QUIZ #3 @ 85 points

Write neatly. Show all work. **Write all responses on separate paper.**

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1. Write the 1st, 2nd, and 20th term of the following sequence: $a_n = (-2)^n n$.

2. Find the first four terms of the sequence given by the following recursive formula:

$$\begin{cases} a_1 = 2 \\ a_2 = 5 \\ a_n = a_{n-1} + a_{n-2}, \quad if \quad n \geq 3 \end{cases}$$

3. Expand and evaluate: $\displaystyle\sum_{k=1}^{6} (-1)^k \cdot k$

4. Use summation notation to rewrite the sequence:
$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots - \frac{1}{128}$$

5. Let $3, -2, -7, -12, \ldots$ be a sequence.

   a) Is this an arithmetic sequence or geometric sequence?
   b) Find the common difference.
   c) Find a formula for the nth term.
   d) Find the sum of the first 30 terms of the sequence.

6) Solve the following system using matrices: Gaussian elimination or Gauss – Jordan method.

$$\begin{cases} x + 3y - 2z - w = 9 \\ 4x + y + z + 2w = 2 \\ -3x - y + z - w = -5 \\ x - y - 3z - 2w = 2 \end{cases}$$

7) Let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 6 & -7 \\ -2 & 1 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix}$$

Do the following operations. If not defined, say so and explain why.

   a) $A - B$ d) AC

   b) $3A + C$ e) AD

   c) BA f) DA

① $a_n = (-2)^n n$

$a_1 = (-2)^1 (1) = -2$

$a_2 = (-2)^2 (2) = 4(2) = 8$

$a_{20} = (-2)^{20} (20)$

② $\begin{cases} a_1 = 2 \\ a_2 = 5 \\ a_n = a_{n-1} + a_{n-2}, \quad n \geq 3 \end{cases}$

$a_3 = a_2 + a_1 \qquad a_4 = a_3 + a_2$

$= 5 + 2 \qquad\qquad = 7 + 5$

$= 7 \qquad\qquad\quad = 12$

$2, 5, 7, 12, \ldots$

③ $\sum\limits_{k=1}^{6} (-1)^k k = (-1)^1 (1) + (-1)^2 (2) +$

$+ (-1)^3 (3) + (-1)^4 (4) + (-1)^5 (5) + (-1)^6 (6)$

$= -1 + 2 - 3 + 4 - 5 + 6 = 3$

④ $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{128} = ?$

$a_1 = 1$

$a_2 = \frac{-1}{2}$

$a_3 = \frac{1}{4} = \frac{1}{2^2} \qquad a_n = \left(\frac{-1}{2}\right)^{n-1}$

$a_4 = \frac{-1}{8} = \left(\frac{-1}{2}\right)^3$

$a_5 = \frac{1}{2^4}$

$? = \sum\limits_{n=1}^{8} \left(\frac{-1}{2}\right)^{n-1}$

⑤ a) arithmetic sequence

b) $d = a_2 - a_1 = -2 - 3 = -5$

c) $a_n = a_1 + (n-1)d$

where $a_1 = 3$

$d = -5$

$a_n = 3 + (n-1)(-5)$

$= 3 - 5n + 5$

$a_n = 8 - 5n$

d) $S_n = \frac{(a_1 + a_n)n}{2}$

$S_{30} = \frac{(a_1 + a_{30})30}{2}$

$\begin{cases} a_1 = 3 \\ a_{30} = 8 - 5(30) = -142 \end{cases}$

$S_{30} = \frac{(3 - 142)15}{}$

$S_{30} = -2085$

(6)

$$\begin{pmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 4 & 1 & 1 & 2 & | & 2 \\ -3 & -1 & 1 & -1 & | & -5 \\ 1 & -1 & -3 & -2 & | & 2 \end{pmatrix}$$

$R_2 \to -4R_1 + R_2$
$R_3 \to 3R_1 + R_3$
$R_4 \to -R_1 + R_4$

$$\begin{pmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 0 & 11 & -9 & -6 & | & 34 \\ 0 & 0 & 47 & 35 & | & -59 \\ 0 & 0 & 0 & \frac{-37}{47} & | & \frac{-37}{47} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 0 & -11 & 9 & 6 & | & -34 \\ 0 & 8 & -5 & -4 & | & 22 \\ 0 & -4 & -1 & -1 & | & -7 \end{pmatrix}$$

$R_3 \longleftrightarrow R_4$
$R_4 \to -R_1$

4th row: $\frac{-37}{47} W = \frac{-37}{47}$ $\boxed{W = 1}$

3rd row: $47 Z + 35 W = -59$
$47 Z + 35 = -59$

$$\begin{pmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 0 & -11 & 9 & 6 & | & -34 \\ 0 & 4 & 1 & 1 & | & 7 \\ 0 & 8 & -5 & -4 & | & 22 \end{pmatrix}$$

$R_2 \to \frac{-1}{11} R_2$
$R_4 \to -2R_3 + R_4$

$\boxed{Z = -2}$

2nd row: $11y - 9Z - 6W = 34$
$11y + 18 - 6 = 34 \Rightarrow \boxed{y = 2}$

$$\begin{pmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 0 & 1 & \frac{-9}{11} & \frac{-6}{11} & | & \frac{34}{11} \\ 0 & 4 & 1 & 1 & | & 7 \\ 0 & 0 & -7 & -6 & | & 8 \end{pmatrix}$$

$R_3 \to -4R_1 + R_3$

1st row: $x + 3y - 2Z - W = 9$
$x + 6 + 4 - 1 = 9 \Rightarrow \boxed{X = 0}$

The solution is $(0, 2, -2, 1)$.

$$\begin{pmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 0 & 1 & \frac{-9}{11} & \frac{-6}{11} & | & \frac{34}{11} \\ 0 & 0 & \frac{47}{11} & \frac{35}{11} & | & \frac{-59}{11} \\ 0 & 0 & -7 & -6 & | & 8 \end{pmatrix}$$

$R_2 \to 11 R_2$
$R_3 \to \frac{11}{47} R_3$

(7)
dim A = 3×3
dim B = 2×3
dim C = 3×3
dim D = 3×2

$$\begin{pmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 0 & 11 & -9 & -6 & | & 34 \\ 0 & 0 & 1 & \frac{35}{47} & | & \frac{-59}{47} \\ 0 & 0 & -7 & -6 & | & 8 \end{pmatrix}$$

$R_3 \to 47R_3$
$R_4 \to 7R_3 + R_4$

a) $A - B =$ not defined

b) $3A + C =$

$$3 \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix} =$$

$$= \begin{pmatrix} 6 & 3 & 0 \\ -3 & 6 & -9 \\ 0 & 15 & -3 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix} =$$

$$= \begin{pmatrix} 7 & 4 & 3 \\ -5 & 9 & -14 \\ 1 & 15 & -4 \end{pmatrix}$$

c) $BA = \begin{pmatrix} 1 & 6 & -7 \\ -2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} =$

$$= \begin{pmatrix} -4 & -22 & -11 \\ -5 & 15 & -6 \end{pmatrix}$$

d) $AC = \begin{pmatrix} 2 & 1 & 0 \\ 7 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ -2 & 3 & -5 \\ 1 & 0 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 5 & 1 \\ -8 & 5 & -10 \\ -11 & 15 & -24 \end{pmatrix}$$

e) $AD = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 2 & -3 \\ 0 & 5 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 3 & 2 \\ -13 & -7 \\ -8 & -2 \end{pmatrix}$$

f) $DA =$ not defined