 \end{array}\right)$

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② $A(2,-1)$

$B(-3,1)$

ⓐ eq. circle of center (h,k) and radius r:

$(x-h)^2+(y-k)^2=r^2$

Center = midpoint of AB

$h=\dfrac{x_A+x_B}{2}=\dfrac{-1}{2}$

$k=\dfrac{y_A+y_B}{2}=\dfrac{0}{2}=0$

Center $(-\frac{1}{2},0)$

$r=\dfrac{AB}{2}$

$(AB)^2=(\Delta x)^2+(\Delta y)^2$

$(AB)^2 = (2-(-3))^2 + (1-(-1))^2$

$= 25 + 4 = 29$

$AB = \sqrt{29} \Rightarrow r = \frac{\sqrt{29}}{2}$

The equation is:

$(x+\frac{1}{2})^2 + y^2 = \frac{29}{4}$

(b) The above equation does not represent y as a function of x because its graph is a circle and it doesn't pass the vertical line test (given an x-value, there are two y-values)

(c) x-int: let $y=0$

$(x+\frac{1}{2})^2 = \frac{29}{4}$ $\quad / \sqrt{\ }$

$\sqrt{(x+\frac{1}{2})^2} = \sqrt{\frac{29}{4}}$

$x + \frac{1}{2} = \pm \frac{\sqrt{29}}{2}$

$x = -\frac{1}{2} \pm \frac{\sqrt{29}}{2}$

The x-int are: $\left(-\frac{1}{2} \pm \frac{\sqrt{29}}{2}, 0\right)$

y-int: let $x=0$

$(\frac{1}{2})^2 + y^2 = \frac{29}{4}$

$y^2 = \frac{28}{4}$

$y^2 = 7$

$\Rightarrow y = \pm\sqrt{7}$

y-int: $(0, \pm\sqrt{7})$

(d) $m = \frac{\Delta y}{\Delta x}$

$m = \frac{1-(-1)}{-3-2} = \frac{2}{-5}$ slope

$(2,-1)$ point on line

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{2}{5}(x-2)$

(e) y is a function of x because the graph is a descending line, so it passes the vertical line test (for every x, there is only one y)

(3) (a) $(t-\frac{1}{2})^2 = \frac{3}{4}$ $\quad / \sqrt{\ }$

$\sqrt{(t-\frac{1}{2})^2} = \sqrt{\frac{3}{4}}$

$t - \frac{1}{2} = \pm \frac{\sqrt{3}}{2}$

$t = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$

(b) $\frac{1}{2}a^2 - 3 = -\frac{1}{4}a$

LCD = 4

$2a^2 - 12 = -a$

$2a^2 + a - 12 = 0$

$a = \frac{-1 \pm \sqrt{1^2 - 4(2)(-12)}}{2(2)}$

$a = \frac{-1 \pm \sqrt{97}}{4}$

(c) $x^3+27=0$

$x^3+3^3=0$

$(x+3)(x^2-3x+9)=0$

$x+3=0 \Rightarrow x=-3$

OR

$x^2-3x+9=0$

$x=\dfrac{3\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}=\dfrac{3\pm\sqrt{-3\cdot 9}}{2}$

$x=\dfrac{3\pm 3\sqrt{3}i}{2}$

(d) $2x^4-7x^2+5=0$

let $x^2=t$

then $x^4=t^2$

The equation becomes:

$2t^2-7t+5=0$

$(2t-5)(t-1)=0$

$2t-5=0$ OR $t-1=0$

$t=\frac{5}{2}$ $\qquad$ $t=1$

$x^2=\frac{5}{2}$ $\qquad$ $x^2=1$

$\sqrt{x^2}=\sqrt{\frac{5}{2}}$ $\qquad$ $\sqrt{x^2}=\sqrt{1}$

$x=\pm\frac{\sqrt{10}}{2}$ $\qquad$ $x=\pm 1$

(4) $f(x)=4x^2+2x+8$

$g(x)=x^2+5$

(a) $f(x+2)=4(x+2)^2+2(x+2)+8$

$=4(x^2+4x+4)+2x+4+8$

$f(x+2)=4x^2+18x+28$

(b) $g(-2x)=(-2x)^2+5$

$g(-2x)=4x^2+5$

(c) $(f\circ g)(x)=f(g(x))$

$=f(x^2+5)$

$=4(x^2+5)^2+2(x^2+5)+8$

$=4(x^4+10x^2+25)+2x^2+10+8$

$(f\circ g)(x)=4x^4+42x^2+118$

(d) $(f-g)(x)=f(x)-g(x)$

$=(4x^2+2x+8)-(x^2+5)$

$=4x^2+2x+8-x^2-5$

$(f-g)(x)=3x^2+2x+3$

(e) $g(x)=x^2+5$

- check symmetry about X-axis:

$y\mapsto -y$ $\qquad$ $-y=x^2+5$

$y=-x^2-5 \neq$ given equation

The graph doesn't have symmetry about X-axis

- check symmetry about y-axis:

$x \mapsto -x \qquad y = (-x)^2 + 5$

$y = x^2 + 5$ is

the given equation

The graph has symmetry about the y-axis

- check symmetry about origin:

$x \mapsto -x \qquad -y = (-x)^2 + 5$

$y \mapsto -y \qquad -y = x^2 + 5$

$y = -x^2 - 5$

$\neq$ given eq

The graph doesn't have symmetry about the origin

(7) $f(x) = 4x^2 + 2x + 8$

f is even iff $f(-x) = f(x)$

f is odd iff $f(-x) = -f(x)$

$f(-x) = 4(-x)^2 + 2(-x) + 8$

$f(-x) = 4x^2 - 2x + 8$

$f(-x) \neq f(x)$

Also, $f(-x) \neq -f(x)$

f is neither

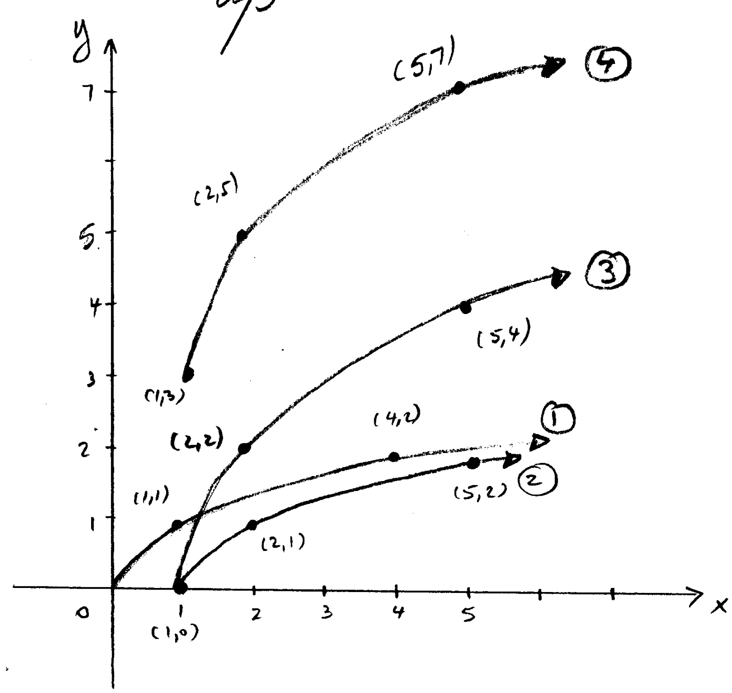
(5) $f(x) = 2\sqrt{x-1} + 3$

1st $y = \sqrt{x}$

2nd $y = \sqrt{x-1}$ shift previous graph 1 unit right

3rd $y = 2\sqrt{x-1}$ vertical stretch of the previous graph by a factor of 2

4th $y = 2\sqrt{x-1} + 3$ shift previous graph 3 units up



(6) (a) Yes, the graph passes the vertical line test

(b) Domain: $x \in [-6, 7]$

Range: $y \in [-2, 4]$

(c) x-n: $(1,0)$ and $(5,0)$

y-n: $(0,1)$

(d) $f(-2) = 3$

(e) $f(x) = -2$ iff $x = 3$

(f) $f(x) > 0$ iff

$x \in [-6, 1) \cup (5, 7]$

(g) $$\begin{aligned}(f \circ f)(3) &= f(f(3)) \\ &= f(-2) \\ &= 3\end{aligned}$$

(h) $y = f(x) + 1$

shift the graph of f 1 unit up