Math 130 Fall 2007 Name:

QUIZ #2 @ 85 points SOLUTIONS

Write neatly. Show all work. **Write all responses on separate paper. Clearly label the exercises.**

1) Let $f(x) = -3x^2 + 24x - 46$.

You may use the given grid to graph. Write all the answers and show ALL your work on separate paper.

a) What type of curve is this?

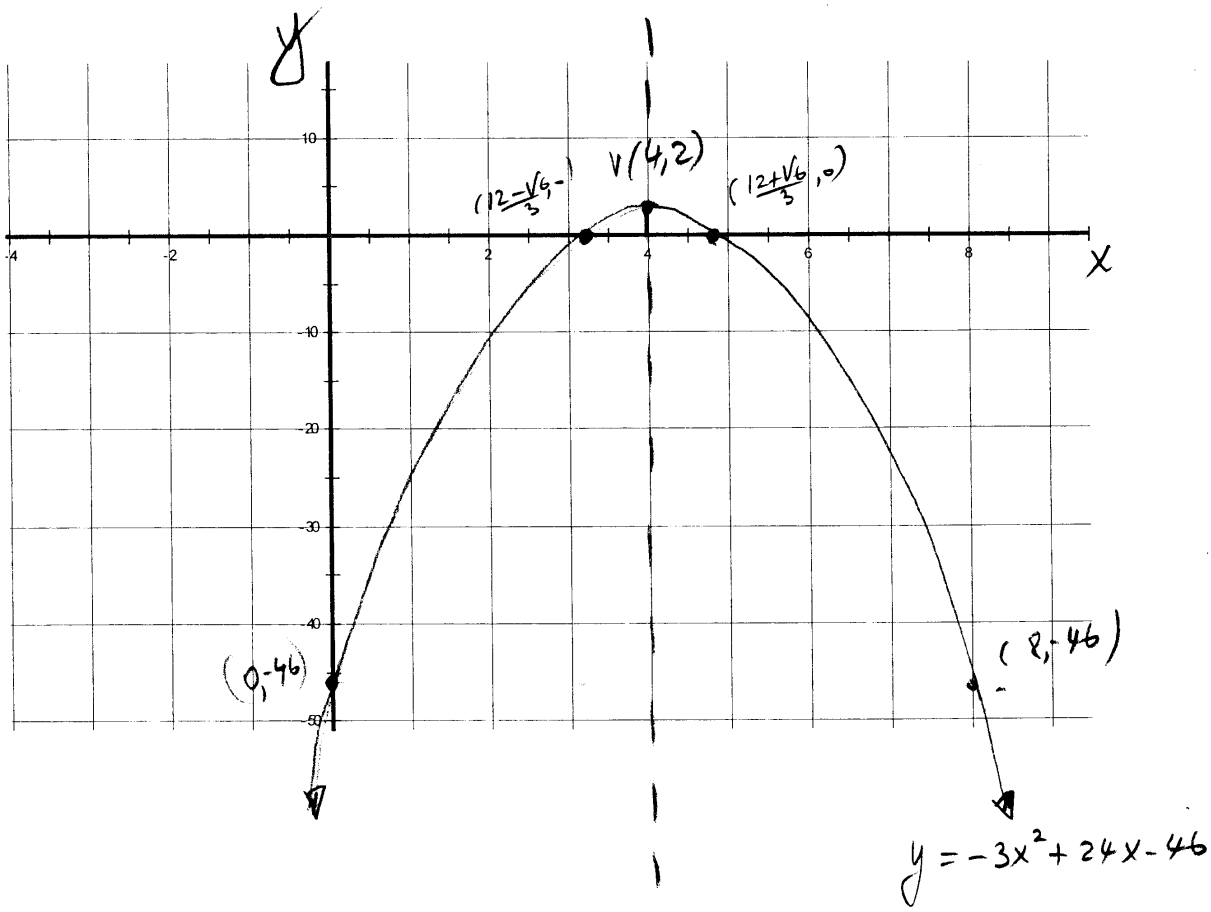
b) What is the y-intercept?

c) What is the vertex ?

d) Find the x- intercept(s) (if any).

e) Sketch its graph. Label the axes, the vertex, and the intercepts.

f) Find the domain and range .



2. Consider the polynomial function

$$f(x) = x^3 + 5x^2 + 2x - 8.$$

Questions $a - g$ below relate to this polynomial function.
You may use the given grid to graph. Write all the answers and show ALL your work on separate paper.

a) Describe the long-term behavior of this function; that is, what happens as $x \to \infty$ and $x \to -\infty$.

b) Use synthetic division to divide $f(x)$ by $x - 3$ and relate dividend, divisor, quotient and remainder in an equation.

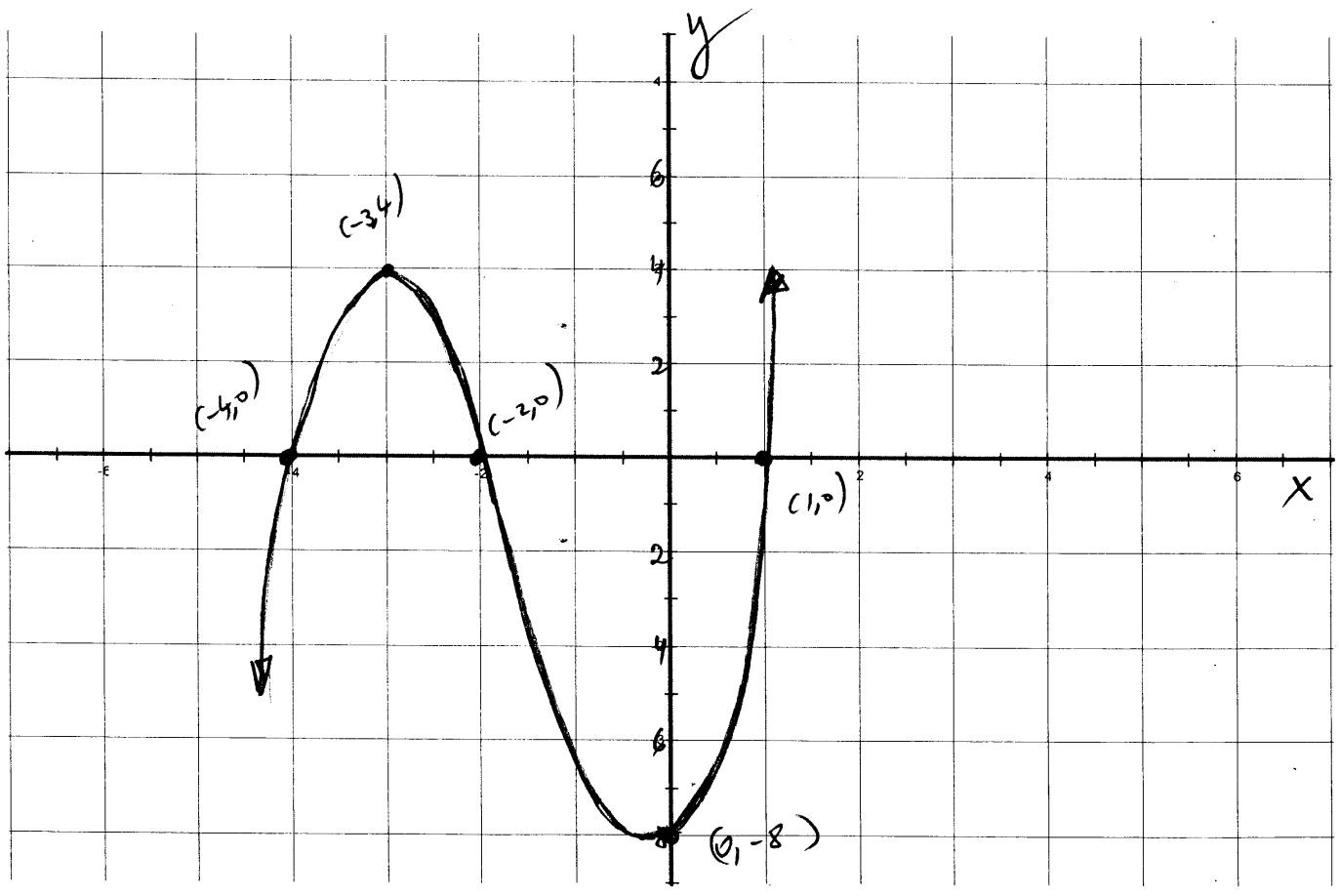
c) Using Descartes' rule of signs, determine the number of positive real zeros and the number of negative real zeros for $f(x)$.

d) State why the condition for the theorem on rational zeros is satisfied and use the theorem on rational zeros to list all possible rational zeros.

e) Find all the real zeros of $f(x)$ and use the zeros to factor f completely.

f) What are the intercepts of the graph of $f(x)$? Write each intercept as an ordered pair.

g) Sketch a graph of $f(x)$ showing how it passes through its intercepts. Plot additional points, as necessary, to get the shape of the graph. Clearly label all the points.



MATH130 - Quiz #2

① $f(x) = -3x^2 + 24x - 46$

(a) parabola that opens downwards ($a = -3 < 0$)

(b) let $x = 0$, then $y = -46$

$\boxed{y\text{-int: } (0, -46)}$

(c) $x_v = \frac{-b}{2a} = \frac{-24}{2(-3)} = 4$

$y_v = -3(4)^2 + 24(4) - 46$

$y_v = 2$

$\boxed{V(4, 2)}$

(d) set $y = 0$, then

$-3x^2 + 24x - 46 = 0 \quad /(-1)$

$3x^2 - 24x + 46 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-24) \pm \sqrt{(-24)^2 - 4(3)(46)}}{2(3)}$

$= \frac{24 \pm \sqrt{24}}{6} = \frac{24 \pm 2\sqrt{6}}{6}$

$= \frac{2(12 \pm \sqrt{6})}{6} = \frac{12 \pm \sqrt{6}}{3}$ $\begin{matrix} \approx 4.8 \\ \approx 3.2 \end{matrix}$

$\boxed{x\text{-int: } \left(\frac{12 \pm \sqrt{6}}{3}, 0\right)}$

(f) $\boxed{\begin{array}{l} x \in \mathbb{R} \\ y \in (-\infty, 2] \end{array}}$

② $f(x) = x^3 + 5x^2 + 2x - 8$

(a) The end behavior is given by the leading term x^3

when $x \to \infty$, $y \to \infty$ (up)

$x \to -\infty$, $y \to -\infty$ (down)

(b)

	1	5	2	−8
3	1	8	26	70 R

$\boxed{f(x) = (x-3)(x^2 + 8x + 26) + 70}$

(c) There is one variation in sign in $f(x)$ $\Rightarrow$ 1 positive real zero

$f(-x) = -x^3 + 5x^2 - 2x - 8$

(1) (2)

There are 2 variations in sign in $f(-x)$ $\Rightarrow$ 2 or 0 negative real zeros

(d) The Rational zeros th. can be applied because all coefficients are integers

Possible rational zeros: $\frac{p}{q} \in \{\pm 1, \pm 2, \pm 4, \pm 8\}$

(e)

	1	5	2	−8
1	1	6	8	0

$f(x) = (x-1)(x^2 + 6x + 8)$

$\boxed{f(x) = (x-1)(x+4)(x+2)}$ (factored form)

the zeros are $\boxed{x = 1, x = -4, x = -2}$ each of multiplicity one.

(f) $\boxed{\begin{array}{l} x\text{-int: } (-4, 0), (-2, 0), (1, 0) \\ y\text{-int: } (0, -8) \end{array}}$